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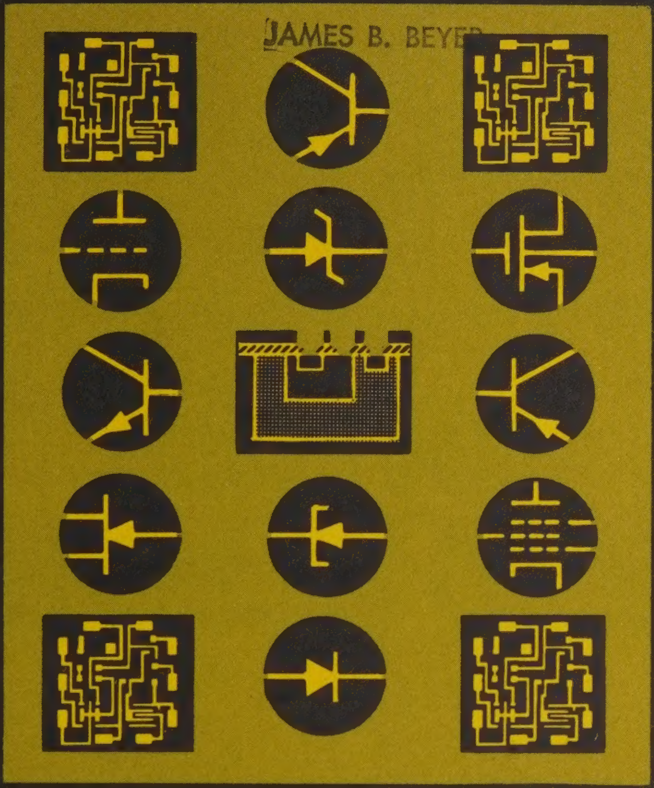
JAMES B. BEYE

Solutions Manual to accompany

Electronic devices and circuits

MILLMAN & HALKIAS

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The solutions correspond to problem statements in the second printing of the text, and where there is a difference from the first printing, the change is indicated by an asterisk.

These solutions have been prepared with the assistance of Professors Millman and Halkias. Their helpfulness is gratefully acknowledged.

N. C. V.

CHAPTER 1

(1-1) a) From Eq. (1-13) we have: $v = 5.93\times10^5 V^{1/2} = 5.93\times10^5 (1000)^{1/2}$

$$= 1.88\times10^7 \text{ m/sec} \quad \longleftarrow$$

b) From Eq. (1-10), $eV = \frac{1}{2}m(v_x^2 - v_{ox}^2)$ or $v_x^2 = v_{ox}^2 + 2\frac{e}{m}V$

or $v_x^2 = (10^5)^2 + 2\,\dfrac{1.6\times10^{-19}\times10^3}{2.01\times1.66\times10^{-27}} = 10.6\times10^{10}$

$$\text{or } v_x = 3.26\times10^5 \text{ m/sec} \quad \longleftarrow$$

(1-2) a) $x = \frac{1}{2}a_x t^2$ or $t = \left(\dfrac{2x}{a_x}\right)^{1/2} = \left(\dfrac{2x}{\frac{e}{m}\mathcal{E}_x}\right)^{1/2} = \left(\dfrac{2\times10^{-2}}{1.76\times10^{11}\times\frac{10^3}{10^{-2}}}\right)^{1/2} = 1.065\times10^{-9}\text{ sec} \quad \longleftarrow$

b) $a_x = \frac{e}{m}\mathcal{E}_x = 1.76\times10^{11}\times\dfrac{10^3}{10^{-2}}\sin 377t = 1.76\times10^{16}\sin 377t$

$v_x = \int_0^t a_x\,dt = 4.67\times10^{13}(1-\cos 377t)$; $x = \int_0^t v_x\,dt = 1.24\times10^{11}(\theta - \sin\theta)$; $\theta = 377t$

with $x = 0.01$ m, $\theta \ll 1$, hence $x = 1.24\times10^{11}\left[\theta - \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)\right]$

or $0.01 = 2.07\times10^{10}\,\theta^3\left(1 - \frac{\theta^2}{20} + \cdots\right) \approx 2.07\times10^{10}\,\theta^3$

Then $\theta \approx 7.85\times10^{-5}$

and $t = \dfrac{\theta}{377} = \dfrac{7.85\times10^{-5}}{377} = 2.08\times10^{-7}$ sec $\longleftarrow$

(1-3) a) 1 eV $= 1.6\times10^{-19}$ J. Hence, the initial energy of the electron is $10^{-16}/1.6\times10^{-19} = 625$ eV. Since the retarding potential is 400 V, the electron reaches the second electrode with an energy of $625 - 400 = 225$ eV $\longleftarrow$

b) 625 V $\longleftarrow$

(1-4) a) $a = -\left(\frac{e}{m}\right)\left(\frac{V_m}{d}\right)\sin\omega t$, $\quad v = v_0 + \int_0^t a\,dt = v_0 + \frac{e}{m}\frac{V_m}{\omega d}(\cos\omega t - 1)$

$x = \int_0^t v\,dt = \left(v_0 - \frac{e}{m}\frac{V_m}{\omega d}\right)t + \frac{e}{m}\frac{V_m}{\omega^2 d}\sin\omega t \quad \longleftarrow$

b) For $v = 0$, $\cos\omega t = 1 - \dfrac{v_0 m\omega d}{eV_m}$, $\quad \mathcal{E} = \dfrac{V_m\sin\omega t}{d} = \dfrac{V_m}{d}(1-\cos^2\omega t)^{1/2}$

or $\mathcal{E} = \dfrac{V_m}{d}\left[\dfrac{2v_0 m\omega d}{eV_m} - \left(\dfrac{v_0 m\omega d}{eV_m}\right)^2\right]^{1/2} = \left(\dfrac{v_0 m V_m \omega}{ed}\right)^{1/2}\left(2 - \dfrac{v_0 m\omega d}{eV_m}\right)^{1/2} \quad \longleftarrow$

(1-5) a) $a = \dfrac{e\mathcal{E}}{m} = (1.76\times10^{11})(10^7 t/3\times10^{-2}) = 5.86\times10^{19}t$ m/sec^2

$v = \int_0^t a\,dt = 2.93\times10^{19}t^2$ and $x = \int_0^t v\,dt = 9.77\times10^{18}t^3$

when $x = 2.8\times10^{-2}$, $t = t_1 = (2.8\times10^{-2}/9.77\times10^{18})^{1/3} = 1.42\times10^{-7}$ sec

$V = 10^7 t_1 = 1.42$ V $\longleftarrow$

b) At reversal time, t_1, $v_1 = (2.93\times10^{19})(1.42\times10^{-7})^2 = 5.88\times10^5$ m/sec. The acceleration after reversal is $a_2 = \frac{e\mathcal{E}}{m} = (1.76\times10^{11})(\frac{50}{3\times10^{-2}}) = 2.93\times10^{14}$ m/sec². If t_2 is the time after reversal then

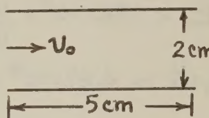
$x = 0.028 + v_1 t_2 - \frac{1}{2} a_2 t_2^2 = 0.028 + 5.88\times10^5 t_2 - 1.465\times10^{14} t_2^2$. If the electron hits the top plate, $x = 0.03$ and solving for t_2 we find an imaginary value. Hence, the electron does not hit the top plate.

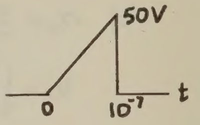
c) At the bottom plate $x = 0$. Solving the quadratic for t_2 we find $t_2 = 1.6\times10^{-8}$ sec. Hence, the total time of flight is $t_1 + t_2$

$= (1.42 + 0.16)\times10^{-7} = 1.58\times10^{-7}$ sec $\longleftarrow$

d) $v_2 = v_1 - a_2 t_2 = 5.88\times10^5 - (2.93\times10^{14})(1.6\times10^{-8}) = -4.09\times10^6$ m/sec. $\longleftarrow$

(1-6) $\mathcal{E} = \frac{50t}{2\times10^{-2}\times10^{-7}} = 2.5\times10^{10}\,t$





$a_y = \frac{e\mathcal{E}}{m} = \frac{1.6\times10^{-19}\times2.5\times10^{10}\,t}{1.65\times10^{-27}} = 2.42\times10^{18}\,t$ m/sec²

$v_y = \int_0^t a_y\,dt = 1.21\times10^{18}\,t^2$ m/sec ; $y = \int_0^t v_y\,dt = 0.4\times10^{18}\,t^3$ m

$\frac{1}{2} m v_{ox}^2 = 1.6\times10^{-19}\times100 = \frac{1}{2}(1.65\times10^{-27})\,v_{ox}^2$ or $v_{ox} = 1.39\times10^5$ m/sec

At $t = 5\times10^{-2}/1.39\times10^5 = 3.59\times10^{-7}$ sec, the ion leaves the plates. At $t = 10^{-7}$ sec $y_1 = 0.4\times10^{18}(10^{-7})^3 = 4\times10^{-4}$ m. From 10^{-7} to 3.59×10^{-7} sec, v_y = constant = $1.21\times10^{18}(10^{-7})^2 = 1.21\times10^4$ m/sec and the distance covered is $y_2 = 1.21\times10^4(2.59\times10^{-7}) = 3.14\times10^{-3}$ m. Total distance is $y_1 + y_2 = (0.4 + 3.14)10^{-3} = 0.354$ cm

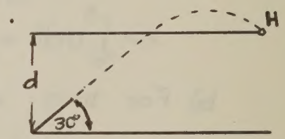
(1-7) a) $a_y = \frac{e\mathcal{E}}{m} = 1.76\times10^{11}\times5\times10^3 = 8.8\times10^{14}$ m/sec² ; $v_0 = 5.93\times10^5 V^{1/2} = 5.93\times10^6$ m/sec

$v_{ox} = 5.93\times10^6 \cos 30° = 5.13\times10^6$ m/sec ; $v_{oy} = 5.93\times10^6 \sin 30° = 2.97\times10^6$ m/sec

$x = v_{ox} t$ or $t = (3\times10^{-2})/(5.13\times10^6) = 5.84\times10^{-9}$ sec $\longleftarrow$

b) $d = v_{oy} t + \frac{1}{2} a_y t^2 = (2.97\times10^6)(5.84\times10^{-9}) + \frac{1}{2}(8.8\times10^{14})(5.84\times10^{-9})^2$

$= 1.735\times10^{-2} + 1.50\times10^{-2} = 3.24$ cm $\longleftarrow$



c) (i) $t = 5.84\times10^{-9}$ sec as above

(ii) $d = v_{oy} t - \frac{1}{2} a_y t^2 = 1.735 - 1.50 = 0.24$ cm $\longleftarrow$

The y-component of velocity at H is

$v_y = v_{oy} - a_y t = 2.97\times10^6 - (8.8\times10^{14})(5.84\times10^{-9})$

$= 2.97\times10^6 - 5.13\times10^6 = -2.16\times10^6$ m/sec

Hence, the path is as indicated above.

(1-8) $V = kx^{4/3}$; $\frac{1}{2} m v^2 = eV = ekx^{4/3} = \frac{1}{2} m (\frac{dx}{dt})^2$

or $\left(\frac{2ek}{m}\right)^{1/2} x^{2/3} = \frac{dx}{dt}$ or $\left(\frac{2ek}{m}\right)^{1/2} \int_0^t dt = \int_0^d x^{-\frac{2}{3}}\,dx$

$\left(\frac{2ek}{m}\right)^{1/2} t = 3x^{1/3}\Big|_0^d$ or $t = \frac{3d^{1/3}}{\left(\frac{2ek}{m}\right)^{1/2}}$ $\longleftarrow$

(1-9) a) $v = 5.93\times10^5 V^{1/2}$ or $V = \left(\frac{3\times10^8}{5.93\times10^5}\right)^2 = 255{,}000$ V

b) From Eq. (1-30), $v = c\left[1 - \frac{1}{(1+v_N^2/2c^2)^2}\right]^{1/2}$

For $v_N = c$, $v = c\left[1 - \frac{1}{(1+\frac{1}{2})^2}\right]^{1/2} = c\left[1 - \frac{1}{2.25}\right]^{1/2} = c\left(\frac{1.25}{2.25}\right)^{1/2}$

$= 0.745\times3\times10^8 = 2.24\times10^8$ m/sec $\longleftarrow$

(1-10) From Eq. (1-26), $eV = mc^2 - m_oc^2$ we find $\frac{m}{m_o} = \frac{eV}{m_oc^2} + 1$

For electrons : $\frac{m}{m_o} = \frac{1.76\times10^{11}\times2\times10^6}{(3\times10^8)^2} + 1 = 3.92 + 1 = 4.92$ $\longleftarrow$

For deuterons : $\frac{m}{m_o} = \frac{1.6\times10^{-19}\times2\times10^6}{1.66\times10^{-27}\times2.01\times(3\times10^8)^2} + 1 = 1.07\times10^{-3} + 1 = 1.001$ $\longleftarrow$

(1-11) $f = \frac{d(mv)}{dt} = \frac{d}{dt}\left(\frac{m_ov}{\sqrt{1-v^2/c^2}}\right) = e\mathcal{E}$; $\frac{m_ov}{\sqrt{1-v^2/c^2}} = e\mathcal{E}t$ since $v=0$ at $t=0$

$\frac{v^2}{1-v^2/c^2} = a^2t^2$ where $a \equiv \frac{e\mathcal{E}}{m_o}$; $v^2\left(1+\frac{a^2t^2}{c^2}\right) = a^2t^2$ or $v = \frac{at}{\left(1+\frac{a^2t^2}{c^2}\right)^{1/2}}$ $\longleftarrow$

$x = \int v\,dt = a\int\frac{t\,dt}{\left(1+\frac{a^2t^2}{c^2}\right)^{1/2}}$. Let $\alpha = \frac{a^2t^2}{c^2}$, then $d\alpha = \frac{2a^2t}{c^2}dt$

$\therefore\ x = \frac{ac^2}{2a^2}\int\frac{d\alpha}{(1+\alpha)^{1/2}} = \frac{c^2}{a}(1+\alpha)^{1/2} + \text{const.}$

When $t=0$, $\alpha=0$ and $x=0$ $\therefore$ const. $= -\frac{c^2}{a}$

Hence, $x = \frac{c^2}{a}\left[\left(1+\frac{a^2t^2}{c^2}\right)^{1/2} - 1\right]$ where $a \equiv \frac{e\mathcal{E}}{m_o}$ $\longleftarrow$

(1-12) a) From Eq. (1-24), $S = \frac{\ell L}{2dV_a} = \frac{1.27\times19.4\times10^{-2}}{2\times0.475\times400} = 0.65$ mm/V $\longleftarrow$

This value is smaller than the experimental value of 0.89 mm/V because the theory in Sec. 1-8 neglects fringing at the edge of the plates.

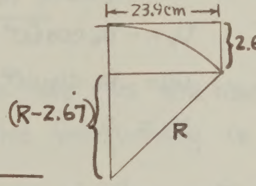
b) 30V will produce a deflection of $(30)(0.89) = 2.67$ cm

From the sketch $R^2 = (23.9)^2 + (R-2.67)^2$ or $R = 108$ cm

Eq. (1-13) gives : $v = 5.93\times10^5 V^{1/2} = 5.93\times10^5(400)^{1/2} = 1.19\times10^7$ m/sec

From Eq. (1-39), $B = \frac{v}{\frac{e}{m}R} = \frac{1.19\times10^7}{1.76\times10^{11}\times1.08} = 6.22\times10^{-5}$ Wb/m^2

$= 0.622$ gauss $\longleftarrow$



c) From Eq. (1-47), $B = \frac{D\sqrt{V_a}}{\ell L\sqrt{\frac{e}{2m}}} = \frac{2.67\sqrt{400}}{1.27\times19.4\times10^{-2}\sqrt{\frac{1.76\times10^{11}}{2}}} = 7.30\times10^{-4}$ Wb/m^2 $= 7.3$ gauss $\longleftarrow$

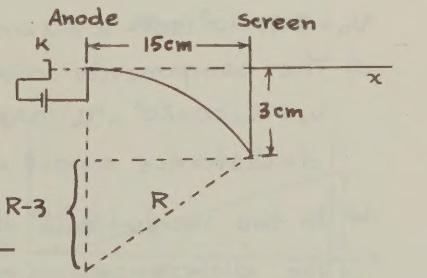
(1-13) From the sketch, $R^2 = (15)^2 + (R-3)^2$

or $R = 39$ cm

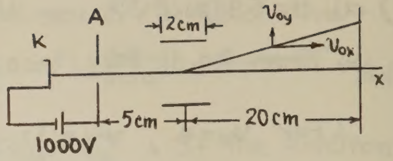
Eq. (1-13) gives: $v_{ox} = 5.93\times10^5(2000)^{1/2} = 2.65\times10^7$ m/sec

From Eq. (1-39), $B = \frac{v_{ox}}{\frac{e}{m}R} = \frac{2.65\times10^7}{0.39\times1.76\times10^{11}}$

$= 3.84\times10^{-4}$ Wb/m^2 $= 3.84$ gauss $\longleftarrow$



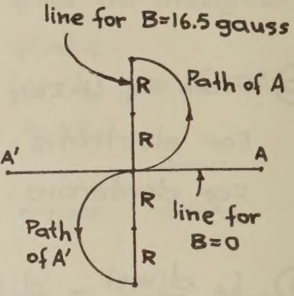
(1-14) a) The theory in Sec. 1-15 on magnetic focusing is applicable with the center of the plates considered as a virtual cathode. From Eq. (1-49)



$$B^2 = \frac{8\pi^2 V_a}{L^2\ e/m} = \frac{8\pi^2 \times 1000}{(0.2)^2 \times 1.76\times10^{11}} = 1.085\times10^{-5}$$

$B = 33\times10^{-4}\ \text{Wb}/m^2 = 33$ gauss ⟵

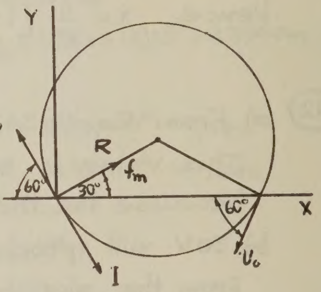
b) The electronic path is a helix of constant pitch. The time to travel down the tube is independent of B and hence is the same as in part (a). The period of the circular motion varies inversely as B and hence is twice that in part (a). Thus each electron travels one half a revolution on its way down the tube. The path of the electrons A and A′ at the two ends of the original 4 cm line is indicated in the sketch. Since each electron moves through an angle of 90°, then the resultant deflection is a straight line perpendicular to the original line for B=0.



The length of the line is 4R where from Eq. (1-45) $R = \frac{m v_{oy}}{eB}$ where v_{oy} is the maximum perpendicular velocity. Since this velocity causes a 2 cm deflection (with B=0), then $v_{oy} = \frac{2}{20} v_{ox} = \frac{5.93\times10^5}{10}(1000)^{1/2} = 1.88\times10^6$ m/sec

$$4R = \frac{4\times1.88\times10^6}{1.76\times10^{11}\times16.5\times10^{-4}} = 2.58\times10^{-2}\ m = 2.58\ cm$$ ⟵

(1-15) a) From the sketch, $x = 2R\cos30° = \sqrt{3}R$. From Eq. (1-13)



$v_o = 5.93\times10^5(100)^{1/2} = 5.93\times10^6$ m/sec. From Eq. (1-39)

$$R = \frac{mv}{eB} = \frac{5.93\times10^6}{1.76\times10^{11}\times5\times10^{-3}} = 0.675\ cm$$

$\therefore\ x = \sqrt{3}R = 1.73\times0.675 = 1.16$ cm, $z = 0$ ⟵

b) The speed is constant along the circular path.

$v_x = -v_o\cos60° = -(5.93\times10^6)(0.5) = -2.97\times10^6$ m/sec, ⟵

$v_y = -v_o\sin60° = -(5.93\times10^6)(0.866) = -5.14\times10^6$ m/sec, ⟵

$v_z = 0$

(1-16) The path is a helix of constant pitch. The circular path is normal to B. The period is independent of the velocity and is the same for both. From Eq. (1-42), $T = \frac{3.57\times10^{-11}}{B} = \frac{3.57\times10^{-11}}{2\times10^{-3}} = 1.785\times10^{-8}$ sec. From Eq. (1-13),

$v_o = 5.93\times10^5(50)^{1/2} = 4.2\times10^6$ m/sec

a) The components along B are: $v_1 = v_{o1}\cos10° = (v_o)(0.9848)$

$v_2 = v_{o2}\cos20° = (v_o)(0.9397)$, $v_1 - v_2 = (v_o)(0.9848 - 0.9397) = (v_o)(0.0451)$

d = distance apart $= (v_1 - v_2)T = (0.0451)(4.2\times10^6)(1.785\times10^{-8}) = 0.338\times10^{-2}$ m

$= 0.338$ cm ⟵

b) In two revolutions the distance is doubled for each and so is the difference i.e. d = 0.676 cm ⟵

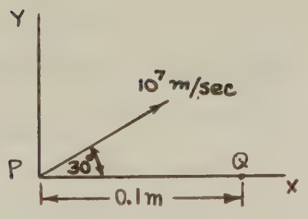
(1-17) The path is a helix of constant pitch.

$v_x = v_{ox} = 10^7 \cos 30° = 8.66\times10^6$ m/sec. The time to go from P to Q is $\frac{0.1}{8.66\times10^6} = 1.15\times10^{-8}$ sec

This must equal an integral multiple n of the period T. From Eq. (1-42)

$$nT = \frac{3.57\times10^{-11} n}{B} = 1.15\times10^{-8}$$

or $B = \frac{3.57\times10^{-11} n}{1.15\times10^{-8}} = 3.1\times10^{-3} n$ Wb/m² $= 3.1 n$ mWb/m² ←



(1-18) $v_{ox} = v_o \cos 30° = 8.66\times10^6$ m/sec

$v_{oy} = v_o \sin 30° = 5.00\times10^6$ m/sec

The path is a helix of constant pitch. The distance along Y is $y = v_{oy}t = (5\times10^6)(5\times10^{-9})$ $= 0.025$ m $= 2.5$ cm ←

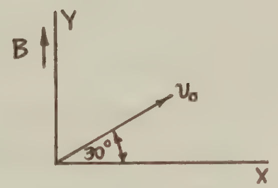
The path in the XZ plane is a circle as shown. The angular velocity is from Eq. (1-40)

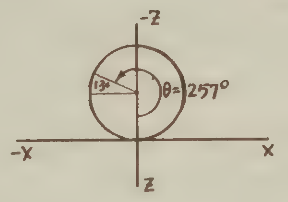
$$\omega = \frac{eB}{m} = (1.76\times10^{11})(5.1\times10^{-3}) = 8.96\times10^8 \text{ rad/sec}$$

and $\theta = \omega t = (8.96\times10^8)(5\times10^{-9}) = 4.48$ rad $= 257°$

$$R = \frac{v_{ox}}{\omega} = \frac{8.66\times10^6}{8.96\times10^8} = 0.966\times10^{-2} \text{ m} = 0.966 \text{ cm}$$

$x = -0.966 \cos 13° = -0.94$ cm, $z = -0.966(1+\sin 13°) = -1.18$ cm ←





(1-19) a) Straight line with constant acceleration. ←

b) Straight line with constant velocity. ←

c) Cycloidal helix with constant pitch. ←

d) Circular helix with constant pitch. ←

(1-20) a) $v = 5.93\times10^5(1000)^{1/2} = 1.87\times10^7$ m/sec; $t = \frac{\ell}{v} = \frac{2\times10^{-2}}{1.87\times10^7} = 1.07\times10^{-9}$ sec is the time the electron is in the region between the plates. If the frequency is f, then the time for one half a cycle is $\frac{1}{2f}$.

$\therefore 1.07\times10^{-9} \leq \frac{1}{2f}$ or $f_{max} = \frac{10^9}{2\times1.07} = 4.67\times10^8$ Hz ←

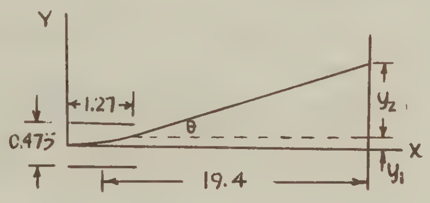
b) If $f = 60$ Hz, then one cycle is $\frac{1}{60}$ sec

$\therefore \frac{t}{1/60} = 60t = (60)(1.07\times10^{-9}) = 6.42\times10^{-8}$ ←

(1-21) $v_x = v_{ox} = 5.93\times10^5(200)^{1/2} = 8.38\times10^6$ m/sec. The time the electron is between the plates is

$$t_1 = \frac{1.27\times10^{-2}}{8.38\times10^6} = 1.515\times10^{-9} \text{ sec}$$

$$a_y = \frac{eV_d}{md} = \frac{(1.76\times10^{11})(60\sin 2\pi\times10^8 t)}{0.475\times10^{-2}} = 2.22\times10^{15}\sin 2\pi\times10^8 t \text{ m/sec}^2$$



(1-21) (Cont'd) $v_y = \int_0^t a_y dt = 3.53\times10^6 (1-\cos 2\pi\times10^8 t)$ m/sec (since $v_{oy}=0$)

$y = \int_0^t v_y dt = 3.53\times10^6 t - 5.63\times10^{-3} \sin 2\pi\times10^8 t$ m ; at $t=t_1$, $y=y_1$

$y_1 = 3.53\times10^6 \times 1.515\times10^{-9} - 5.63\times10^{-3} \sin(2\pi\times10^8\times1.515\times10^{-9}) = 5.35\times10^{-3} - 4.58\times10^{-3} = 0.077$ cm

The velocity components with which the electron leaves the plates are $v_{x1} = v_{ox}$ and $v_{oy} = 3.53\times10^6 [1-\cos(2\pi\times10^8\times1.515\times10^{-9})] = 1.475\times10^6$ m/sec

After leaving the plates the velocity is constant because the field is zero. The distance to the screen is $19.4 - \frac{1}{2}(1.27) = 18.8$ cm $= 0.188$ m

The time t_2 to reach the screen is $t_2 = \frac{0.188}{8.38\times10^6} = 2.24\times10^{-8}$ sec.

$y_2 = v_{y1} t_2 = 1.475\times10^6 \times 2.24\times10^{-8} = 3.31\times10^{-2}$ m $= 3.31$ cm

As a check : $\tan\theta = \frac{v_{y1}}{v_{x1}} = \frac{1.475\times10^8}{8.38\times10^8} = 0.176$

$y_2 = 0.188\tan\theta = 0.188\times0.176 = 3.31\times10^{-2}$ m as obtained above.

The total deflection is $D = y_1 + y_2 = 0.077 + 3.31 = 3.39$ cm. ←

If the center of the plates is assumed to be a virtual cathode, then $y = 19.4\tan\theta = 19.4\times0.176 = 3.42$ cm which is approximately correct.

(1-22) (Refer to the sketch in prob. 1-21)

$v_x = 8.38\times10^6$ m/sec ; $t_1 = 1.515\times10^{-9}$ sec from prob. 1-21

$a_y = \frac{eV_d}{md} = 1.76\times10^{11} \frac{4\times10^{10} t}{0.475\times10^{-2}} = 1.48\times10^{24} t$ m/sec² ; $v_y = \int_0^t a_y dt = 0.74\times10^{24} t^2$, ($v_{oy}=0$)

$y = 0.247\times10^{24} t^3$; $y_1 = 0.247\times10^{24}\times(1.515\times10^{-9})^3 = 8.6\times10^{-4}$ m $= 0.086$ cm

The velocity components with which the electron leaves the plates are : $v_{x1} = v_{ox}$ and $v_{y1} = 0.74\times10^{24}\times(1.515\times10^{-9})^2 = 1.7\times10^6$ m/sec.

After leaving the plates the velocity remains constant because the field is zero. The distance to the screen is $19.4 - \frac{1}{2}(1.27) = 18.8$ cm.

The time t_2 to reach the screen is $t_2 = \frac{0.188}{8.38\times10^6} = 2.24\times10^{-8}$ sec

$y_2 = v_{y1} t_2 = 1.7\times10^6\times2.24\times10^{-8} = 3.81\times10^{-2}$ m $= 3.81$ cm.

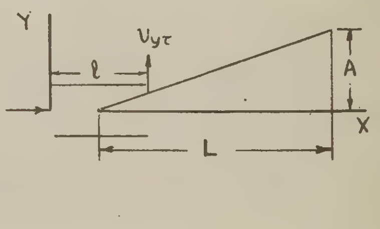
The total deflection is $D = y_1 + y_2 = 0.086 + 3.81 = 3.9$ cm ←

As a check on y_2 : $\tan\theta = \frac{v_{y1}}{v_{x1}} = \frac{1.7\times10^6}{8.38\times10^6} = 0.203$

$y_2 = 0.188\tan\theta = 0.188\times0.203 = 3.81$ cm as above.

If the center of the plates had been assumed to be a virtual cathode, then $y = 19.4\tan\theta = 19.4\times0.203 = 3.94$ cm which is approximately correct.

(1-23) a) Assume $\mathcal{E}\sin(\omega t+\theta)$ is the electric field acting on the electron. Each electron enters at $t=0$ but for different values of θ. The angle θ giving the maximum deflection is to be found later.



$$a_y = \frac{dv_y}{dt} = \frac{e\mathcal{E}\sin(\omega t+\theta)}{m}$$

(1-23) (Cont'd) $v_y = \frac{e\mathcal{E}}{m\omega}[\cos\theta - \cos(\omega t+\theta)]$ since when $t=0$, $v_y=0$

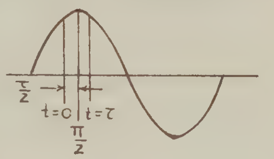
Assume that the center of the plates is a virtual cathode so that $A = L\frac{v_{y\tau}}{v_o}$ where $v_{y\tau} = v_y$ for $t=\tau=\frac{\ell}{v_o}$, the transit time.

$$\therefore A = \frac{e\mathcal{E}L}{m\omega v_o}[\cos\theta - \cos(\omega\tau+\theta)]$$

To find A_{max}, we differentiate with respect to θ and set $\frac{dA}{d\theta}=0$.

Thus, $-\sin\theta + \sin(\omega\tau+\theta) = 0$ or $\sin\theta = \sin(\omega\tau+\theta)$

$\therefore \theta = \pi - (\omega\tau+\theta)$ or $\theta = \frac{\pi}{2} - \frac{\omega\tau}{2}$. This makes sense, since the electron will be between the plates during the time when the field is greatest, namely from $\theta = \frac{\pi}{2} - \frac{\omega\tau}{2}$ to $\theta = \frac{\pi}{2} + \frac{\omega\tau}{2}$ (since the transit angle is $\omega\tau$).



$$A = \frac{e\mathcal{E}L}{m\omega v_o}\left[\cos\left(\frac{\pi}{2}-\frac{\omega\tau}{2}\right) - \cos\left(\frac{\pi}{2}+\frac{\omega\tau}{2}\right)\right] \doteq \frac{e\mathcal{E}L}{m\omega v_o}\left(2\sin\frac{\omega\tau}{2}\right)$$

If $\tau \ll T$, then the electric field may be considered constant and equal to $\mathcal{E}$; $v_{y\tau} = \frac{e\mathcal{E}\tau}{m}$ and $A_o = v_{y\tau}\frac{L}{v_o} = \frac{e\mathcal{E}\tau L}{m v_o}$. The same result may be obtained by letting $\omega \to 0$ in the expression for A.

$$\therefore \frac{A}{A_o} = \frac{2e\mathcal{E}L}{m\omega v_o}\,\frac{1}{\frac{e\mathcal{E}\tau L}{m v_o}}\sin\frac{\omega\tau}{2} \quad\text{or}\quad A = A_o\frac{\sin\frac{\omega\tau}{2}}{\left(\frac{\omega\tau}{2}\right)} \quad \text{QED} \qquad \longleftarrow$$

b) Let $\alpha = \frac{\omega\tau}{2}$. Then $\frac{\sin\alpha}{\alpha} = 0.9$; For small α, $\sin\alpha = \alpha - \frac{\alpha^3}{6} + \dots$

$\therefore \frac{\sin\alpha}{\alpha} = 1 - \frac{\alpha^2}{6} = 0.9$ or $\alpha^2 = 0.6$ and $\alpha = 0.774$ rad $= 44.3°$

We use this as a first approximation for α in $\sin\alpha$. Thus

$\alpha = \frac{\sin\alpha}{0.9} = \frac{\sin 44.3°}{0.9} = 0.778$ rad. This is close enough.

Now $\tau = \frac{\ell}{v_o}$ and $v_o = 5.93\times10^5(1000)^{1/2} = 1.88\times10^7$ m/sec, so $\tau = \frac{10^{-2}}{1.88\times10^7} = 5.3\times10^{-10}$

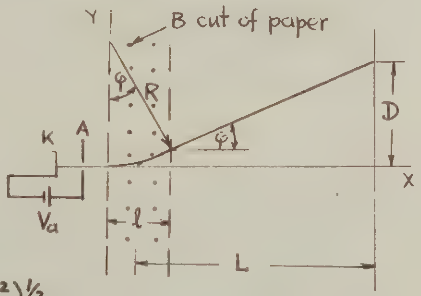
$\alpha = \frac{\omega\tau}{2} = \pi f\tau \doteq 0.778$ or $f = \frac{0.778}{\pi\times5.3\times10^{-10}} = 467$ MHz ⟵

(1-24) From sketch we have: $D = L\tan\varphi$

where $\varphi = \sin^{-1}\frac{\ell}{R}$, $\tan\varphi = \frac{\sin\varphi}{\cos\varphi}$, $\sin\varphi = \frac{\ell}{R}$

$\cos\varphi = \sqrt{1-\ell^2/R^2} = \frac{1}{R}\sqrt{R^2-\ell^2}$, $\tan\varphi = \frac{\ell}{\sqrt{R^2-\ell^2}}$

$R = \frac{mv}{eB}$; Then $D = \frac{\ell L}{\sqrt{R^2-\ell^2}}$



Consider $\sqrt{R^2-\ell^2} = \left(\frac{m^2v^2}{e^2B^2} - \ell^2\right)^{1/2} = \frac{1}{eB}(m^2v^2 - e^2B^2\ell^2)^{1/2}$

From Eq. (1-12), $v^2 = \frac{2eV_a}{m}$ so $\sqrt{R^2-\ell^2} = \frac{1}{eB}(2emV_a - e^2B^2\ell^2)^{1/2} = \frac{1}{B}\left[2\frac{m}{e}V_a - (B\ell)^2\right]^{1/2}$

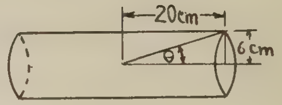
$$\text{Hence, } D = \frac{\ell LB}{\sqrt{\frac{2mV_a}{e} - (B\ell)^2}} = \ell LB\sqrt{\frac{e}{2mV_a - e(B\ell)^2}} = \ell LB\sqrt{\frac{e/m}{2V_a - \frac{e}{m}(B\ell)^2}} \quad \text{QED}$$

(1-25) From Eq. (1-49), $B = K V_a^{1/2} n$; $B = \left(\frac{400}{900}\right)^{1/2}(2.5) = 1.67$ mWb/m² ←

Since B is proportional to n, then the next higher value is $B = 2 \times 1.67 = 3.34$ mWb/m² ←

(1-26) At the center of the solenoid $B = N_0 N I \cos\theta$ Wb/m²

where N = turns/m and $N_0 = 4\pi \times 10^{-7}$ H/m



$$B = 4\pi \times 10^{-7} \times 24 \times \frac{100}{2.54} \times \left(\frac{20}{\sqrt{20^2+6^2}}\right) I = 1.135 \times 10^{-3} I \text{ Wb/m}^2$$

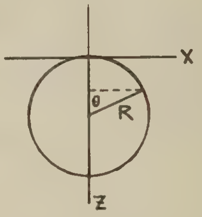
From Eq. (1-49) $\frac{e}{m} = \frac{8\pi^2 V_a n}{L^2 B^2}$ or $I = \frac{1}{1.135 \times 10^{-3}} \left(\frac{8V_a}{e/m}\right)^{1/2} \frac{\pi n}{L} = 1.43\, n$

For $n = 1, 2$ and 3, $I = 1.43$, 2.86 and 4.29 A ←

For $n = 4$, $I = 5.72$A which exceeds the rating of the solenoid.

(1-27) a) $x = R\sin\theta$, $z = R(1-\cos\theta)$

On the screen $y = L$ and the time to reach the screen is $t = \frac{L}{v_{oy}}$ ∴ $\theta = \omega t$ where from Eq. (1-40) $\omega = \frac{eB}{m}$. Thus, $\theta = \frac{eB}{m}\frac{L}{v_{oy}} \equiv \alpha$.

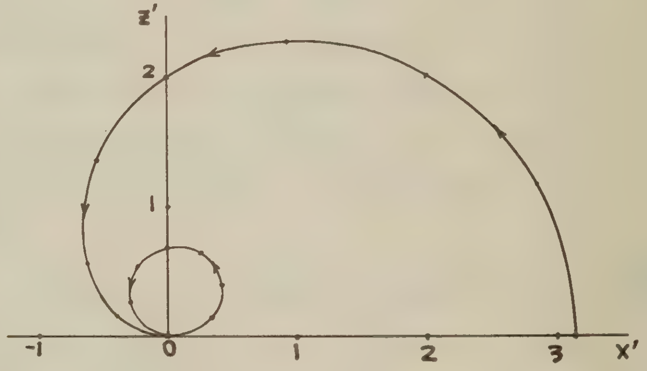


From Eq. (1-39), $R = \frac{m v_{ox}}{eB} = \frac{v_{ox} L}{v_{oy}\alpha}$

Hence, $x = \frac{v_{ox}L}{v_{oy}\alpha}\sin\alpha$ and $z = \frac{v_{ox}L}{v_{oy}\alpha}(1-\cos\alpha)$ QED ←

b) If $x' = \frac{\pi v_{oy}}{L v_{ox}} x$ and $z' = \frac{\pi v_{oy}}{L v_{ox}} z$, then $x' = \frac{\pi}{\alpha}\sin\alpha$ and $z' = \frac{\pi}{\alpha}(1-\cos\alpha)$

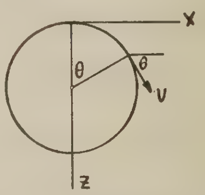
α	x'	z'	α	x'	z'
0	3.14	0.00	2π	0.00	0.00
$\pi/4$	2.83	1.17	$9\pi/4$	0.31	0.13
$\pi/2$	2.00	2.00	$5\pi/2$	0.40	0.40
$3\pi/4$	0.94	2.28	$11\pi/4$	0.26	0.62
π	0.00	2.00	3π	0.00	0.67
$5\pi/4$	−0.56	1.36	$13\pi/4$	−0.22	0.52
$3\pi/2$	−0.67	0.67	$7\pi/2$	−0.29	0.29
$7\pi/4$	−0.40	0.16	$15\pi/4$	−0.19	0.08
2π	0.00	0.00	4π	0.00	0.00



(1-28) a) From Eq. (1-13) $v_0 = 5.93 \times 10^5 (300)^{1/2} = 1.027 \times 10^7$ m/sec

$v_{ox} = 1.027 \times 10^7 \cos 60° = 0.514 \times 10^7$ m/sec

$v_{oy} = 1.027 \times 10^7 \cos 30° = 0.89 \times 10^7$ m/sec



$a_y = \frac{e\mathcal{E}}{m} = 1.76 \times 10^{11} \times 5 \times 10^3 = 8.8 \times 10^{14}$ m/sec²

$y = v_{oy}t + \frac{1}{2}a_y t^2$; $4.4 \times 10^{14} t^2 + 0.89 \times 10^7 t - 0.016 = 0$

Solving we find $t = 1.68 \times 10^{-9}$ sec. From Eq. (1-39)

$R = \frac{v_{ox}}{B\, e/m} = \frac{0.514 \times 10^7}{1.2 \times 10^{-3} \times 1.76 \times 10^{11}} = 2.435$ cm. From Eq. (1-40) $\omega = \frac{Be}{m} = 1.2 \times 10^{-3} \times 1.76 \times 10^{11}$

or $\omega = 2.11 \times 10^8$ rad/sec ; $\theta = \omega t = 2.11 \times 10^8 \times 1.68 \times 10^{-9} = 0.354$ rad $= 20.3°$

$x = R\sin\theta = 2.435 \sin 20.3° = 0.845$ cm, $z = R(1-\cos\theta) = 2.435(1-\cos 20.3°) = 0.153$ cm

$y = 1.6$ cm ←

(1-28) (Cont'd)

b) $v_x = v_{ox}\cos\theta = (0.514\times10^{7})\cos 20.3° = 4.81\times10^{6}$ m/sec ←

$v_y = a_y t + v_{oy} = (8.8\times10^{14})(1.68\times10^{-9}) + 0.89\times10^{7} = (1.48+8.9)\times10^{6} = 10.4\times10^{6}$ m/sec ←

$v_z = v_{ox}\sin\theta = (0.514\times10^{7}\sin 20.3° = 1.79\times10^{6}$ m/sec ←

c) If $\mathcal{E}$ is reversed, a_y is negative and $y = v_{oy}t + \frac{1}{2}a_y t^2$ becomes $-4.4\times10^{14}t^2 + 0.89t - 0.016 = 0$. Solving for t we find $t = 2\times10^{-9}$ sec and $t = 18.1\times10^{-9}$ sec.

The smaller value of time must be used since at this instant the plate is exposed. If the plate were absent then the electron would continue up the Y-axis, reverse and return to $y = 1.6$ cm at the end of 18.1×10^{-9} sec.

$\theta = \omega t = (2.11\times10^{8})(2\times10^{-9}) = 0.422$ rad $= 24.2°$

$x = R\sin\theta = 2.435\sin 24.2° = 0.998$ cm ←

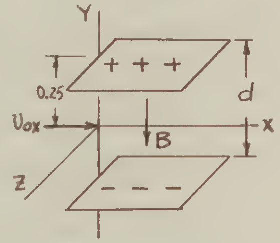
$z = R(1-\cos\theta) = 2.435(1-\cos 24.2°) = 0.216$ cm, $y = 1.6$ cm ←

$v_x = v_{ox}\cos\theta = 0.514\times10^{7}\cos 24.2° = 4.68\times10^{6}$ m/sec ←

$v_y = a_y t + v_{oy} = (-8.8\times10^{14})(2\times10^{-9}) + 8.9\times10^{6} = 7.14\times10^{6}$ m/sec ←

$v_z = v_{ox}\sin\theta = 0.514\times10^{7}\sin 24.2° = 2.1\times10^{6}$ m/sec ←

(1-29) a) $d = 0.5$ cm, $V_d = 200$V, $B = 0.1$ Wb/m², $v_{ox} = 4\times10^{6}$ m/sec



The path is a helix of varying pitch.

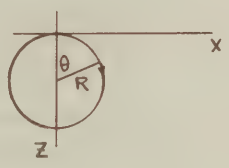
$v_y = \frac{e\mathcal{E}}{m}t = 1.76\times10^{11}\times\frac{200}{5\times10^{-3}}t = 7.04\times10^{15}t$

$y = 3.52\times10^{15}t^2$; The time to reach the positive plate is $t = \left(\frac{2.5\times10^{-3}}{3.52\times10^{15}}\right)^{1/2} = (71\times10^{-20})^{1/2} = 8.43\times10^{-10}$ sec

$\omega = \frac{eB}{m} = (1.76\times10^{11})(0.01) = 1.76\times10^{9}$ rad/sec

$\theta = \omega t = (1.76\times10^{9})(8.43\times10^{-10}) = 1.484$ rad $= 85°$

$R = \frac{v_{ox}}{\omega} = \frac{4\times10^{6}}{1.76\times10^{9}} = 2.27\times10^{-3}$ m $= 0.227$ cm



$x = R\sin\theta = (0.227)\sin 85° = 0.226$ cm ←

$z = R(1-\cos\theta) = (0.227)(1-\cos 85°) = 0.207$ cm ←

b) $v_x = \frac{dx}{dt} = \omega R\cos\omega t = v_{ox}\cos\theta = (4\times10^{6})(0.087) = 0.348\times10^{6}$ m/sec ←

$v_z = \frac{dz}{dt} = \omega R\sin\omega t = v_{ox}\sin\theta = (4\times10^{6})(0.996) = 3.98\times10^{6}$ m/sec ←

$v_y = (7.04\times10^{15})(8.43\times10^{-10}) = 5.94\times10^{6}$ m/sec ←

(1-30) If T is the period then the distance covered in the first revolution is $y_1 = \frac{1}{2}a_y T^2$ and in the first two revolutions is $y_2 = \frac{1}{2}a_y(2T)^2$. Hence the distance travelled during the second revolution is $y_2 - y_1 = \frac{3}{2}a_y T^2$. From Eq. (1-42) $T = \frac{2\pi m}{eB} = \frac{2\pi\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.1} = 6.53\times10^{-7}$ sec

$a_y = \frac{e\mathcal{E}}{m} = \frac{1.6\times10^{-19}\times10^{4}}{1.67\times10^{-27}} = 9.64\times10^{11}$ m/sec²

$\therefore\ y_2 - y_1 = \frac{3}{2}(9.64\times10^{11})(6.53\times10^{-7})^2 = 0.618 = 61.8$ cm ←

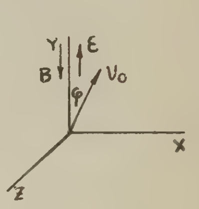
(1-31) $v_y = v_{oy} - a_y t$; If $v_y = 0$ at $t = T$, then $T = \frac{v_{oy}}{a_y}$. From Eq. (1-13) we have:

$v_o = 5.93\times10^5 (300)^{1/2} = 1.025\times10^7$ m/sec; $v_{oy} = v_o \cos 60^\circ = 5.13\times10^6$ m/sec

$a_y = \frac{e\mathcal{E}}{m} = 1.76\times10^{11}\times10^4 = 1.76\times10^{15}$ m/sec² ; $T = \frac{5.13\times10^6}{1.76\times10^{15}} = 2.92\times10^{-9}$ sec

From Eq. (1-42), $B = \frac{3.57\times10^{-11}}{2.92\times10^{-9}} = 1.22\times10^{-2}$ Wb/m² $\longleftarrow$

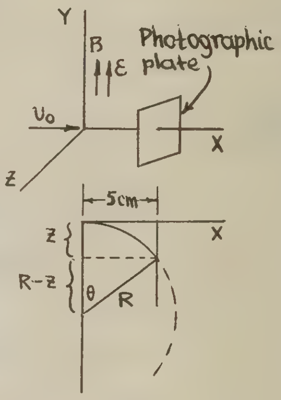
(1-32) $y = v_{oy}t - \frac{1}{2}a_y t^2$; When $y = 0$, $t = \frac{2v_{oy}}{a_y}$. If the electron is to return to the origin, then t must be a multiple of the period T or $t = nT$. From Eq. (1-41), $T = \frac{2\pi m}{eB}$



$$\therefore \frac{2v_{oy}}{a_y} = \frac{2\pi m n}{eB} = \frac{2v_o\cos\varphi}{\frac{e\mathcal{E}}{m}} \quad \text{or} \quad \frac{n\pi}{B} = \frac{v_o\cos\varphi}{\mathcal{E}}$$

or $n\pi\mathcal{E} = Bv_o\cos\varphi$ where n is an integer $\longleftarrow$

(1-33) a) The path is a helix of varying pitch. From Eqs. (1-12) & (1-13)



$v_o = 5.93\times10^5 (V \frac{m_e}{m_H})^{1/2} = 5.93\times10^5 (\frac{150}{1836})^{1/2} = 1.7\times10^5$ m/sec

From Eq. (1-39) $R = \frac{mv}{eB} = \frac{1.67\times10^{-27}\times1.7\times10^5}{1.6\times10^{-19}\times3\times10^{-2}} = 5.88\times10^{-2}$ m

In order to find when the ion strikes the plate we must first find θ because $\theta = \omega t$. From the sketch

$\sin\theta = \frac{5}{5.88} = 0.852$ and $\theta = 58.4^\circ$

$t = \frac{\theta}{\omega} = \frac{\theta}{v_o/R} = \frac{58.4^\circ}{57.3} \times \frac{5.88\times10^{-2}}{1.7\times10^5} = 3.52\times10^{-7}$ sec $\longleftarrow$

b) From the sketch: $(5.88 - z)^2 + 5^2 = (5.88)^2$ or $z = 3.1$ cm

$a_y = \frac{e\mathcal{E}}{m} = \frac{1.6\times10^{-19}\times2\times10^4}{1.67\times10^{-27}} = 1.93\times10^{12}$ m/sec²

$y = \frac{1}{2}a_y t^2 = \frac{1}{2}(1.93\times10^{12})(3.52\times10^{-7})^2 = 0.119$ m $= 11.9$ cm ; $x = 5$ cm $\longleftarrow$

c) Plate perpendicular to Y-axis: $y = \frac{1}{2}a_y t^2$

$t = \left(\frac{2y}{a_y}\right)^{1/2} = \left(\frac{2\times5\times10^{-2}}{1.93\times10^{12}}\right)^{1/2} = 2.27\times10^{-7}$ sec; $\theta = \omega t = \frac{v_o}{R}t = \frac{1.7\times10^5\times2.27\times10^{-7}}{5.88\times10^{-2}} = 0.657$ rad $= 37.7^\circ$

$x = R\sin\theta = 5.88\sin 37.7^\circ = 3.6$ cm ; $z = R(1-\cos\theta) = 5.88(1-\cos 37.7^\circ) = 1.23$ cm; $y = 5$cm

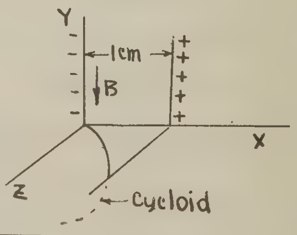
d) Plate perpendicular to Z-axis: $z = 5 = 5.88(1-\cos\theta)$, $\theta = 81.4^\circ = 1.42$ rad

$t = \frac{\theta}{\omega} = \frac{\theta}{v_o}R = \frac{1.42\times5.88\times10^{-2}}{1.7\times10^5} = 4.91\times10^{-7}$ sec

$y = \frac{1}{2}a_y t^2 = \frac{1}{2}(1.93\times10^{12})(4.91\times10^{-7})^2 = 0.232$ m $= 23.2$ cm; $x = R\sin\theta = 5.82$cm; $z = 5$cm

(1-34) a) The path is a common cycloid in the XZ plane.

$\mathcal{E} = \frac{100}{10^{-2}} = 10^4$ V/m



From Eq. (1-53) : $\omega = \frac{eB}{m} = 1.76\times10^{11}\times1.68\times10^{-3} = 2.96\times10^8$ rad/sec

$u = \frac{\mathcal{E}}{B} = \frac{10^4}{1.68\times10^{-3}} = 5.95\times10^6$ m/sec

From Eq. (1-58) : $Q = \frac{u}{\omega} = \frac{5.95\times10^6}{2.96\times10^8} = 2.01$ cm

From Eq. (1-59) : $x = Q(1-\cos\theta) = 2.01(1-\cos\theta) = 1 \quad \therefore \theta = 60^\circ$

$z = Q(\theta - \sin\theta) = 2.01(\frac{\pi}{3} - 0.866) = 0.36$ cm $\longleftarrow$

b) $\theta = \frac{\pi}{3} = \omega t \quad \therefore t = \frac{\pi/3}{2.96}\times10^{-8} = 3.54\times10^{-9}$ sec. $\longleftarrow$

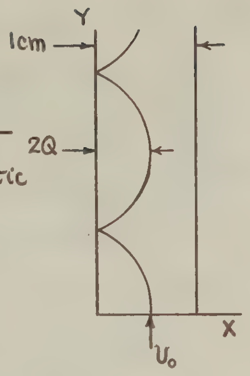
(1-35) a) From the sketch we see that $2Q=0.5$ cm or $Q=0.25=\frac{u}{\omega}$ and from Eq. (1-53) $u=\frac{\mathcal{E}}{B}=\frac{10^4}{B}$; $\omega=\frac{eB}{m}=1.76\times10^{11}B$

so $0.25=\dfrac{10^4}{1.76\times10^{11}B^2}$ or $B=4.77\times10^{-3}$ Wb/m² $=47.7$ gauss ←

In order for the initial force to be along $-X$, the magnetic field must be out of the plane of the paper.

b) From Eq. (1-56), $v_x=u\sin\omega t$, $v_z=u-u\cos\omega t$; when $v_x=0$, $v_z=0$ (cusp) or $v_z=2u$ (at $x=2Q$)

$\therefore v_0=2u=\dfrac{2\times10^4}{B}=\dfrac{2\times10^4}{4.77\times10^{-3}}=4.2\times10^6$ m/sec ←



(1-36) a) The motion is a cycloid in the X-Z plane as shown with constant velocity along Y. From Eq. (1-12)

$v_0=(2\frac{e}{m}V)^{1/2}=\left(2\,\dfrac{1.6\times10^{-19}\times10^2}{1.67\times10^{-27}}\right)^{1/2}=1.39\times10^5$ m/sec

$t=\dfrac{0.14}{1.39\times10^5}=1.01\times10^{-6}$ sec . From Eq. (1-53)

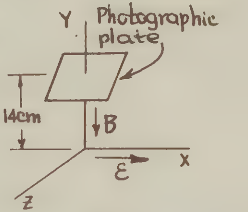
$\omega=\dfrac{eB}{m}=\dfrac{1.6\times10^{-19}\times0.05}{1.67\times10^{-27}}=4.82\times10^6$ rad/sec ; $u=\dfrac{\mathcal{E}}{B}=\dfrac{5\times10^3}{0.05}=10^5$

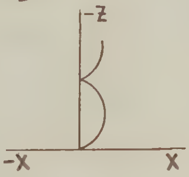
From Eq. (1-58), $Q=\dfrac{u}{\omega}=\dfrac{10^5}{4.82\times10^6}=2.07$ cm

$\theta=\omega t=4.82\times10^6\times1.01\times10^{-6}=4.87$ rad $=279°$

Eq. (1-59) $\begin{cases} x=Q(1-\cos\theta)=2.07(1-\cos279°)=1.75\text{ cm} & \longleftarrow \\ -z=Q(\theta-\sin\theta)=2.07(4.87-\sin279°)=12.1\text{ cm} & \longleftarrow \end{cases}$

$y=14$ cm ←





b) $2Q=2(2.07)=4.14$ cm and since the plate is at 14 cm, the ion does not strike.

c) The distance along the $-z$ axis per cycle is $2\pi Q=13$ cm. Hence the $-Z$ distance in the second cycle is $14-13=1$ cm and the angle θ' in the second cycle is given by $1=2.07(\theta'-\sin\theta')$ or $\theta'=85°$.

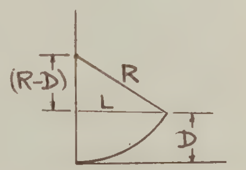
Hence $x=Q(1-\cos\theta')=2.07(1-\cos85°)=1.89$ cm ←

$t_2=\dfrac{\theta'}{\omega}=\dfrac{85°}{57.3°}\,\dfrac{1}{4.82\times10^6}=3.05\times10^{-7}$ sec in the second cycle. The time for one cycle $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{4.82\times10^6}=13\times10^{-7}$ sec . Total time before hitting plate is

$t=t_2+T=16.05\times10^{-7}$ sec ; $y=v_{0y}t=1.39\times10^5\times16.05\times10^{-7}=22.5$ cm ←

$z=14$ cm

(1-37) In order for the net force to be zero $e\mathcal{E}=Bev$ or $v=\dfrac{\mathcal{E}}{B}$. From Eq. (1-53), $\dfrac{e}{m}=\dfrac{v}{RB}=\dfrac{\mathcal{E}}{RB^2}$ where R is the radius of the circular path and is found from the sketch to be $R=\dfrac{D^2+L^2}{2D}$ QED ←



(1-38) From Eq. (1-56), $v_x=u\sin\theta$, $v_z=u-u\cos\theta$; $v^2=v_x^2+v_z^2=u^2\sin^2\theta+u^2-2u^2\cos\theta+u^2\cos^2\theta$

or $v^2=u^2(\sin^2\theta+\cos^2\theta)+u^2-2u^2\cos\theta=2u^2(1-\cos\theta)=4u^2\sin^2\frac{\theta}{2}$

$\therefore v=2u\sin\theta/2$ QED ←

$v=\dfrac{ds}{dt}$ where s is the distance along the path. $\therefore s=\int v\,dt=2u\int_0^t\sin\frac{\omega t}{2}\,dt$

or $s=-4\frac{u}{\omega}\cos\frac{\omega t}{2}\Big|_0^t=4Q(1-\cos\frac{\theta}{2})$ where $Q=\frac{u}{\omega}$. QED ←

(1-39) Eqs. (1-54) : $\frac{dv_x}{dt} = \omega u - \omega v_z$, $\frac{dv_z}{dt} = +\omega v_x$;

$\frac{d^2v_x}{dt^2} = -\omega\frac{dv_z}{dt} = -\omega^2 v_x$ $\therefore$ $v_x = A\cos\omega t + C\sin\omega t$, $\frac{dv_x}{dt} = -\omega A\sin\omega t + \omega C\cos\omega t$

At $t=0$, $v_x = v_{ox}$ and $\frac{dv_x}{dt} = \omega u - \omega v_{oz}$ $\therefore$ $A = v_{ox}$ and $\omega u - \omega v_{oz} = \omega C$

or $C = u - v_{oz}$; $\therefore$ $v_x = v_{ox}\cos\omega t + (u - v_{oz})\sin\omega t = \frac{dx}{dt}$. Integrating,

$x = \frac{v_{ox}}{\omega}\sin\omega t - \left(\frac{u-v_{oz}}{\omega}\right)\cos\omega t + D$; @ $t=0$, $x=0$ or $D = \frac{u-v_{oz}}{\omega}$

$\therefore$ $x = \frac{v_{ox}}{\omega}\sin\omega t + \left(\frac{u-v_{oz}}{\omega}\right)(1-\cos\omega t)$ QED $\longleftarrow$

$\frac{dv_y}{dt} = 0$, $v_y = v_{oy} = \frac{dy}{dt}$ $\therefore$ $y = v_{oy}t$ $\longleftarrow$

$\frac{dv_z}{dt} = \omega v_x = \omega v_{ox}\cos\omega t + \omega(u - v_{oz})\sin\omega t$; $v_z = v_{ox}\sin\omega t - (u - v_{oz})\cos\omega t + E$

@ $t=0$, $v_z = v_{oz} = -u + v_{oz} + E$ or $E = u$; $v_z = v_{ox}\sin\omega t - (u - v_{oz})\cos\omega t + u = \frac{dz}{dt}$

$z = -\frac{v_{ox}}{\omega}\cos\omega t - \left(\frac{u - v_{oz}}{\omega}\right)\sin\omega t + ut + F$. @ $t=0$, $z=0$ $\therefore$ $F = \frac{v_{ox}}{\omega}$

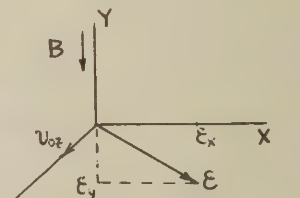
$\therefore$ $z = \frac{v_{ox}}{\omega}(1-\cos\omega t) - \left(\frac{u-v_{oz}}{\omega}\right)\sin\omega t + ut$ QED $\longleftarrow$

(1-40) a) In order to remain in the YZ plane, the force in the x-direction must equal zero: $f_x = -e\mathcal{E}_x - Bev_{oz} = 0$

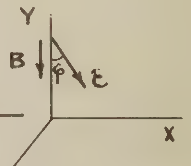
or $v_{oz} = -\frac{\mathcal{E}_x}{B}$ $\longleftarrow$

b) The only component of force present is $f_y = +e\mathcal{E}_y$. Hence, $v_z = v_{oz} = \text{constant}$; $y = \frac{1}{2}\frac{e}{m}\mathcal{E}_y t^2$, $z = -\frac{\mathcal{E}_x}{B}t$ $\longleftarrow$

c) The path is a parabola: $y = \frac{1}{2}\frac{e\,\mathcal{E}_y z^2}{m\,v_{oz}^2}$ $\longleftarrow$

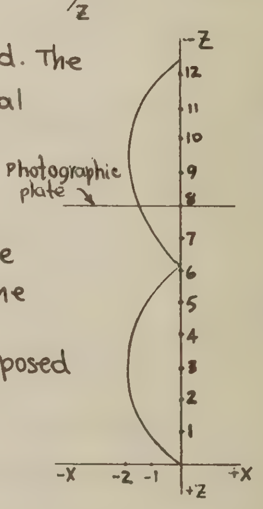


(1-41) $\mathcal{E}_x = \mathcal{E}\sin\varphi$, $\mathcal{E}_y = -\mathcal{E}\cos\varphi$. Due to B and $\mathcal{E}_x$ the motion is a common cycloid in the XZ plane as given by Eq. (1-59). The motion along Y is given by $y = \frac{a_y t^2}{2} = \frac{e\mathcal{E}\cos\varphi}{2m}t^2$. The path might be called a cycloidal helix of variable pitch. $\longleftarrow$



(1-42) The projection of the path in the XZ plane is a common cycloid. The orientation of this cycloid is found by considering the initial effects of the field on the electron. Thus, first the electric field accelerates it in the −X direction. Applying the direction rule of Sec. 1-11 for an electronic velocity along −X and a magnetic field along −Y gives a magnetic force along −Z. Hence, the projection of the path is as shown. The Y velocity is constant.

It is now seen that the photographic plate will not be exposed unless it is located along the −Z axis as shown.



Eq. (1-53) $\begin{cases} u = \frac{\mathcal{E}\sin\varphi}{B} = \frac{5\times10^3\sin20°}{10^{-3}} = 1.71\times10^6\ \text{m/sec} \\ \omega = \frac{eB}{m} = 1.76\times10^{11}\times10^{-3} = 1.76\times10^8\ \text{rad/sec} \end{cases}$

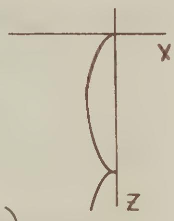
(1-42) (Cont'd) From Eq. (1-58), $Q = \frac{u}{\omega} = \frac{1.71\times10^{6}}{1.76\times10^{8}} = 0.971\times10^{-2}\,m = 0.971\ cm$

The distance traveled along the $-Z$ direction between cusps is $2\pi Q = 6.1$ cm. Thus, the plate is located beyond the first cycle as shown in the sketch. The angle θ' in the second cycle is given by the equation $8.0 - 6.1 = (0.971)(\theta' - \sin\theta')$ cm. This equation is solved graphically by plotting $\theta' - 1.95$ vs. θ' and $\sin\theta'$ vs. θ' and finding the point of intersection. The result is $\theta' = 2.53$ rad or 145°.

Hence, $\theta = \theta' + 2\pi = 2.53 + 6.28 = 8.81$ rad. From Eq. (1-59), $x = -(0.971)(1-\cos145°)$ or $x = -1.76$ cm. The time t it takes the electron to reach the plate is $t_1 = \frac{\theta}{\omega} = \frac{8.81}{1.76\times10^{8}} = 5.01\times10^{-8}$ sec ; $a_y = \frac{e\mathcal{E}_y}{m} = 1.76\times10^{11}\times5\times10^{3}\cos20° = 8.28\times10^{14}\,m/sec^2$

$$\therefore\ y = \tfrac{1}{2}a_y t_1^2 = \tfrac{1}{2}(8.28\times10^{14})(5.01\times10^{-8})^2 = 1.04\,m \quad \leftarrow$$

(1-43) a) Cycloidal helix of variable pitch along the Y-axis. The cycloids are in the XZ plane. The electron starts up and then goes down the Y-axis.



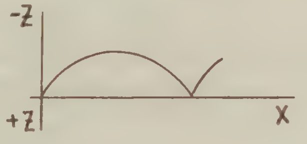
b) From Eq. (1-59), $-x = Q(1-\cos\theta)$ and $z = Q(\theta - \sin\theta)$.

$y = v_{oy}t - \frac{1}{2}a_y t^2$; $v_{oy} = 5.93\times10^{5}(400)^{1/2} = 1.19\times10^{7}\,m/sec$

For $\theta = 0, 2\pi, 4\pi$, etc then $x = 0$. $\theta = 2\pi n$ where n is an integer. When $y = 0$, $t = \frac{2v_{oy}}{a_y}$, $\theta = \omega t = \left(\frac{eB}{m}\right)\left(\frac{2v_{oy}}{a_y}\right) = \left(\frac{eB}{m}\right)\left(\frac{2v_{oy}}{\frac{e\mathcal{E}_y}{m}}\right)$

or $\theta = \frac{2v_{oy}B}{\mathcal{E}_y} = 2\pi n = \frac{2\times1.19\times10^{7}B}{10^{4}\cos60°}$ or $B = 1.32n\times10^{-3}\,Wb/m^2 = 13.2n$ gauss $\leftarrow$

(1-44) The motion is a cycloid in the XZ plane as shown, with constant acceleration along Y.



$a_y = \frac{e\mathcal{E}}{m} = 1.76\times10^{11}\times5\times10^{3}\sin30° = 4.4\times10^{14}\,m/sec^2$

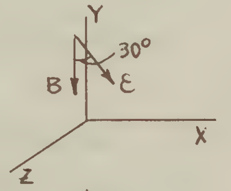
$y = \frac{1}{2}a_y t^2 = \frac{1}{2}(4.4\times10^{14})(5\times10^{-9})^2 = 0.55$ cm $\leftarrow$

Eq. (1-53) $\begin{cases} \omega = \frac{eB}{m} = 1.76\times10^{11}\times10^{-3} = 1.76\times10^{8}\ rad/sec \\ u = \frac{\mathcal{E}_x}{B} = \frac{5\times10^{3}\cos30°}{10^{-3}} = 4.33\times10^{6}\ m/sec \end{cases}$. From Eq. (1-58), $Q = \frac{u}{\omega} = 2.46$ cm

$\theta = \omega t = 1.76\times10^{8}\times5\times10^{-9} = 0.88$ rad $= 50.4°$. Eqs. (1-59) modified for the present coordinate system, are : $x = Q(\theta - \sin\theta) = 2.46(0.88 - \sin50.4°) = 0.27$ cm $\leftarrow$

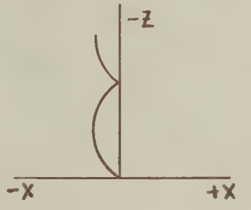
$z = -Q(1-\cos\theta) = -2.46(1-\cos50.4°) = -0.89$ cm $\leftarrow$

(1-45) Assume that B and $\mathcal{E}$ are in the XY plane. The path is a helix in the XZ plane as shown, with constant acceleration along Y.



Eqs. (1-53) $\begin{cases} \omega = \frac{eB}{m} = 1.76\times10^{11}\times10^{-2} = 1.76\times10^{+9}\ rad/sec \\ u = \frac{\mathcal{E}_x}{B} = \frac{10^{5}\sin30°}{10^{-2}} = 5\times10^{6}\ m/sec \end{cases}$

Eqs. (1-58) $\begin{cases} Q = \frac{u}{\omega} = 2.82\times10^{-3}\,m = 0.282\ cm \\ \theta = \omega t = 1.76\times10^{9}\times10^{-9} = 1.76\ rad = 101° \end{cases}$



Eqs. (1-59) $\begin{cases} -x = Q(1-\cos\theta) = 0.282(1-\cos101°) = 0.34\ cm \\ -z = Q(\theta - \sin\theta) = 0.282(1.76 - \sin101°) = 0.22\ cm \end{cases}$

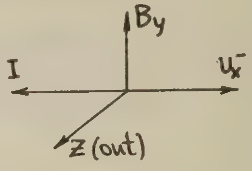
$a_y = \frac{e\mathcal{E}_y}{m} = 1.76\times10^{11}\times10^{5}\cos30° = 1.54\times10^{16}\,m/sec^2$

$y = \frac{1}{2}a_y t^2 = 0.77$ cm ; $d^2 = x^2 + y^2 + z^2 = 0.756$ $\therefore d = 0.87$ cm $\leftarrow$

(1-46) a) $f_x = m\frac{dv_x}{dt} = -e\mathcal{E}_x = e\frac{\partial V}{\partial x}$; $f_y = m\frac{dv_y}{dt} = -e\mathcal{E}_y = e\frac{\partial V}{\partial y}$; $f_z = m\frac{dv_z}{dt} = -e\mathcal{E}_z = e\frac{\partial V}{\partial z}$ $\longleftarrow$

b) $m\frac{dv_x}{dt} = e\frac{\partial V}{\partial x} + ev_z B_y - ev_y B_z$; $m\frac{dv_y}{dt} = e\frac{\partial V}{\partial y} + ev_x B_z - ev_z B_x$ $\longleftarrow$ $\longleftarrow$

$$m\frac{dv_z}{dt} = e\frac{\partial V}{\partial z} + ev_y B_x - ev_x B_y$$



The magnetic forces are obtained as follows: Consider v_x and B_y. The direction rule of Sec. 1-11 gives a force in the $-z$ direction. Similarly, v_x and B_z gives a force in the $+Y$ direction. Also, v_x and B_x gives zero force. In the notation of vector analysis:
$$m\frac{d\vec{v}}{dt} = e\nabla V + e\vec{B}\times\vec{v} \quad \longleftarrow$$

(1-47) a) From Eq. (1-40), $f = \frac{1}{2\pi}\frac{eB}{m} = \frac{1}{2\pi}\frac{1.6\times10^{-19}\times0.9}{1.67\times10^{-27}} = 1.38\times10^{7}\,\text{Hz} = 13.8\text{ MHz}$ $\longleftarrow$

b) The number of energy kicks is $\frac{6\times10^{6}}{6\times10^{4}} = 100$. Since there are two kicks per cycle, the total no. of cycles is 50. The transit time is
$$50T = \frac{50}{1.38\times10^{7}} = 3.62\times10^{-6}\text{ sec} = 3.62\ \mu\text{sec} \quad \longleftarrow$$

c) From Eq. (1-12), $v = \left(2\frac{eV}{m}\right)^{1/2} = \left(\frac{2\times1.6\times10^{-19}\times6\times10^{6}}{1.66\times10^{-27}}\right)^{1/2} = 3.4\times10^{7}$ m/sec

Eq. (1-39) gives: $R = \frac{mv}{eB} = \frac{1.66\times10^{-27}\times3.4\times10^{7}}{1.6\times10^{-19}\times0.9} = 0.391\text{ m} = 39.1\text{ cm}$ $\longleftarrow$

(1-48) a) From Eqs. (1-39) and (1-27), $v = \omega R = \frac{eBR}{\frac{m_o}{\sqrt{1-\beta^2}}}$ where $\beta = \frac{v}{c}$

or $\frac{\beta}{\sqrt{1-\beta^2}} = \frac{eBR}{m_oc} = \frac{1.6\times10^{-19}\times1.3\times0.5}{1.67\times10^{-27}\times3\times10^{8}}$. Solving, we get $\beta = 0.204$

Then $v = \beta c = 0.204\times3\times10^{8} = 6.12\times10^{7}$ m/sec.
$$f = \frac{1}{2\pi}\frac{eB}{m_o}\sqrt{1-\beta^2} = \frac{1}{2\pi}\frac{1.6\times10^{-19}\times1.3}{1.67\times10^{-27}}(1-0.021+\ldots) = 19.4\text{ MHz} \quad \longleftarrow$$

b) $V = \frac{m_oc^2}{e}\left(\frac{m}{m_o}-1\right) = \frac{m_oc^2}{e}\left(\frac{1}{\sqrt{1-\beta^2}}-1\right) = 0.021\frac{m_oc^2}{e}$
$$= \frac{0.021\times1.67\times10^{-27}\times9\times10^{16}}{1.6\times10^{-19}} = 19.7\text{ MeV} \quad \longleftarrow$$

c) Total no. of revolutions $= \frac{3.3\times10^{-6}}{T} = 3.3\times10^{-6}f$
$$= 3.3\times10^{-6}\times19.4\times10^{6} = 64$$ or 128 passages between the dees.

Since the total energy is 19.7 MeV, then the energy per passage is $\frac{19.7\times10^{6}}{128} = 154$ keV $\longleftarrow$

CHAPTER 2

(2-1) a) Angular momentum $= mvr = \frac{nh}{2\pi}$ where n is an integer.

From Eq. (2-1) $v = \left(\frac{e^2}{4\pi m\epsilon_o r}\right)^{1/2}$. Hence, $\frac{mer}{(4\pi m\epsilon_o r)^{1/2}} = \frac{nh}{2\pi}$

or $r = \frac{n^2h^2}{4\pi^2}\,\frac{4\pi m\epsilon_o}{m^2e^2} = \frac{h^2\epsilon_o n^2}{\pi m e^2}$. For $n=1$, we have:

$$r = \frac{(6.625\times10^{-34})^2\,\frac{10^{-9}}{36\pi}}{\pi(9.11\times10^{-31})(1.6\times10^{-19})^2} = 5.3\times10^{-11}\ m = 0.53\ \text{Å} \qquad \text{QED}$$

b) Eq. (2-3), $W_n = -\frac{e^2}{8\pi\epsilon_o r} = -\frac{e^2(\pi m e^2)}{8\pi\epsilon_o^2 h^2 n^2} = -\frac{me^4}{8h^2\epsilon_o^2 n^2}$

c) Using Eq. (2-4), $\frac{1}{\lambda} = \frac{f}{c} = \frac{W_2 - W_1}{hc} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$ where

$$R = \frac{me^4}{8\epsilon_o^2h^3c} = \frac{(9.1\times10^{-31})(1.6\times10^{-19})^4(36\pi\times10^9)^2}{8\,(6.62\times10^{-34})^3(3\times10^8)} = 1.1\times10^7\ m^{-1} \qquad \text{QED}$$

(2-2) From Eq. (2-4), $\lambda = \frac{c}{f} = \frac{ch}{W_2 - W_1}$ cm $= \frac{ch(10^8)}{W_2 - W_1}$ Å ; If E is in eV and W in J, then $W = 1.6\times10^{-19}E$.

Hence, $\lambda = \frac{3\times10^{10}\times6.625\times10^{-34}\times10^8}{1.6\times10^{-19}(E_2 - E_1)} = \frac{12{,}400}{E_2 - E_1}$ QED

(2-3) From Eq. (2-1), $v = \left(\frac{e^2}{4\pi m\epsilon_o r}\right)^{1/2}$; Hence, $T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{e}(4\pi m\epsilon_o)^{1/2}$. From Eq. (2-3), $r = \frac{e^2}{8\pi\epsilon_o(-W)}$ $\therefore\ T = \frac{2\pi}{e}(4\pi m\epsilon_o)^{1/2}\frac{e^3}{[8\pi\epsilon_o(-W)]^{3/2}} = \frac{m^{1/2}e^2}{4\sqrt{2}\,\epsilon_o(-W)^{3/2}}$ QED

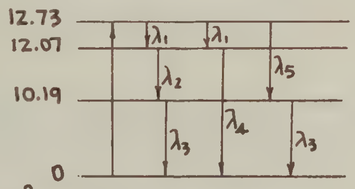
(2-4) From Eq. (2-7), $\Delta E = \frac{12{,}400}{1400} = 8.86$ eV. In Fig. 2-1, the 1850 Å line is from 6.71 eV to zero. Hence, the second photon must be from 8.86 to 6.71 or $\Delta E = 2.15$ eV. $\therefore\ \lambda = \frac{12{,}400}{2.15} = 5770$ Å ⟵

(2-5) From Fig. 2-1 we find that the 2537 Å line results in a transition from the 4.88 eV level. From Eq. (2-7), $\Delta E = \frac{12{,}400}{4078} = 3.05$ eV.

Hence, the initial energy jump must have been from zero to 3.05+4.88 =7.93 eV level, which checks with one of the levels in Fig. 2-1.

$\therefore\ \lambda = \frac{12{,}400}{7.93} = 1560$ Å ⟵

(2-6) From Eq. (2-7), $\Delta E = \frac{12{,}400}{972} = 12.73$ eV



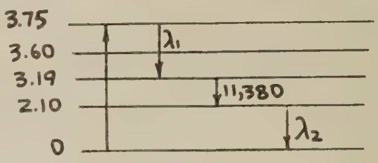
Referring to the sketch we see that three possibilities exist:

(1) λ_1, λ_2 and λ_3 (2) λ_1 and λ_4 (3) λ_3 and λ_5

where $\lambda_1 = \frac{12{,}400}{0.66} = 18{,}800$ Å, $\lambda_2 = \frac{12{,}400}{1.88} = 6{,}600$ Å

$\lambda_3 = \frac{12{,}400}{10.19} = 1220$ Å, $\lambda_4 = \frac{12{,}400}{12.07} = 1030$ Å, $\lambda_5 = \frac{12{,}400}{2.54} = 4880$ Å. ⟵

(2-7) From Eq. (2-7), $\Delta E = \frac{12{,}400}{3300} = 3.75$ eV which corresponds to one of the levels. $\Delta E = \frac{12{,}400}{11{,}380} = 1.09$ eV which corresponds to the difference between the 3.19 eV and 2.10 eV levels. Hence, there must be a jump from 3.75 to 3.19 and then from 2.10 to zero.



a) $\lambda_1 = \frac{12{,}400}{(3.75-3.19)} = 22{,}200$ Å ←

$\lambda_2 = \frac{12{,}400}{2.10} = 5{,}896$ Å ←

b) 3.75 to 3.19 eV and 2.10 to 0. ←

(2-8) a) Mercury does not have an energy level at 5.00 eV and hence the cold vapor will be transparent to the 5.00 eV photon.

b) From Fig. 2-1, there are two energy levels below 5.00eV, namely, 4.66 and 4.88. Hence, the atom may be excited into either of these two states. From the 4.88 eV level the atom may return to the zero state with the emission of the 2537 Å line.

c) Since the 5.46 eV level is metastable, the cold vapor will be transparent to the 5.46 eV photon.

(2-9) a) From Eq. (2-7) the energy of the electron is $\frac{12{,}400}{5893} = 2.11$ eV. From Eq. (1-13), $v = 5.93\times10^5 V^{1/2} = 5.93\times10^5 (2.11)^{1/2} = 8.61\times10^5$ m/sec ←

b) The energy needed to excite the 2537 Å line is $12{,}400/2537 = 4.88$ eV. Since the electrons have only 2.11 eV energy, they cannot excite the 2537 Å line.

(2-10) a) The ionization potential of Ne is 21.5 V. From eq. (1-13),

$$v = 5.93\times10^5 (21.5)^{1/2} = 2.75\times10^6 \text{ m/sec}$$ ←

b) $\lambda = 12{,}400/21.5 = 577$ Å ; $f = \frac{c}{\lambda} = \frac{3\times10^8}{577\times10^{-10}} = 5.2\times10^{15}$ Hz ←

(2-11) a) From Eq. (2-7), $E = 12{,}400/0.5 = 24{,}800$ V ←

b) $\lambda = 12{,}400/60\times10^3 = 0.207$ Å ←

(2-12) From Fig. 2-1 the lowest metastable state of mercury is 4.66 eV. As a result of the impact, the electron loses $7.73 - 4.66 = 3.07$ eV. Its energy drops to $10 - 3.07 = 6.93$ eV $= 6.93\times1.6\times10^{-19} = 1.11\times10^{-18}$ J ←

(2-13) Argon resonance radiation corresponds to an energy of 11.6 eV and sodium ionization potential is 5.12 eV. The electron is ejected with $11.6 - 5.12 = 6.48$ eV.

From Eq. (1-13), $v = 5.93\times10^5 (6.48)^{1/2} = 1.51\times10^6$ m/sec ←

(2-14) a) $E = hf = 6.626\times10^{-34}\times10^{7}\frac{J}{qu.} = \frac{6.626\times10^{-27}}{1.6\times10^{-19}}\ eV/qu. = 4.14\times10^{-8}\ eV/quantum \longleftarrow$

b) $1000\ W = 1000\ J/sec = N\times6.626\times10^{-27}\ J$

$\therefore N = \frac{10^{3}}{6.626\times10^{-27}} = 1.51\times10^{29}\ /sec \qquad \longleftarrow$

c) One cycle $= 10^{-7}$ sec $\quad \therefore N\times10^{-7} = 1.51\times10^{22}/cycle \qquad \longleftarrow$

d) $E = pc$ or $p = \frac{E}{c} = \frac{6.626\times10^{-27}}{3\times10^{8}} = 2.209\times10^{-35}\ \frac{kg\text{-}m}{sec} \qquad \longleftarrow$

(2-15) a) From Eq. (2-8) $\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}}{1\times1} = 6.626\times10^{-34}\ m \qquad \longleftarrow$

b) Using Eqs. (2-8) and (1-12) we have: $\lambda = \frac{h}{m\left(\frac{2eV}{m}\right)^{1/2}} = \frac{h}{(2meV)^{1/2}}$

$\lambda = \frac{6.626\times10^{-34}}{(2\times9.1\times10^{-31}\times1.6\times10^{-19}\times10)^{1/2}} = 3.88\times10^{-10}\ m \qquad \longleftarrow$

(2-16) a) From Eqs. (2-8) and (1-12), $\lambda = \frac{h}{(2meV)^{1/2}} = \frac{6.626\times10^{-34}}{(2\times9.1\times10^{-31}\times1.6\times10^{-19}\times300)^{1/2}} = 7.1\times10^{-11}\ m$

Since vacuum tube dimensions are in mm, the particle is classical.

b) $\lambda = \frac{6.626\times10^{-34}}{(2\times9.1\times10^{-31}\times1.6\times10^{-19}\times2.5\times10^{4})^{1/2}} = 7.65\times10^{-12}\ m$ (neglecting relativistic corrections)

Again the particle is classical. $\longleftarrow$

c) $\lambda = \frac{h}{mv}$. But $mvr = \frac{nh}{2\pi}$ $\quad \therefore mv = \frac{nh}{2\pi r}$ or $\lambda = \frac{2\pi r}{n}$. For small values of n, $\lambda \sim$ dimensions of system. Therefore, in this case the particle is quantum mechanical. $\longleftarrow$

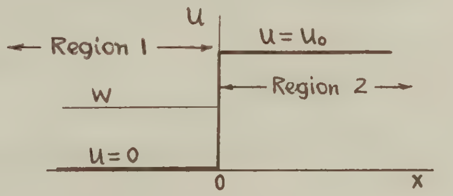
(2-17) a) Ψ must be single-valued in order for the probability density function $|\Psi|^2$ to be physically meaningful. Similarly, if Ψ is not continuous, the probability of finding the particle in a small region around the point of discontinuity is not defined.

b) The Schrödinger equation: $\nabla^2\Psi + V\Psi = E\Psi$ involves the second spatial derivative of Ψ. Thus, if $\frac{\partial\Psi}{\partial x}$ is not continuous, $\frac{\partial^2\Psi}{\partial x^2}$ may not be defined.

c) Since $|\Psi|^2$ is the probability density function, the condition $\int_{\text{all space}} |\Psi|^2 d\tau = 1$ locates the particle to be somewhere in space.

d) Since $\int_{-\infty}^{\infty} |\Psi|^2 d\tau = 1$, Ψ must be finite everywhere.

(2-18) a) In region 1, Eq. (2-14) may be written as $\frac{d^2\psi}{dx^2} = -\frac{8\pi^2 m}{h^2} W\psi$ whose solution is $\psi_1 = C\sin ax + D\cos ax$



(2-18) (Cont'd) Differentiating twice ψ_1 we get $\frac{d^2\psi_1}{dx^2} = -a^2(C\sin ax + D\cos ax)$ or $\frac{d^2\psi_1}{dx^2} = -a^2\psi_1$. Comparing we have $a = \left(\frac{8\pi^2 mW}{h^2}\right)^{1/2}$ $\leftarrow$

In region 2: solution $\psi_2 = Ae^{-x/2d_o} + Be^{x/2d_o}$

$\frac{d^2\psi_2}{dx^2} = \frac{1}{4d_o^2}\left(Ae^{-x/2d_o} + Be^{x/2d_o}\right) = \frac{1}{4d_o^2}\psi_2$. Substituting in Eq. (2-14)

$\frac{1}{4d_o^2}\psi_2 = \frac{8\pi^2 m}{h^2}(U_o - W)\psi_2$ or $d_o = \frac{h}{4\pi}\left[\frac{1}{2m(U_o-W)}\right]^{1/2}$ $\leftarrow$

b) Since ψ must remain finite as $x \to \infty$, then $B = 0$. Since $\psi_1(0) = \psi_2(0)$, then $D = A$

$\therefore \psi_1 = C\sin ax + A\cos ax$ and $\psi_2 = Ae^{-x/2d_o}$

$\frac{d\psi_1}{dx} = aC\cos ax - aA\sin ax$; $\frac{d\psi_2}{dx} = -\frac{A}{2d_o}e^{-x/2d_o}$

Since $\left.\frac{d\psi_1}{dx}\right|_{x=0} = \left.\frac{d\psi_2}{dx}\right|_{x=0}$, then $aC = -\frac{A}{2d_o}$ or $C = \frac{-A}{2ad_o}$

$\therefore \psi_1 = A\left(\cos ax - \frac{1}{2ad_o}\sin ax\right)$ and $\psi_2 = Ae^{-x/2d_o}$

c) The fact that ψ_2 is non zero means that there is a finite probability that the electron can penetrate through the barrier even though $W < U_o$.

CHAPTER 3

(3-1) The atomic weight times the weight of an atom of unit atomic weight equals the weight of the atom under consideration. Hence, AM is the weight of one atom and

$$n = \left(\frac{\text{atoms}}{AM\ \text{kg}}\right)\left(\nu\ \frac{\text{electrons}}{\text{atom}}\right)\left(d\ \frac{\text{kg}}{m^3}\right) = \frac{d\nu}{AM}\ \text{electrons}/m^3$$

If η = number of atoms per molecule, then $A\eta$ = molecular weight = weight of one mole in g and

$$n = \left(\frac{1}{A\eta}\frac{\text{mole}}{g}\right)\left(A_o\frac{\text{molecules}}{\text{mole}}\right)\left(\eta\frac{\text{atoms}}{\text{molecule}}\right)\left(10^3\frac{g}{kg}\right)\left(d\frac{kg}{m^3}\right)\left(\nu\frac{\text{electrons}}{\text{atom}}\right) = \frac{A_o d\nu \times 10^3}{A}\ \text{el}/m^3$$

QED

(3-2) $n = \frac{d\nu}{AM}$ (from Prob. 3-1). Assume $\nu = 1$. Then

$$n = \left(8.9\frac{g}{cm^3}\right)\left(10^{-3}\frac{kg}{g}\right)\left(10^6\frac{cm^3}{m^3}\right)\left(\frac{1\ \text{atom}}{63.57\times 1.66\times 10^{-27}\ kg}\right)\left(1\frac{\text{electron}}{\text{atom}}\right) = 8.41\times 10^{28}\ \text{electrons}/m^3$$

From Eq. (3-3) $\sigma = ne\mu = \left(8.41\times 10^{28}\frac{\text{electrons}}{m^3}\right)\left(1.6\times 10^{-19}\frac{\text{coul}}{\text{electron}}\right)\left(34.8\frac{cm^2}{V\text{-}sec}\right)\left(10^{-4}\frac{m^2}{cm^2}\right)$

$= 4.7\times 10^7 \frac{C}{m\text{-}V\text{-}sec} = 4.7\times 10^7\ (\text{ohm}-m)^{-1}$ $\leftarrow$

ⓐ (3-3) From Prob. 3-1, $n = \frac{d\nu}{AM} = (2.7\frac{g}{cm^3})(3\frac{electrons}{atom})(10^{-3}\frac{kg}{g})(\frac{1 atom}{26.97\times1.66\times10^{-27}})(10^6\frac{cm^3}{m^3})$

$= 18\times10^{28}$ electrons/m^3

Using Eq. (3-3), $\mu = \frac{\sigma}{ne} = \frac{1}{ne\rho}$ or

$$\mu = \frac{1}{(18\times10^{28}\frac{electrons}{m^3})(1.6\times10^{-19}\frac{C}{electron})}\cdot\frac{10^2\frac{cm}{m}}{3.44\times10^{-6}\text{ ohm-cm}} = 10^{-3}\frac{m^2}{V\text{-sec}} \leftarrow$$

(3-4) a) From Eq. (3-2), $v = \frac{J}{ne} = \left[\frac{2A}{(\frac{\pi}{4})(1.03)^2 mm\times10^{-6}\frac{m^2}{mm^2}}\right]\left(\frac{1}{8.4\times10^{28}\frac{electrons}{m^3}}\right)\left(\frac{1}{1.6\times10^{-19}\frac{C}{electron}}\right)$

$= 1.78\times10^{-4}\frac{A\text{-}m}{C} = 1.78\times10^{-4}$ m/sec $\leftarrow$

b) The resistance per meter $= \frac{6.51\ \Omega}{(1000 ft)(12\frac{in}{ft})(2.54\frac{cm}{in})(10^{-2}\frac{m}{cm})} = 0.0213\ \Omega/m \leftarrow$

$\mathcal{E} = (0.0213)(2) = 0.0426$ V/m

Hence, from Eq. (3-1), $\mu = \frac{v}{\mathcal{E}} = \frac{1.78\times10^{-4} m/sec}{0.0426 V/m} = 4.17\times10^{-3}\frac{m^2}{V\text{-sec}}$

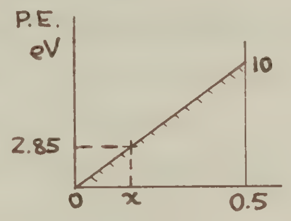
c) From Eq. (3-3), $\sigma = ne\mu$

$= (8.4\times10^{28})(1.6\times10^{-19})(4.17\times10^{-3}) = 5.61\times10^{7}\ (\Omega\text{-}m)^{-1} \leftarrow$

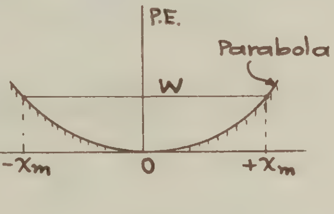
(3-5) a) From Eq. (1-13), $V = \left(\frac{v}{5.93\times10^5}\right)^2 = \left(\frac{10^8}{5.93\times10^5}\right)^2 = 2.85$ V

$x = \frac{2.85}{10}(0.5) = 0.143$ cm $\leftarrow$

b) $v = 5.93\times10^5 V^{1/2} = 5.93\times10^5(10)^{1/2} = 1.88\times10^8$ m/sec $\leftarrow$



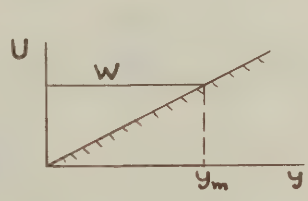
(3-6) Potential energy, $U = -\int_0^x f dx = \int_0^x kx dx = \frac{1}{2}kx^2$
with $U = 0$ at $x = 0$.
For a total energy W, the particle is trapped between x_m and $-x_m$ where $x_m = (\frac{2W}{k})^{1/2}$



(3-7) a) Potential energy, $U = mgy$. For $W = \frac{1}{2}mv_0^2 = U$, the velocity v is zero and the particle reverses its direction. At $U = W$, $y = y_m$ or

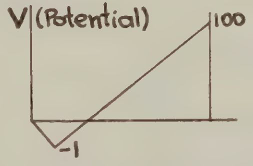
$\frac{mv_0^2}{2} = mgy_m \quad \therefore\ y_m = \frac{v_0^2}{2g}$ QED

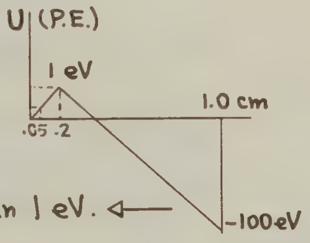
b) From the sketch it is clear that $U = W$ corresponds to a collision with the barrier.



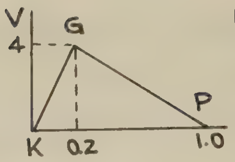
(3-8) a) $W = (\frac{0.05}{0.2})(1) = 0.25$ eV $\leftarrow$

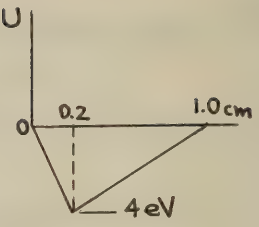
b) The electron must have an energy greater than the maximum of the potential energy barrier or W must be greater than 1 eV. $\leftarrow$





(3-9) a) From the sketch we see that for any $W>0$ there is no collision with the potential energy barrier.





b) Between K and G

$a_1 = \frac{e\mathcal{E}}{m} = 1.76\times10^{11}\frac{4}{2\times10^{-3}} = 3.52\times10^{14}$ m/sec²

$x_1 = \frac{1}{2}a_1t_1^2 = 1.76\times10^{14}t_1^2 = 2\times10^{-3}$ $\therefore$ $t_1 = 3.37\times10^{-9}$ sec = 3.37 nsec

At G, $v = v_0 = a_1t_1 = (3.52\times10^{14})(3.37\times10^{-9}) = 11.85\times10^{5}$ m/sec. As a check:

$v = 5.93\times10^{5}(4)^{1/2} = 11.85\times10^{5}$ m/sec.

Between G and P, $a_2 = -(1.76\times10^{11})\frac{4}{8\times10^{-3}} = -0.88\times10^{14}$ m/sec²

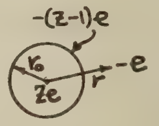
$x_2 = v_0t + \frac{1}{2}a_2t^2 = 8\times10^{-3} = 11.85\times10^{5}t - 0.44\times10^{14}t^2$. Solving gives $t = t_2 = 13.45$ nsec

Total time is $t_1 + t_2 = 3.37 + 13.45 = 16.82$ nsec ⟵

c) $v = v_0 + a_2t_2 = 11.85\times10^{5} - (0.88\times10^{14})(13.45\times10^{-9}) = 0$ ⟵

Note: This result is obvious since the plate is at the cathode potential.

(3-10) a) For $r > r_0$, the total charge enclosed by a sphere of radius r is $+e$ and by Gauss' law



$4\pi r^2\epsilon_0\mathcal{E} = e$ or $\mathcal{E} = \frac{e}{4\pi r^2\epsilon_0}$;

$U = -eV = +\int e\mathcal{E}dr = -\frac{e^2}{4\pi\epsilon_0 r} + C_1$. Since $U=0$ at $r=\infty$, then $C_1 = 0$

and $4\pi\epsilon_0 U = -\frac{e^2}{r}$ QED

For $r < r_0$, the total charge enclosed by a sphere of radius r is $+Ze$ and by Gauss' law $\mathcal{E} = \frac{Ze}{4\pi\epsilon_0 r^2}$; $U = \int e\mathcal{E}dr = -\frac{Ze^2}{4\pi\epsilon_0 r} + C_2$

at $r = r_0$, $U = \frac{-e^2}{4\pi\epsilon_0 r_0}$ and $C_2 = \frac{(Z-1)e^2}{4\pi\epsilon_0 r_0}$

Hence, $4\pi\epsilon_0 U = -\frac{Ze^2}{r} + (Z-1)\frac{e^2}{r_0}$ QED

b) $y = \frac{4\pi\epsilon_0 U r_0}{e^2} = -\frac{Zr_0}{r} + (Z-1)$

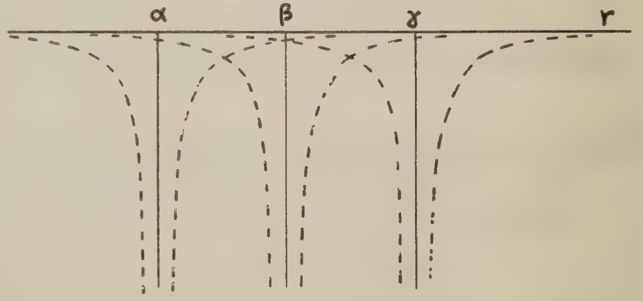
For $Z = 11$ and $x = \frac{r}{r_0}$

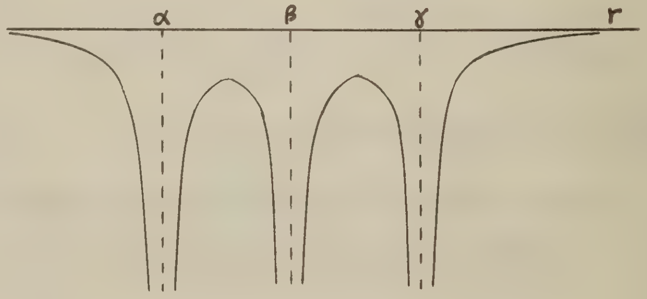
we get:

$y = -\frac{11}{x} + 10$ for $x \leq 1$

and

$y = -\frac{1}{x}$ for $x \geq 1$





Note: The upper plot shows U due to the individual ions. The lower plot shows the resultant U.

(3-11) a) At 0°K $dn_E = \rho_E\, dE = \gamma E^{1/2} dE$, Eqs. (3-6) and (3-11). From Eq. (3-9) $\gamma = 6.82\times10^{27}$. Take $E = 8.55$ eV. $dE = 8.6 - 8.5 = 0.1$; $E_F = 8.95$ eV p. 64.

$dn_E = (6.82\times10^{27})(8.55)^{1/2}(0.1) = 2\times10^{27}$ electrons/m³ ⟵

b) At 2500°K, $y \equiv e^{\frac{(E-E_F)}{kT}} = e^{\frac{(8.55-8.95)}{8.62\times10^{-5}\times2500}} = 0.156$

From Eqs. (3-6) and (3-7)

$$dn_E = \frac{\gamma E^{1/2} dE}{1+y} = \frac{2\times10^{27}}{1.156} = 1.73\times10^{27} \text{ electrons/m}^3 \longleftarrow$$

(3-12) a) $n = 6.02\times10^{23}\,\frac{\text{molecules}}{\text{mole}} \times \frac{1\text{ mole}}{26.97\text{ g}} \times 2.7\,\frac{\text{g}}{\text{cm}^3} \times 3\,\frac{\text{electrons}}{\text{atom}} \times \frac{1\text{ atom}}{\text{molecule}} \times 10^6\,\frac{\text{cm}^3}{\text{m}^3}$

$= 1.81\times10^{29}$ electrons/m³

From Eq. (3-13), $E_F = 3.64\times10^{-19}\times(1.81\times10^{29})^{2/3} = 11.6$ eV ⟵

b) $n = 6.02\times10^{23}\,\frac{\text{molecules}}{\text{mole}} \times \frac{1\text{ mole}}{107.9\text{ g}} \times 10.5\,\frac{\text{g}}{\text{cm}^3} \times 1\,\frac{\text{electron}}{\text{molecule}} \times 10^6\,\frac{\text{cm}^3}{\text{m}^3}$

$= 5.87\times10^{28}$ electrons/m³

$\therefore\ E_F = 3.64\times10^{-19}(5.87\times10^{28})^{2/3} = 5.5$ eV ⟵

(3-13) a) $E_{av} = \dfrac{\text{Energy of all electrons}}{\text{Total no. of electrons}}$; but the energy of dn_E electrons is $E\,dn_E$

The energy of all the electrons is $\int E\,dn_E$ and the total no. of electrons is $\int dn_E$ $\quad\therefore\ E_{av} = \dfrac{\int E\,dn_E}{\int dn_E}$

b) From Eqs. (3-6) and (3-11)

$$E_{av} = \frac{\int \rho_E E\,dE}{\int \rho_E\,dE} = \frac{\int_0^{E_F} \gamma E^{3/2} dE}{\int_0^{E_F} \gamma E^{1/2} dE} = \frac{\frac{2}{5}E_F^{5/2}}{\frac{2}{3}E_F^{3/2}} = \frac{3}{5}E_F \qquad \text{QED}$$

(3-14) From Eq. (3-29), $I_{th} = SA_0T^2e^{-E_W/kT} = SA_0(1500)^2e^{-\frac{E_W}{8.62\times10^{-5}\times1500}}$

$10^4 I_{th} = SA_0(2000)^2 e^{-\frac{E_W}{8.62\times10^{-5}\times2000}}$

Taking the ratio of these two equations:

$10^4 = \left(\frac{20}{15}\right)^2 e^{(7.74-5.8)E_W}$ from which we get $E_W = 4.45$ eV ⟵

(3-15) a) From Eq. (3-31), $\dfrac{dI_{th}}{I_{th}} = \left(2 + \dfrac{E_W}{kT}\right)\dfrac{dT}{T} = \left(2 + \dfrac{4.52}{8.62\times10^{-5}\times2310}\right)\dfrac{20}{2310} = 21.4\%$ ⟵

b) From Eq. (3-29)

$I_{th} = SA_0(2300)^2 e^{-\frac{4.52}{8.62\times10^{-5}\times2300}}$; $2I_{th} = SA_0(T^2)\, e^{-\frac{4.52}{8.62\times10^{-5}T}}$

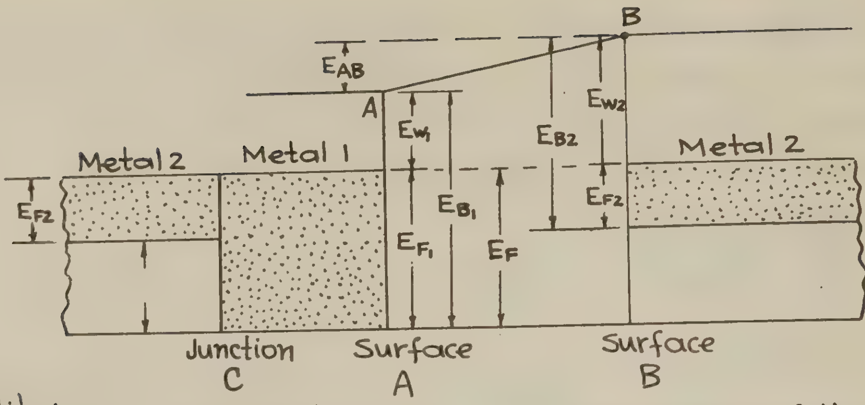
The ratio of these equations gives: $2 = \left(\frac{T}{2300}\right)^2 e^{-\frac{52{,}400}{T}+22.8}$

Taking the logarithm to the base 10 and simplifying we have:

$$\frac{22{,}800}{T} = 9.6 + 2\log\frac{T}{2300}$$

This is solved by trial and error to obtain T = 2370°K ⟵

(3-16) a)



b) Equilibrium is attained when the Fermi levels of the two metals are located at the same energy level E_F. Then from the diagram above we see that $E_{AB} = E_{W2} - E_{W1}$. QED

(3-17) From Eq. (3-33), $I = I_{th} e^{-V_r/V_T}$; $0.1\, I_{th} = I_{th} e^{-\frac{11,600}{2000} V_r}$

or $e^{5.8 V_r} = 10$ $\therefore V_r = 0.40$ V ⟵

(3-18) From Eq. (3-33), $I = I_{th} e^{-V_r/V_T}$ or $\frac{I}{I_{th}} = e^{-(4.75-4.50)\frac{(11,600)}{2000}} = e^{-1.45} = 0.235$ ⟵

(3-19) The effective retarding potential is $5.00 - 3.00 = 2.00$ V. From Eq. (1-13) $v = 5.93 \times 10^5 V^{1/2} = 5.93 \times 10^5$ or $V = 1$ V. ∴ The electron travels half the distance from K to A, or 1 cm ⟵

(3-20) From Eq. (3-33), $I_1 = I_{th} e^{-V_r/V_T}$; If the retarding potential is decreased by 0.2 V, the current is $I_2 = I_{th} e^{-(V_r - 0.2)/V_T}$. Dividing these two equations we have:

$$\frac{I_2}{I_1} = e^{\frac{(0.2)(11,600)}{1000}} = e^{2.32} = 10 \qquad \text{QED}$$

(3-21) a) At zero applied voltage, the true potential is 0.5 V retarding. From Eq. (3-33) $I = I_{th} e^{-V_r/V_T} = 10 e^{-\frac{0.5 \times 11,600}{1500}} = 10^{-3.87} = 0.21\ \mu A$ ⟵

b) $1 = 10 e^{-\frac{11,600 V_r}{1500}}$ or $V_r = 0.3$ = net retarding voltage. ∴ The applied voltage is $0.5 - 0.3 = 0.2$ V accelerating. ⟵

c) $\frac{I}{I_{th}} = e^{-V_r/V_T} = e^{-\frac{(1+0.5)(11,600)}{1500}} = e^{-11.6} = 9.3 \times 10^{-6}$ ⟵

(3-22) From Eq. (3-40), $\frac{I}{I_{th}} = e^{\frac{0.44 \mathcal{E}^{1/2}}{2500}} = 1.01$. Since for $x \ll 1$, $\ln(1+x) \approx x$ then $\ln(1.01) \approx 0.1 = 0.44 \mathcal{E}^{1/2}/2500$ or $\mathcal{E} = 3,240$ V/m ⟵

(3-23) a, c, e, f

(3-24) a, e

CHAPTER 4

(4-1) From Eq. (3-29), $I_{th} = SA_oT^2e^{-\frac{E_W}{kT}}$ where A_o and E_W are given in Table 4-1.
$I_{T\text{-}W} = S\,(3.0)(1840)^2\,e^{-\frac{2.63}{8.62\times10^{-5}\times1840}}$; $I_W = 143\times10^{-6} = S\,(60.2)(1840)^2\,e^{-\frac{4.52}{8.62\times10^{-5}\times1840}}$
Taking the ratio $\frac{I_{T\text{-}W}}{I_W} = \frac{3.0}{60.2}e^{11.9}$ or $I_{T\text{-}W} = 143\times10^{-6}\times\frac{3.0}{60.2}e^{11.9} = 7.13\times10^{-6}e^{11.9}$
or $I_{T\text{-}W} = 1.03\ A$ ⟵

(4-2) $P = 5.8\times10^4\ W/m^2$. If 10% is lost by conduction, then the total input power density is $(1.1)(5.8\times10^4) = 6.38\times10^4\ W/m^2$ and the total input power is $6.38\times10^4\times1.8\times10^{-4} = 11.5\ W$
From Eq. (3-29), $I_{th} = SA_oT^2e^{-E_W/kT} = 1.8\times10^{-4}\times100\times(1100)^2\,e^{-\frac{1}{8.62\times10^{-5}\times1100}}$
$= 2.18\times10^4\,e^{-10.55}$
or $\log I_{th} = 4.338 - 0.434\times10.55 = -0.241$ $\therefore$ $I_{th} = 0.575\ A$ ⟵
Cathode efficiency $= \frac{\text{emission current}}{\text{input power}} = \frac{0.575}{11.5} = 50\ mA/W$ ⟵

(4-3) The surface area is $S = (2)(0.014+0.004)(4.5)(2.54)^2(10^{-4}) = 10.45\times10^{-5}\ m^2$
From Eq. (3-29), $I_{th} = SA_oT^2e^{-E_W/kT} = 10.45\times10^{-5}\times100\times(1200)^2\times e^{-\frac{1}{8.62\times10^{-5}\times1200}}$
or $I_{th} = 0.98\ A$ ⟵

(4-4) From Eq. (3-29) and Table 4-1, $J_{th} = A_oT^2e^{-\frac{E_W}{kT}} = 60.2\times10^4\times(2300)^2\,e^{-\frac{4.52}{8.62\times10^{-5}\times2300}}$
$= 407\ A/m^2$
From Eq. (3-33), $J = J_{th}\,e^{-V_r/V_T}$
or $10^{-2} = 407\,e^{-\frac{11{,}600V_r}{2300}}$. Taking logs : $-2 = 2.61 - (0.434)(5.03)V_r$
or $V_r = 2.10\ V$ ⟵

(4-5) From Eq. (3-33), $I = I_{th}\,e^{-V_r/V_T}$; $0.235 I_{th} = I_{th}\,e^{-\frac{11{,}600}{2000}V_r}$ or $e^{-5.8V_r} = 0.235$
or $V_r = 0.25\ V$. Now $E_r = eV_r = 0.25\ eV$ $\therefore$ $E_W = 4.75 - 0.25 = 4.50\ eV$ ⟵

(4-6) From Eq. (4-10), $\frac{V^{3/2}}{x^2} = \text{const.} = \frac{V_P^{3/2}}{d^2}$ or $V = V_P\left(\frac{x}{d}\right)^{4/3}$ QED

From Eq. (4-3), $v = \left(\frac{2eV}{m}\right)^{1/2} = \left(\frac{2eV_P}{m}\right)^{1/2}\left(\frac{x}{d}\right)^{2/3} = v_P\left(\frac{x}{d}\right)^{2/3}$ where $v_P = \left(\frac{2eV_P}{m}\right)^{1/2}$
and v_P is the velocity when $x = d$ (at the plate)
From Eqs. (4-2) and (4-10), $\rho = \frac{J}{v} = \frac{\frac{4}{9}\left(2\frac{e}{m}\right)^{1/2}\epsilon_o\frac{V_P^{3/2}}{d^2}}{\left(\frac{2eV_P}{m}\right)^{1/2}\left(\frac{x}{d}\right)^{2/3}} = \rho_P\left(\frac{x}{d}\right)^{-2/3}$

where $\rho_P = \frac{4}{9}\epsilon_o\frac{V_P}{d^2} = \frac{4}{9}\,\frac{1}{36\pi\times10^9}\,\frac{V_P}{d^2} = V_P/(81\pi d^2\times10^9)$
and ρ_P is the charge density at $x = d$. QED

(4-7) From Prob. 4-6 under space-charge conditions

$v = v_P\left(\frac{x}{d}\right)^{2/3} = \frac{dx}{dt}$ or $\int x^{-2/3}\,dx = \frac{v_P}{d^{2/3}}\int dt$; $3x^{1/3} = \frac{v_P}{d^{2/3}}\,t$

If at $x=d$, $t=T$ = transition time, then $T = 3\frac{d^{1/3}d^{2/3}}{v_P} = \frac{3d}{v_P}$ QED

For zero space charge the acceleration is constant and the average speed is $\frac{1}{2}v_P$. Hence, $d=\frac{1}{2}v_P T$ or $T=\frac{2d}{v_P}$.

(4-8) From Eqs. (4-10) and (4-11), $V = V_P\left(\frac{x}{d}\right)^{4/3}$. The slope of the line tangent to this curve at $x=d$ is

$$\left.\frac{dV}{dx}\right|_{x=d} = \left.\frac{4}{3}\frac{V_P}{d^{4/3}}\,x^{1/3}\right|_{x=d} = \frac{4}{3}\frac{V_P}{d}$$

The equation of this tangent is $V = \frac{4}{3}\frac{V_P}{d}x + B$ where B is determined so that $V=V_P$ at $x=d$. $\therefore B = V_P - \frac{4}{3}V_P = -\frac{1}{3}V_P$

Thus, $V = \frac{1}{3}V_P\left(\frac{4x}{d}-1\right)$ and when $V=0$, $x=\frac{d}{4}$ QED.

(4-9) a) From Eq. (4-11), $I = 2.33\times10^{-6}\,S\,V_P^{3/2}/d^2$ or $\frac{2}{1} = \left(\frac{V_P}{100}\right)^{3/2}$ $\therefore V_P = 159$ V ←

b) $\frac{I}{10} = (2)^{3/2}$ or $I = 28.3$ mA ←

c) $I = 2.33\times10^{-6}(2S)\frac{(100)^{3/2}}{(d/2)^2}$; $10 = 2.33\times10^{-6}S\frac{(100)^{3/2}}{d^2}$

Taking the ratio of these two equations, we have $\frac{I}{10} = 8$

or $I = 80$ mA ←

(4-10) From Eq. (4-11), $J = 2.33\times10^{-6}\frac{V_P^{3/2}}{d^2} = \frac{2.33\times10^{-6}\times(100)^{3/2}}{4\times10^{-4}} = 5.83\ \text{A/m}^2 = \text{const.}$

Since $J=\rho v$, we must find v at $x=1$ cm. From Eq. (4-10), $V=V_P\left(\frac{x}{d}\right)^{4/3}$.

At $x=\frac{d}{2}$, $V = 100\left(\frac{1}{2}\right)^{4/3} = 39.7$ V and $v = 5.93\times10^{5}(39.7)^{1/2} = 3.73\times10^{6}$ m/sec.

$\rho = \frac{J}{v} = \frac{5.83}{3.73\times10^{6}} = 1.57\times10^{-6}\ \text{C/m}^3$ ←

(4-11) For an applied voltage of 50 V, the thermionic emission current will be collected as the temperature is increased from zero until the temperature becomes high enough so that the thermionic emission current equals the space-charge current. Then the field at the cathode becomes zero. From Eqs. (3-29) and (4-12),

$$I_{th} = GV_P^{3/2} = SA_0T^2e^{-\frac{E_W}{kT}} \quad \text{or} \quad G(50)^{3/2} = SA_0(1000)^2e^{-\frac{12,000}{1000}} \qquad (1)$$

Now, if V_P remains at 50 V and the temperature is increased, the electric field will remain zero at the cathode and space-charge limited current will be collected. Now let T be increased to 1200°K and held fixed. Then, if V_P is increased, the maximum V_P for which the field remains zero at the cathode is the one for which the space-charge current equals the 1200°K thermionic emission current.

(4-11) (Cont'd) $$GV_P^{3/2} = SA_0(1200)^2 e^{-\frac{12,000}{1200}} \qquad (2)$$

Dividing (2) by (1) we have

$$\left(\frac{V_P}{50}\right)^{3/2} = \left(\frac{1200}{1000}\right)^2 e^{-10+12} = 1.44e^2 = 10.65$$

or $V_P = (50)(10.65)^{2/3} = (50)(4.85) = 242$ V ⟵

(4-12) a) The supply voltage across each tube is 25V. ⟵
The space-charge current is from Eq. (4-11), $I = 20\left(\frac{25}{100}\right)^{3/2} = 2.5$ mA ⟵

b) The voltage across each tube is $V_P = 150$V ⟵
Hence, the space-charge current exceeds 20mA. Thus, the temperature-limited current will be collected, $I = 20$ mA ⟵

c) The tube at 2300°K can deliver only 20mA. Hence, the current through the other tube in series is also 20 mA. The tube at 3000°K is operating under space-charge limited conditions and, hence, to deliver 20 mA, the voltage across this tube is 100 V.
∴ the voltage across the 2300°K tube is 300−100= 200 V. ⟵

(4-13) a) For a vacuum diode under space-charge limited conditions Eq. (4-12) applies, namely, $I_P = GV_P^n$. Taking the logarithm of both sides, we have:

$$\log I_P = \log G + n \log V_P$$

Thus, a plot of $\log I_P$ vs. $\log V_P$ is a straight line whose slope is n.

b) From such a log-log plot we find that for the 5Y3-GT diode $n = 1.46$. ⟵
Thus, this diode satisfies Eq. (4-12) with $n = \frac{3}{2}$ very closely.

(4-14) a) From Eq. (4-11), $J = 2.33\times10^{-6}\frac{V_P^{3/2}}{d^2} = 2.33\times10^{-6}\frac{(100)^{3/2}}{10^{-4}} = 23.3$ A/m²
$P = JV_P = 23.3\times100 = 2330$ W/m² ⟵

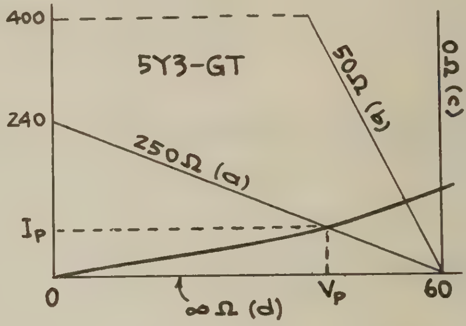
b) J is multiplied by $4^{3/2} = 8$. V_P is multiplied by 4.
∴ P is multiplied by $8\times4 = 32$ ⟵

(4-15) From Eq. (4-12), $I_P = GV_P^{3/2}$ ∴ $G = \frac{5\times10^{-3}}{100^{3/2}} = 5\times10^{-6}$
Plate dissipation $= I_PV_P = (5\times10^{-6})V_P^{5/2} = 16$ or $V_P = 400$V ⟵

(4-16) From Eq. (4-11), $I = (S)(2.33\times10^{-6})\frac{V_P^{3/2}}{d^2}$; If in a plane-parallel diode all dimensions are enlarged by the same factor K, the area is multiplied by K² and d² is multiplied by K². Hence, I remains the same.

(4-17) At $v_p = 50\,V$, $i_p \doteq 100\,mA$ $\quad \therefore\ R = \frac{v_p}{i_p} = \frac{50}{0.1} = 500\,\Omega$ ←

$R_f = r_p = \frac{dv_p}{di_p}$. A tangent to the V-I characteristic at $V_p = 50V$ gives $R_f = r_p = 350\,\Omega$ ←



(4-18) a) From the load line through $v_p = 60V$, $i_p = 0$ and $v_p = 0$, $i_p = \frac{60}{.250K} = 240\,mA$, we find

$I_p = 75\,mA$, $V_p = 42\,V$ ←

b) The load line is $v_p = V_{PP} - i_p R_L$ and for $I_p = 0.4\,A$, $v_p = 60 - 0.4 \times 50 = 40\,V$. The L.L. is drawn as shown and we find

$I_p = 112\,mA$, $V_p = 54\,V$ ←

c) From a vertical L.L. we find, $I_p = 126\,mA$, $V_p = 60V$ ←

d) From a horizontal L.L., $I_p = 0$, $V_p = 0$. ←

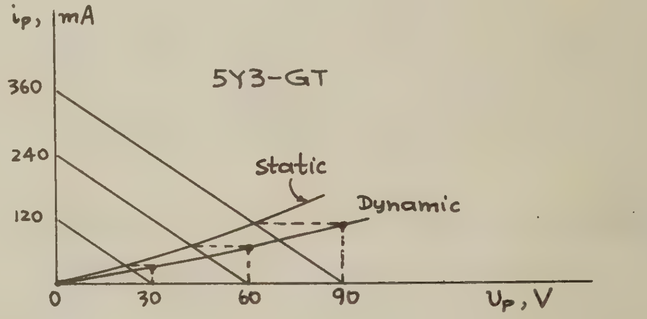
(4-19) The equation describing the load line is $v_p = 120 - i_p R_L$. If we plot the load line for each load resistor on the V-I characteristics of 5Y3-GT (Fig. D-1), we find:

a) $R_L = 500\,\Omega$, $I_p = 124\,mA$, $V_p = 58\,V$; b) $R_L = 200\,\Omega$, $I_p = 200\,mA$, $V_p = 80V$ ←

c) $R_L = 0\,\Omega$, $I_p = 350\,mA$, $V_p = 120V$; d) $R_L = \infty$, $I_p = 0$, $V_p = 0$ ←

(4-20)

The construction of the dynamic characteristic is indicated in the sketch.



(4-21) From Eq. (4-12), $I_p = G V_p^{3/2}$; $\quad G = \frac{0.1}{(100)^{3/2}} = 10^{-4}$. $\quad R = \frac{V_p}{I_p} = \frac{V_p}{G V_p^{3/2}} = 10^4 V_p^{-1/2}$

$r_p = \frac{dV_p}{dI_p} = \left(\frac{3}{2} G V_p^{1/2}\right)^{-1} = \left(\frac{3}{2}\frac{1}{R}\right)^{-1}$ or $r_p = \frac{2}{3} R$

a) For $V_p = 100V$, $R = 10^4 (100)^{-1/2} = 1000\,\Omega = 1K$, $r_p = 667\,\Omega$ ←

b) For $V_p = 50V$, $R = 10^4 (50)^{-1/2} = 1410\,\Omega$, $r_p = 940\,\Omega$ ←

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CHAPTER 5

(5-1) Assuming that μ is independent of temperature, then it follows from Eq. (5-2) that σ varies as n_i. From Eq. (5-3),

$$n_i = A_o^{1/2} T^{3/2} e^{-E_{GO}/2kT} \quad \text{or} \quad \ln n_i = \ln A_o^{1/2} + \frac{3}{2}\ln T - \frac{E_{GO}}{2kT}$$

$$\frac{dn_i}{n_i} = \frac{3}{2}\frac{dT}{T} + \frac{E_{GO}}{2kT^2}dT = \left(\frac{3}{2} + \frac{E_{GO}}{2kT}\right)\frac{dT}{T} \qquad @\ T = 300^\circ K,\ kT = 0.026$$

For Ge: $\dfrac{dn_i}{n_i} = \left(1.5 + \dfrac{0.785}{0.052}\right)\left(\dfrac{1}{300}\right)(100\%) = 5.5\ \%$ per degree ←

(5-2) Using the result of Prob. 5-1 and Table 5-1, we have

For Si: $\dfrac{dn_i}{n_i} = \left(1.5 + \dfrac{1.21}{0.052}\right)\left(\dfrac{1}{300}\right)(100\%) = 8.3\%$ per degree ←

(5-3) From Eq. (5-8), $n = \gamma\displaystyle\int_{E_c}^{\infty}(E-E_c)^{1/2} e^{-\frac{(E-E_F)}{kT}} dE$. Let $x \equiv \dfrac{E-E_c}{kT}$, $dx = \dfrac{dE}{kT}$

$E - E_F = (E-E_c) + (E_c - E_F)$. $\therefore\ n = \gamma(kT)^{3/2}\displaystyle\int_0^{\infty} x^{1/2} e^{-x} e^{-\frac{(E_c-E_F)}{kT}} dx$

$$= \gamma(kT)^{3/2} e^{-\frac{(E_c-E_F)}{kT}} \underbrace{\int_0^{\infty} x^{1/2} e^{-x} dx}_{y}$$

From a table of integrals we find that

$$y = \int_0^{\infty} x^{m-1} e^{-x} dx = \Gamma(m) \quad \text{where } \Gamma \text{ is the Gamma function.}$$

For $m = \frac{3}{2}$, $y = \Gamma(\frac{3}{2}) = \frac{1}{2}\sqrt{\pi}$

$$\therefore\ n = \frac{\gamma(kT)^{3/2}\sqrt{\pi}}{2} e^{-\frac{(E_c-E_F)}{kT}} = N_c e^{-\frac{(E_c-E_F)}{kT}}$$

where $N_c = \dfrac{\gamma(kT)^{3/2}\sqrt{\pi}}{2}$. From Eq. (3-9), $\gamma = \dfrac{4\pi}{h^3}(2m_n)^{3/2}(1.60\times10^{-19})^{3/2}$

so $N_c = \dfrac{2\pi^{3/2}}{h^3}(2m_n kT)^{3/2}(1.60\times10^{-19})^{3/2} = 2\left(\dfrac{2\pi m_n kT}{h^2}\right)^{3/2}(1.60\times10^{-19})^{3/2}$

where k is in eV/°K.

(5-4) From Eq. (5-13), $p = \displaystyle\int_{-\infty}^{E_V}\gamma(E_V-E)^{1/2} e^{-\frac{(E_F-E)}{kT}} dE$. Let $x \equiv \dfrac{E_V - E}{kT}$, $dx = -\dfrac{dE}{kT}$

$E_F - E = (E_V - E) + (E_F - E_V)$

$$p = -\gamma\int_{\infty}^{0} x^{1/2}(kT)^{3/2} e^{-x} e^{-\frac{(E_F-E_V)}{kT}} dx = \gamma(kT)^{3/2} e^{-\frac{(E_F-E_V)}{kT}} \underbrace{\int_0^{\infty} x^{1/2} e^{-x} dx}_{y}$$

From a table of integrals we find that

$y = \displaystyle\int_0^{\infty} x^{m-1} e^{-x} dx = \Gamma(m)$ where Γ is the Gamma function.

for $m = 3/2$, $\Gamma(3/2) = \frac{\sqrt{\pi}}{2}$ $\quad\therefore\ p = \dfrac{\gamma(kT)^{3/2}\sqrt{\pi}}{2} e^{-\frac{E_F-E_V}{kT}} = N_V e^{-\frac{(E_F-E_V)}{kT}}$

where $N_V = 2\left(\dfrac{2\pi m_p kT}{h^2}\right)^{3/2}$

⑤-⑤ From Eqs. (5-10) and (5-16) we have:

$$\frac{kT}{2}\ln\frac{N_c}{N_v} = \frac{3}{4}kT\ln\frac{m_n}{m_p} = \frac{3}{4}(26)\ln 2 \text{ meV} = 13.5 \text{ meV}$$

⑤-⑥ a) From Eq. (5-10), $N_c = 2\left(\frac{2\pi m_n kT}{h^2}\right)^{3/2}(1.60\times10^{-19})^{3/2} = \alpha\left(\frac{m_n}{m}\right)^{3/2}T^{3/2}$

where $\alpha \equiv 2\left(\frac{2\pi k m}{h^2}\right)^{3/2}(1.60\times10^{-19})^{3/2} = 2\left(\frac{2\pi\times8.62\times10^{-5}\times9.1\times10^{-31}}{(6.626\times10^{-34})^2}\right)^{3/2}(16)^{3/2}(10^{-30})$

or $\alpha = 4.82\times10^{21}$ $\quad\therefore\ N_c = 4.82\times10^{21}\left(\frac{m_n}{m}\right)^{3/2}T^{3/2}$ .... Eq. (5-20)

From Eqs. (5-18) and (5-19) we have $n_i^2 = np = N_c N_v e^{-E_G/kT}$
where N_c and N_v are given by Eq. (5-10), namely,
$N_c = \alpha\left(\frac{m_n}{m}\right)^{3/2}T^{3/2}$ and $N_v = \alpha\left(\frac{m_p}{m}\right)^{3/2}T^{3/2}$

$$\therefore\ n_i^2 = \alpha^2\left(\frac{m_n m_p}{m^2}\right)^{3/2}T^3 e^{-E_G/kT} = (4.82\times10^{21})^2\left(\frac{m_n m_p}{m^2}\right)^{3/2}T^3 e^{-\frac{E_G}{kT}}$$

or $n_i^2 = 2.33\times10^{43}\left(\frac{m_n m_p}{m^2}\right)^{3/2}T^3 e^{-E_G/kT}$ ..... Eq. (5-21)

b) From Eq. (5-20)

$$\left(\frac{m_n m_p}{m^2}\right)^{3/2} = \frac{n_i^2}{2.33\times10^{43}T^3}e^{E_G/kT} = 1.59\times10^{-51}n_i^2 e^{38.5E_G} \quad \text{at } T = 300^\circ K$$

or $\left(\frac{m_n m_p}{m^2}\right) = 1.36\times10^{-34}n_i^{4/3}e^{25.6E_G}$. Using Table 5-1, we have:

For Ge

$$\frac{m_n m_p}{m^2} = 1.36\times10^{-34}(2.5\times10^{19})^{4/3}e^{25.6\times0.72} = 10.4\times10^{-1} = 1.04$$

For Si

$$\frac{m_n m_p}{m^2} = 1.36\times10^{-34}(1.5\times10^{16})^{4/3}e^{25.6\times1.1} = 8.9\times10^{-1} = 0.89$$

⑤-⑦ a) Use CGS units. From Eq. (5-2), $\sigma = n_i e(\mu_n + \mu_p)$. From Table 5-1:
or $\sigma = (2.5\times10^{13})(1.6\times10^{-19})(3800+1800) = 0.0224\ (\Omega\text{-cm})^{-1}$

$\therefore\ \rho = \frac{1}{\sigma} = 44.6\ \Omega\text{-cm}$ ←

b) $n_{Ge} = 6.02\times10^{23}\frac{\text{atoms}}{\text{mole}}\times1\frac{\text{mole}}{72.6\text{ g}}\times5.32\frac{\text{g}}{\text{cm}^3} = 4.41\times10^{22}\text{ atoms/cm}^3$
If there is 1 donor atom per 10^8 Ge atoms, then $N_D = 4.41\times10^{14}$ atoms/cm³
From Eq. (5-25) $n \approx N_D$ and from Eq. (5-27), $p = \frac{n_i^2}{N_D} = \frac{(2.5\times10^{13})^2}{4.41\times10^{14}}$
or $p = 1.42\times10^{12}$ holes/cm³. Since n is two orders of magnitude greater than p, we can neglect p. Then $\sigma = ne\mu_n$
or $\sigma = 4.41\times10^{14}\times1.6\times10^{-19}\times3800 = 0.268\ (\Omega\text{-cm})^{-1}$

$\therefore\ \rho = \frac{1}{\sigma} = 3.72\ \Omega\text{-cm}$ ←

(5-8) a) Use CGS units. From Eq. (5-2), $\sigma = n_i e(\mu_n + \mu_p)$. Using Table 5-1,

$\sigma = (1.5\times10^{10})(1.6\times10^{-19})(1300+500) = 4.32\times10^{-6}\ (\Omega\text{-cm})^{-1}$

$\therefore \rho = \frac{1}{\sigma} = 231{,}000\ \Omega\text{-cm} \longleftarrow$

b) $n_{Si} = 6.02\times10^{23}\ \frac{\text{atoms}}{\text{mole}} \times 1\ \frac{\text{mole}}{28.1\ \text{g}} \times 2.33\ \frac{\text{g}}{\text{cm}^3} = 5\times10^{22}\ \text{atoms/cm}^3$

If there is 1 donor atom per 10^8 Si atoms, then $N_D = 5\times10^{14}\ \text{atoms/cm}^3$

From Eq. (5-25), $n \approx N_D$ and from Eq. (5-27) $p = \frac{n_i^2}{N_D} = \frac{(1.5\times10^{10})^2}{5\times10^{14}} = 4.5\times10^5$

Since $p << n$, then $\sigma = ne\mu_n = 5\times10^{14}\times1.6\times10^{-19}\times1300 = 1.04\times10^{-1}\ (\Omega\text{-cm})^{-1}$

$\therefore \rho = \frac{1}{\sigma} = 9.6\ \Omega\text{-cm} \longleftarrow$

(5-9) a) Use CGS units. From Eq. (5-19), $np = n_i^2 = (2.5\times10^{13})^2 = 6.25\times10^{26}$

From Eq. (5-24), $p - n = N_A - N_D = (3-2)\times10^{14} = 10^{14}$ or $p = n + 10^{14}$

then $n(n+10^{14}) = 6.25\times10^{26}$, or $n = 5.8\times10^{12}$ electrons/cm³ $\longleftarrow$

and $p = n + 10^{14} = 1.06\times10^{14}$ holes/cm³ $\longleftarrow$

$\therefore$ the sample is p-type

b) If $N_A = N_D$, then $p - n = 0$ or $p = n = n_i = 2.5\times10^{13}\ \text{cm}^{-3}$ $\longleftarrow$

The sample is intrinsic by compensation.

c) From Eq. (5-23), $n_i^2 = A_o T^3 e^{-E_{G0}/kT}$ and Table 5-1, at T=300°K

$A_o = \frac{n_i^2}{T^3} e^{E_{G0}/kT} = \frac{6.25\times10^{26}}{27\times10^6} e^{\frac{0.785}{0.026}} = 2.31\times10^{19} e^{30.2} = 2.91\times10^{32}$

@ T=400°K, $n_i^2 = 2.91\times10^{32}(400)^3 e^{-\frac{0.785}{8.62\times10^{-5}\times400}} = 2.57\times10^{30}$

$np = n(n+10^{14}) = 2.57\times10^{30}$. Solving for n we have

$n = -\frac{10^{14}}{2} \pm \frac{\sqrt{10^{28}+10.28\times10^{30}}}{2}$ or $n \approx -\frac{10^{14}}{2} + \sqrt{2.57\times10^{30}} = -\frac{10^{14}}{2} + n_i$

where $n_i = 1.61\times10^{15}$

So, $n = 1.61\times10^{15} - 0.05\times10^{15} = 1.56\times10^{15}$ electrons/cm³ $\longleftarrow$

$p = n + 10^{14} = 1.56\times10^{15} + 0.1\times10^{15} = 1.66\times10^{15}$ holes/cm³ $\longleftarrow$

Hence, the sample at T=400°K is essentially intrinsic, since

$p \approx n \approx n_i$

(5-10) From Eq. (5-2), $\sigma = pe\mu_p$ since $n_p << p_p$. Using Table 5-1:

$p_p = \frac{\sigma}{e\mu_p} = \frac{100}{1.6\times10^{-19}\times1800} = 3.47\times10^{17}$ holes/cm³ $\longleftarrow$

From Eq. (5-28), $n_p = \frac{n_i^2}{p_p} = \frac{(2.5\times10^{13})^2}{3.47\times10^{17}} = 1.8\times10^9$ electrons/cm³ $\longleftarrow$

(5-11) From Eq. (5-2), $\sigma = n_n e\mu_n$, since $n_n >> p_n$

$n_n = \frac{\sigma}{\mu_n e} = \frac{0.1}{1300\times1.6\times10^{-19}} = 4.81\times10^{14}$ electrons/cm³ $\longleftarrow$

From Eq. (5-27), $p_n = \frac{n_i^2}{n_n} = \frac{(1.5\times10^{10})^2}{4.81\times10^{14}} = 4.68\times10^5$ holes/cm³ $\longleftarrow$

(5-12) From Eq. (5-1), $J=(n\mu_n+p\mu_p)e\mathcal{E}$. In order to find n and p, we first find n_i from Eq. (5-2): $n_i = \frac{\sigma_i}{e(\mu_n+\mu_p)} = \frac{1}{60(1.6\times10^{-19})(3800+1800)}$

$= 1.86\times10^{13}$ electrons/cm³

From Eq. (5-19) : $pn = n_i^2 = (1.86\times10^{13})^2$ (1)

Eq. (5-24) : $p-n = N_A-N_D = 7\times10^{13}-10^{14} = -3\times10^{13}$ (2)

Solving (1) and (2) simultaneously, we get $n=3.88\times10^{13}$, $p=0.88\times10^{13}$

Hence, $J=[(3.88)(3800)+(0.88)(1800)]\times10^{13}(1.6\times10^{-19})(2) = 52.3\ \mathrm{mA/cm^2}$ ←

(5-13) a) The concentration of Ge atoms is

$N = 6.02\times10^{23}\,\frac{\text{atoms}}{\text{mole}}\times\frac{1\,\text{mole}}{72.6\,\text{g}}\times\frac{5.32\,\text{g}}{\text{cm}^3} = 4.41\times10^{22}$ atoms/cm³

$\therefore N_D = (10^{-8})\times4.41\times10^{22} = 4.41\times10^{14}/\text{cm}^3$. From Eq. (5-20), we have:

$N_C = 4.82\times10^{15}\left(\frac{m_n}{m}\right)^{3/2}T^{3/2}\ /\text{cm}^3$. If $m_n=\frac{1}{2}m$ and $T=300°\text{K}$,

$N_C = 4.82\times10^{15}\frac{1}{\sqrt{8}}(300)^{3/2} = 8.87\times10^{18}/\text{cm}^3$

From Eq. (5-30), $E_F-E_C = -kT\ln\frac{N_C}{N_D} = -8.62\times10^{-5}\times300\ln\frac{8.87\times10^{18}}{4.41\times10^{14}}$

$= -0.026\times9.9 = -0.257$ eV ←

$\therefore E_F$ is below E_C

b) $N_D = (10^{-3})\times4.41\times10^{22} = 4.41\times10^{19}/\text{cm}^3$

$\therefore E_F-E_C = -0.026\ln\frac{8.87\times10^{18}}{4.41\times10^{19}}\ \text{eV} = (-0.026)\times(-1.6) = +0.0415$ eV ←

$\therefore E_F$ is above E_C.

c) If $N_D=N_C$, then $E_F=E_C$. Thus, $N_D = \left(\frac{\text{Donor concentration}}{\text{Ge atom}}\right)\times4.41\times10^{22}$

or Donor conc. per Ge. atoms $= 20\times10^{-5}$

Hence, the doping level should be 20 donor atoms per 10^5 Ge atoms.

(5-14) a) The concentration of Si atoms is

$N = 6.02\times10^{23}\,\frac{\text{atoms}}{\text{mole}}\times\frac{1\,\text{mole}}{28.1\,\text{g}}\times\frac{2.33\,\text{g}}{\text{cm}^3} = 5\times10^{22}$ atoms/cm³

$\therefore N_A = (10^{-8})(5\times10^{22}) = 5\times10^{14}/\text{cm}^3$. Analogously to Eq. (5-20) we have

for $N_V = 4.82\times10^{15}\left(\frac{m_p}{m}\right)^{3/2}T^{3/2}$ where $m_p=0.6\,m$ and $T=300°\text{K}$

$N_V = 4.82\times10^{15}(0.6\times300)^{3/2} = 1.17\times10^{19}/\text{cm}^3$

From Eq. (5-31), $E_F-E_V = kT\ln\frac{N_V}{N_A} = 0.026\ln\frac{1.17\times10^{19}}{5\times10^{14}} = 0.026\times10.0$

$= 0.26$ eV

or $E_F-E_V = 0.26$ eV $\therefore E_F$ is above E_V ←

(5-14) b) $N_A = \frac{5\times10^{22}}{5\times10^{3}} = 10^{19}\ /\text{cm}^3$ $\therefore E_F - E_V = 0.026 \ln \frac{1.17\times10^{19}}{10^{19}}$ eV

or $E_F - E_V = 0.0406$ eV and E_F is above E_V ←

c) $E_F = E_V$ when $N_A = N_V$

or $5\times10^{22}\left(\begin{array}{c}\text{Acceptor concentration}\\ \text{per \# of Si atoms}\end{array}\right) = 1.17\times10^{19}$; Acc. conc. $= 2.34\times10^{-4}$

∴ The doping level should be 2.34 acceptor atoms per 10^4 Si atoms.

(5-15) From Prob. 5-14, the concentration of Si is 5×10^{22} atoms/cm³

Then, $N_D = \frac{5\times10^{22}}{2\times10^{8}} = 2.5\times10^{14}\ /\text{cm}^3$.

From Eq. (5-30), $E_F = E_C$ when $N_D = N_C$, where N_C is given by Eq. (5-20): $N_C = 4.82\times10^{15}\left(\frac{m_n}{m}\right)^{3/2} T^{3/2}$; $m_n = m$

Hence, $4.82\times10^{15}\,T^{3/2} = 2.5\times10^{14}$, or $T = 0.14\,^\circ\text{K}$! ←

(5-16) From Eq. (5-31), $E_F = E_V$ when $N_A = N_V$, where N_V is given by

$N_V = 4.82\times10^{15}\left(\frac{m_p}{m}\right)^{3/2} T^{3/2} = 4.82\times10^{15}(0.4)^{3/2}(300)^{3/2}$

$= 4.82\times10^{15}\times120\sqrt{120} = 6.33\times10^{18}$.

∴ Doping concentration, $N_A = 6.33\times10^{18}$ atoms/cm³ ←

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CHAPTER 6

(6-1) a) The concentration of Ge atoms is

$N = 6.02\times10^{23}\ \frac{\text{atoms}}{\text{mole}} \times \frac{1\,\text{mole}}{72.6\,\text{g}} \times \frac{5.32\,\text{g}}{\text{cm}^3} = 4.41\times10^{22}$ atoms/cm³

Then, $N_A = 10^{-8}N = 4.41\times10^{14}/\text{cm}^3$ and $N_D = 10^3 N_A = 4.41\times10^{17}/\text{cm}^3$

From Eq. (6-8) and Table 5-1,

$$E_o = kT \ln \frac{N_D N_A}{n_i^2} = 0.026 \ln \frac{4.41\times10^{17}\times4.41\times10^{14}}{(2.5\times10^{13})^2} = 0.33\ \text{eV}$$ ←

b) The concentration of Si atoms is

$N = 6.02\times10^{23}\ \frac{\text{atoms}}{\text{mole}} \times \frac{1\,\text{mole}}{28.1\,\text{g}} \times \frac{2.33\,\text{g}}{\text{cm}^3} = 5\times10^{22}$ atoms/cm³

Then, $N_A = 10^{-8}N = 5\times10^{14}/\text{cm}^3$ and $N_D = 10^3 N_A = 5\times10^{17}/\text{cm}^3$

Hence, $E_o = 0.026 \ln \frac{5\times10^{17}\times5\times10^{14}}{(1.5\times10^{10})^2} = 0.72$ eV ←

(6-2) a) $\rho = \frac{1}{\sigma} = \frac{1}{N_A e \mu_p} = 2\ \Omega\text{-cm}$ or $N_A = \frac{1}{2\times1.6\times10^{-19}\times1800} = 1.74\times10^{15}/\text{cm}^3$

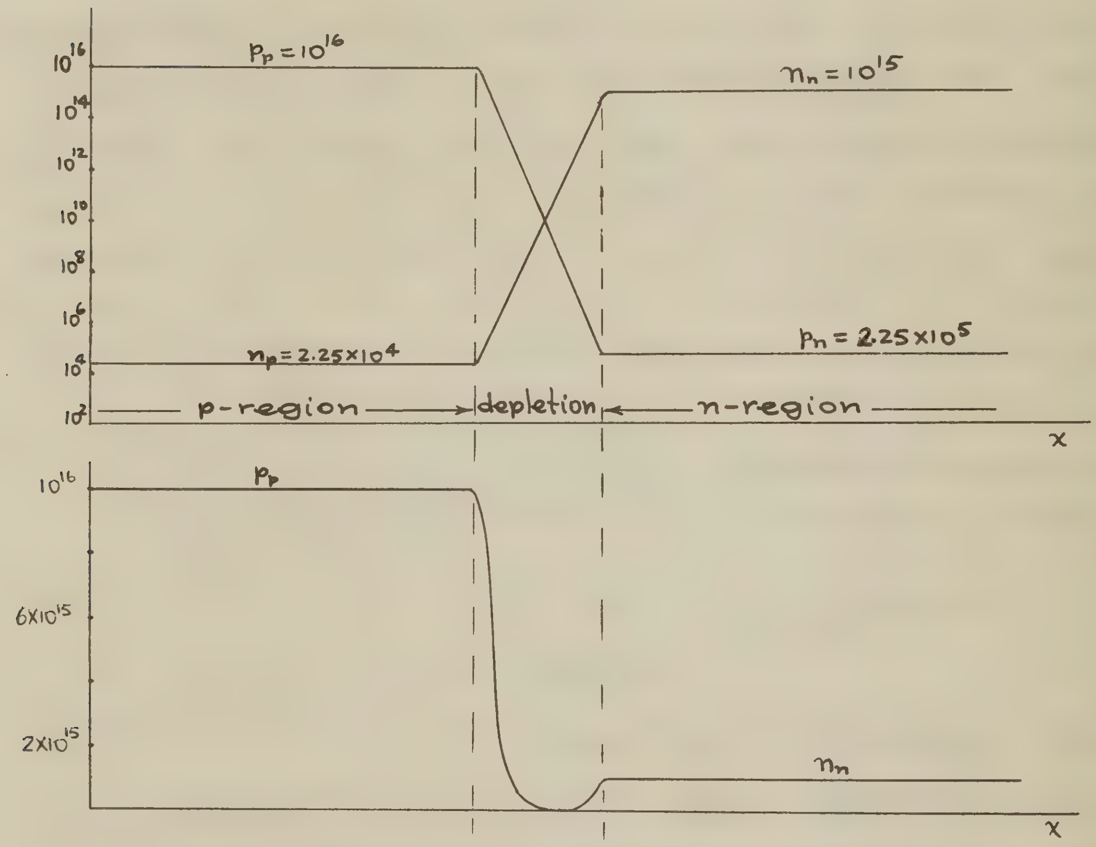
Similarly, $N_D = \frac{1}{1\times1.6\times10^{-19}\times3800} = 1.65\times10^{15}/\text{cm}^3$

From Eq. (6-8), $E_o = 0.026 \ln \frac{1.65\times10^{15}\times1.74\times10^{15}}{(2.5\times10^{13})^2} = 0.22 \text{ eV}$ $\leftarrow$

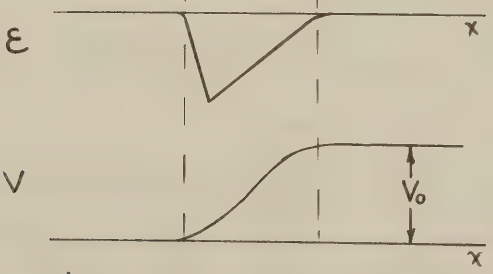
b) $N_A = \frac{1}{2\times1.6\times10^{-19}\times500} = 6.25\times10^{15}/\text{cm}^3$; $N_D = \frac{1}{1\times1.6\times10^{-19}\times1300} = 4.8\times10^{15}/\text{cm}^3$

Then, $E_o = 0.026 \ln \frac{4.8\times10^{15}\times6.25\times10^{15}}{(1.5\times10^{10})^2} = 0.667 \text{ eV}$ $\leftarrow$

(6-3) a)



b)



(6-4) The sketches are similar to those of Prob. 6-3 (a) and (b) with $n_p = \frac{(2.5\times10^{13})^2}{10^{16}} = 6.25\times10^{10}/\text{cm}^3$, $p_n = \frac{(2.5\times10^{13})^2}{10^{15}} = 6.25\times10^{11}/\text{cm}$ and $n_i = 2.5\times10^{13}/\text{cm}^3$

(6-5) From Eq. (6-19), $I_{pn}(0)=\dfrac{AeD_pP_n(0)}{L_p}$ and $I_{np}(0)=\dfrac{AeD_nN_p(0)}{L_n}$

or $\dfrac{I_{pn}(0)}{I_{np}(0)}=\dfrac{[AeD_pP_n(0)]/L_p}{[AeD_nN_p(0)]/L_n}$. Using Eq. (6-23), $P_n(0)=p_{no}(e^{V/V_T}-1)$

and $N_p(0)=n_{po}(e^{V/V_T}-1)$.

Then, $\dfrac{I_{pn}(0)}{I_{np}(0)}=\dfrac{D_pP_{no}}{L_p}\cdot\dfrac{L_n}{D_nn_{po}}=\dfrac{\mu_pP_{no}L_n}{\mu_nn_{po}L_p}$ since $D_p=\mu_pV_T$, $D_n=\mu_nV_T$

From Eq. (5-19), $np=n_i^2$ or $p_{no}=\dfrac{n_i^2}{n_n}$ and $n_{po}=\dfrac{n_i^2}{p_p}$

Hence, $\dfrac{I_{pn}(0)}{I_{np}(0)}=\dfrac{\mu_pp_peL_n}{\mu_nn_neL_p}=\dfrac{\sigma_pL_n}{\sigma_nL_p}$ where $\sigma_p=\mu_pp_pe$ and $\sigma_n=\mu_nn_ne$

(6-6) From Eqs. (6-27) and (5-19), the hole saturation current, I_{po}, is

$I_{po}=\dfrac{AeD_pp_{no}}{L_p}=\dfrac{AeD_p}{L_p}\dfrac{n_i^2}{n_n}$. From Eq. (5-33), $D_p=\mu_pV_T$ and from

Eq. (5-2), $\sigma_i=n_i(\mu_n+\mu_p)e$, $\sigma_n=\mu_nen_n$

$$\therefore\ I_{po}=\frac{Ae\mu_pV_T}{L_p\frac{\sigma_n}{\mu_ne}}\cdot\frac{\sigma_i^2}{(\mu_n+\mu_p)^2e^2}=\frac{AV_T\mu_n\mu_p}{(\mu_n+\mu_p)^2L_p}\frac{\sigma_i^2}{\sigma_n}=AV_T\frac{b}{(1+b)^2}\frac{\sigma_i^2}{L_p\sigma_n}\quad\text{where } b\equiv\frac{\mu_n}{\mu_p}$$

I_{no} is obtained by replacing p by n. Then b becomes $\frac{1}{b}$ and

$$\frac{b}{(1+b)^2}=\frac{1/b}{(1+1/b)^2}=\frac{b}{(1+b)^2}\qquad\therefore\ I_o=I_{po}+I_{no}=AV_T\frac{b\sigma_i^2}{(1+b)^2}\left(\frac{1}{L_p\sigma_n}+\frac{1}{L_n\sigma_p}\right)$$

(6-7) $I_o=AV_T\dfrac{b\sigma_i^2}{(1+b)^2}\left(\dfrac{1}{L_p\sigma_n}+\dfrac{1}{L_n\sigma_p}\right)$; $A=4\times10^{-6}\,m^2$, $V_T=0.026\,V$, $b=\dfrac{3800}{1800}=2.11$

$L_p\sigma_n=0.15\times0.1=0.015\,(\Omega)^{-1}$, $L_n\sigma_n=0.15\,(\Omega)^{-1}$

From Eq. (5-2), $\sigma_i=n_i(\mu_n+\mu_p)e=2.5\times10^{13}(3800+1800)(1.6\times10^{-19})=2.24\,(\Omega\text{-}m)^{-1}$

Then, $I_o=4\times10^{-6}\,m^2\times0.026\,V\times\dfrac{2.11}{(3.11)^2}\dfrac{(2.24)^2}{(\Omega\text{-}m)^2}\left(\dfrac{1}{0.015}+\dfrac{1}{0.15}\right)\Omega=8.34\,\mu A$ ⟵

(6-8) $A=4\times10^{-2}\,cm^2$, $b=\dfrac{1300}{500}=2.6$, $L_p\sigma_n=10^{-2}\times10^{-2}\,(\Omega)^{-1}=L_n\sigma_n$, $\sigma_i=1.5\times10^{10}(1800)(1.6\times10^{-19})$

$=4.32\times10^{-6}\,(\Omega\text{-}cm)^{-1}$

Then, $I_o=4\times10^{-2}\,cm^2\times0.026\,V\times\dfrac{2.6}{(3.6)^2}\dfrac{(4.32)^2\times10^{-12}}{(\Omega\text{-}cm)^2}\cdot\times2\times10^4$

$=104\times10^{-5}\times3.74\times10^{-8}\times2=77.8\times10^{-12}A=77.8\,pA$ ⟵

(6-9) From Prob. 6-6, let $y\equiv\dfrac{I_o}{AV_T}=\dfrac{b\sigma_i^2}{(1+b)^2}\left(\dfrac{1}{L_p\sigma_n}+\dfrac{1}{L_n\sigma_p}\right)$

For Si : $b=\dfrac{\mu_n}{\mu_p}=\dfrac{1300}{500}=2.6$. From Eq. (5-2), $\sigma_i=n_i(\mu_n+\mu_p)e$

$=1.5\times10^{10}(1800)(1.6\times10^{-19})$

or $\sigma_i=0.432\times10^{-5}$

Then, $y_{Si}=\dfrac{2.6}{(3.6)^2}(0.432)^2\times10^{-10}\times2\times10^4=7.5\times10^{-8}$

For Ge : $b=\dfrac{3800}{1800}=2.11$, $\sigma_i=2.5\times10^{13}(5600)(1.6\times10^{-19})=2.24\times10^{-2}$

$y_{Ge}=\dfrac{2.11}{(3.11)^2}\times(2.24)^2\times10^{-4}\times2\times10=2.26\times10^{-3}\quad\therefore\ \dfrac{y_{Ge}}{y_{Si}}=\dfrac{2.26\times10^{-3}}{7.5\times10^{-8}}=3\times10^4$ ⟵

(6-10) a) From Eq. (3-34), $V_T = \dfrac{T}{11{,}600} = 0.026\ \mathrm{V}$ at room temperature.

Using Eq. (6-26), $I = I_o\,(e^{V/V_T} - 1)$: $-0.9 I_o = I_o\,(e^{V/0.026} - 1)$ or $e^{V/0.026} = 0.1$

or $V = (0.026)(-2.3) = -0.060\ \mathrm{V}$ ←

b) $\dfrac{e^{50/26} - 1}{e^{-50/26} - 1} = \dfrac{e^{1.92} - 1}{e^{-1.92} - 1} = \dfrac{6.82 - 1}{0.147 - 1} = -6.83$ ←

c) for $V = 0.1$, $I = 10(e^{100/26} - 1) = 10(e^{3.84} - 1) = 455\ \mu\mathrm{A} = 0.455\ \mathrm{mA}$ ←

$V = 0.2$, $I = 10(e^{200/26} - 1) = 10(e^{7.68} - 1) = 21{,}700\ \mu\mathrm{A} = 21.7\ \mathrm{mA}$ ←

$V = 0.3$, $I = 10(e^{11.52} - 1) = 1.01\times10^{6}\ \mu\mathrm{A} = 1.01\ \mathrm{A}$ ←

(6-11) a) $I = 10^{-5}(e^{V/.026} - 1)$. From Prob. 6-10 we have:

(.1V, .455mA), (0.2V, 21.7mA) also for V=0, I=0.

Additional points:

$V = 0.15\mathrm{V}$, $I = 10^{-5}(e^{150/26} - 1) = 3.2\ \mathrm{mA}$

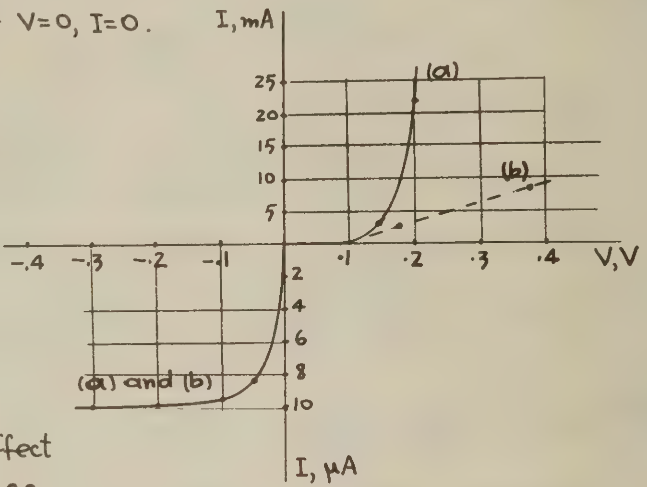
$= 0.05\mathrm{V}$, $I = 10^{-5}(e^{50/26} - 1) = 0.058\ \mathrm{mA}$

$= -0.05\mathrm{V}$, $I = 10^{-5}(e^{-50/26} - 1) = -8.54\ \mu\mathrm{A}$

$= -0.10\mathrm{V}$, $I = 10^{-5}(e^{-100/26} - 1) = -9.78\ \mu\mathrm{A}$

$= -0.15\mathrm{V}$, $I = 10^{-5}(e^{-150/26} - 1) = -9.96\ \mu\mathrm{A}$

$= -0.20\mathrm{V}$, $I = 10^{-5}(e^{-200/26} - 1) = -9.97\ \mu\mathrm{A}$



b) $I = .455\ \mathrm{mA}$, $V = 0.1 + (.455\times25) = 0.111\ \mathrm{V}$

$I = 3.2\ \mathrm{mA}$, $V = 0.15 + (3.2\times25) = 0.158\ \mathrm{V}$

$I = 8\ \mathrm{mA}$, $V = 0.17 + (8\times25)\times10^{-3} = 0.37\ \mathrm{V}$

The 25-Ω resistor has negligible effect on the reverse characteristic, since the current flowing is very small.

(6-12) a) From Eq. (6-31), $\log I = \log I_o + 0.434\dfrac{V}{\eta V_T}$; At $T = 25°\mathrm{C} = 298°\mathrm{K}$, $V_T \approx 0.026\mathrm{V}$

In Fig. 6-9 for T=25°C : $V = 0.75\mathrm{V}$ when $I = 10\ \mathrm{mA} = 10^{-2}\ \mathrm{A}$.
$V = 0.40\mathrm{V}$ when $I = 0.01\ \mathrm{mA} = 10^{-5}\ \mathrm{A}$

Thus, $-2 = \log I_o + 0.434\dfrac{0.75}{(0.026)\eta}$ (1) ; $-5 = \log I_o + 0.434\dfrac{0.40}{(0.026)\eta}$ (2)

Subtracting (2) from (1) : $3 = \dfrac{0.434}{0.026}\,\dfrac{0.35}{\eta}$ or $\eta = 1.94$ ←

b) At $T = -55°\mathrm{C} = 220°\mathrm{K}$, $V_T = \dfrac{220}{11{,}600} = 0.019\ \mathrm{V}$; At $T = 150°\mathrm{C} = 423°\mathrm{K}$, $V_T = 0.0364$

From Fig. 6-9 : $V = 0.91\mathrm{V}$ when $I = 10\mathrm{mA} = 10^{-2}\mathrm{A}$ and $V = 0.55$ @ 150°C
$V = 0.66\mathrm{V}$ when $I = 0.01\mathrm{mA} = 10^{-5}\mathrm{A}$ and $V = 0.1$ @ 150°C

−55°C : Thus, $-2 = \log I_o + \dfrac{0.434}{0.019}\,\dfrac{0.91}{\eta}$ (1) ; $-5 = \log I_o + \dfrac{0.434}{0.019}\,\dfrac{0.66}{\eta}$ (2)

150°C : and $-2 = \log I_o + \dfrac{0.434}{0.0364}\,\dfrac{0.55}{\eta}$ (3) ; $-5 = \log I_o + \dfrac{0.434}{0.0364}\,\dfrac{0.1}{\eta}$ (4)

Subtracting (2) from (1) : $3 = \dfrac{0.434}{0.019}\,\dfrac{0.25}{\eta}$ $\therefore \eta = 1.90$ ←, (3)−(4) $3 = \dfrac{0.434}{0.0364}\,\dfrac{0.45}{\eta}$ $\therefore \eta = 1.79$ ←

(6-13) Refer to Fig. 6-10a. Note that the ordinate is a log scale, so load lines are curves and must be plotted in area of interest.

For R=10M, we plot the L.L. from 100V to 10 μA. The intersection with the 25°C curve is at 5.5 μA, 45V $\longleftarrow$

For R=1M, the L.L. goes from 100 μA to 100V. Intersection is at ≈ 25 μA and 75 V. $\longleftarrow$

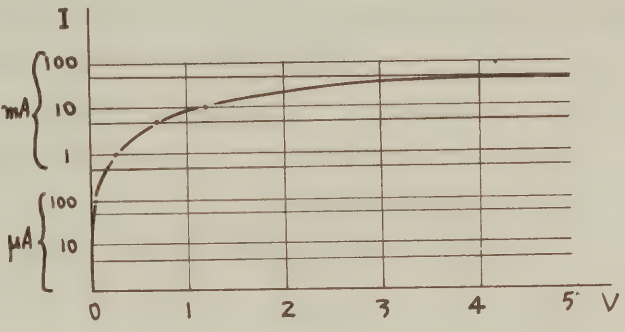
For R=100K, the L.L. goes from 100 V to 1mA. Intersection is at ≈ 70 μA and 93 V. $\longleftarrow$

(6-14) From Eq. (6-26), $V_D = V_T \ln\left(1+\frac{I}{I_o}\right)$

For the resistor, $V_R = IR = 100I$

Then, $V = V_D + V_R$; $I_o = 5\,\mu A$, $V_T = 0.026\,V$

I	V_D	V_R	V
10 μA	0.0286V	0.001V	0.0296V
50	.0625	.005	.0675
100	.079	.01	.089
500	.120	.05	.17
1 mA	.138	.10	.238
5	.180	.50	.68
10	.198	1.0	1.198
50	.240	5.0	5.24



(6-15) a) From Eq. (6-35) for Ge, $I_o = KT^2 e^{-0.785/V_T}$

At $T = 80°C = 353°K$, $V_T = \frac{353}{11{,}600} = 0.0304\,V$ and $I_o = K(353)^2 e^{-\frac{0.785}{0.0304}}$ (1)

At $T = 25°C = 298°K$, $V_T = \frac{298}{11{,}600} = 0.0257\,V$ and $I_o = K(298)^2 e^{-\frac{0.785}{0.0257}}$ (2)

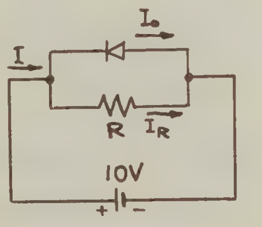
The ratio of (1) to (2) is $\frac{(353)^2 e^{-25.8}}{(298)^2 e^{-30.5}} = (1.18)^2 e^{4.7} = 154$ $\longleftarrow$

b) For Si, $I_o = KT^{3/2} e^{-1.21/2V_T}$

At $T = 150°C = 423°K$, $V_T = \frac{423}{11{,}600} = 0.0364\,V$ and $I_o = K(423)^{3/2} e^{-\frac{1.21}{0.0364\times 2}}$ (3)

At $T = 25°C = 298°K$, $V_T = 0.257\,V$ $\quad I_o = K(298)^{3/2} e^{-\frac{1.21}{0.0257\times 2}}$ (4)

The ratio of (3) to (4) is $\frac{(423)^{3/2} e^{-16.6}}{(298)^{3/2} e^{-23.5}} = (1.45)^{3/2} e^{6.9} = 1745$ $\longleftarrow$



(6-16) $I = I_o + I_R \quad \therefore \frac{dI}{dT} = \frac{dI_o}{dT}$

$= I_o + \frac{10}{R}$

From Eq. (6-36) for Ge $\frac{1}{I_o}\frac{dI_o}{dT} = 0.11$; $\frac{1}{I}\frac{dI}{dT} = 0.07$ (given)

Taking the ratio $\frac{\frac{1}{I_o}\frac{dI_o}{dT}}{\frac{1}{I}\frac{dI}{dT}} = \frac{I}{I_o} = \frac{0.11}{0.07}$ or $I_o = 0.636I$

Then, $I_R = I - 0.636I = 0.364I = \frac{0.364}{0.636} I_o = 0.572(5\,\mu A) = 2.86\,\mu A$ $\therefore R = \frac{10}{2.86\times 10^{-6}} = 3.5M$ $\longleftarrow$

(6-17) At thermal equilibrium at $(25°+10°) = 35°C$, $P_{in} = P_{out} = (10°\text{ above ambient})(0.1\,mW/°C)$ or $P_{out} = 1\,mW$. At 35°C, I_o has doubled from its value at 25°C, i.e. $I_o = 10\,\mu A$.

$P_{out} = VI_o = 10^{-3} = V(10\times 10^{-6})$ $\quad \therefore V = 100\,V$ $\longleftarrow$

(6-18) a) From Eq. (6-33), $I = I_o(e^{V/\eta V_T} - 1) \approx I_o e^{V/\eta V_T}$ since $e^{V/\eta V_T} \gg 1$. From Eq. (6-35) $I_o = KT^m e^{-V_{GO}/\eta V_T}$. $\therefore I = KT^m e^{(V-V_{GO})/\eta V_T} = KT^m e^{e(V-V_{GO})/\eta kT}$ since $V_T = \frac{kT}{e}$. Thus, $\ln I = \ln K + m \ln T + \frac{e(V-V_{GO})}{\eta kT}$ and

$$\frac{d}{dT}(\ln I) = \frac{1}{I}\frac{dI}{dT} = \frac{m}{T} - \frac{e(V-V_{GO})}{\eta kT^2} \approx \frac{V_{GO}-V}{\eta T V_T} \qquad \text{QED}$$

b) For Ge : $\eta = 1$, $V_{GO} = 0.785\,V$ $\therefore \frac{1}{I}\frac{dI}{dT} = \frac{0.785-0.2}{1\times300\times0.026} = 0.075$ or 7.5 % /°K

For Si : $\eta = 2$, $V_{GO} = 1.21\,V$ $\therefore \frac{1}{I}\frac{dI}{dT} = \frac{1.21-0.6}{2\times300\times0.026} = 0.039$ or 3.9% /°K

(6-19) From Eq. (6-33), $I = I_o(e^{V/\eta V_T} - 1)$; At 25°C = 298°K, $V_T = 0.0257\,V$

For Si, $\eta = 2$. Then for $V = 0.4V$, $I = I_o(e^{\frac{0.4}{2\times0.0257}} - 1) = I_o(e^{7.79} - 1)$ or $I = 2200\,I_o$. At 150°C = 423°K, $V_T = \frac{423}{11{,}600} = 0.0364V$, $\frac{V}{\eta V_T} = 5.5$ and $I_o' = (1.07)^{150-25} I_o = (1.07)^{125} I_o \doteq 4800 I_o$. Thus at 150°C, $I' = I_o'(e^{5.5} - 1)$ or $I' = (4800 I_o)(245)$.

$$\therefore \frac{I'}{I} = \frac{4800\times245}{2200} = 534 \quad \longleftarrow$$

From Fig. 6-9, when $V = 0.4V$, $I = 0.01\,mA$ @25°C and $I' \doteq 2\,mA$ @150°C.

$$\therefore \frac{I'}{I} = \frac{2}{0.01} = 200$$

(6-20) From Eq. (6-33), $I = I_o(e^{V/\eta V_T} - 1)$; For Ge $\eta = 1$. Then $\frac{dI}{dV} = \frac{I_o}{V_T} e^{V/V_T} = \frac{1}{r}$ or $r = \frac{V_T}{I_o} e^{-V/V_T}$ where @ $T = 125°C = 398°K$ $V_T = \frac{398}{11{,}600} = 0.0343V$

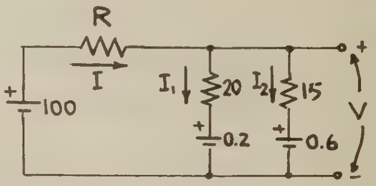
a) For $V = +0.2V$, $r = \frac{34.3\times10^{-3}}{30\times10^{-6}} e^{-\frac{0.2}{0.0343}} = 1.14\times10^3 e^{-5.83} = 3.36\,\Omega$ $\longleftarrow$

b) For $V = -0.2V$, $r = 1.14\times10^3 e^{5.83} = 0.388\,M$ $\longleftarrow$

(6-21) a) $R = 10K$

Approximate solution: $I \approx \frac{100}{R} = \frac{100}{10^4} = 10mA$

Assume that the Si diode is not conducting. Then, $V = 0.2 + 20\times10\times10^{-3} = 0.4\,V$ $\therefore$ The Si diode is indeed not conducting, i.e. $I_2 = 0$.



Exact solution: $I_1 = I = \frac{100-0.2}{10{,}020} = 9.96\,mA$ $\longleftarrow$

b) $R = 1K$. Now both diodes conduct. From the circuit, we have:

$20I_1 - 15I_2 = 0.4$ (1), $1020I_1 + 1000I_2 = 99.8$ (2) Solving (1) & (2) simultaneously, we get $I_1 = 53.7\,mA$, $I_2 = 44.9\,mA$ $\longleftarrow$

(6-22) From p. 136 (Text) we see that for an abrupt junction diode $C_T \approx \frac{\lambda}{V^{1/2}}$ where λ = constant and V = reverse-bias voltage across the diode. When $V = 5V$, $C_T = 20pF$ $\therefore \lambda = 20\sqrt{5}$. When $V = 6V$, $C_T = 20\sqrt{5/6} = 18.25\,pF$ This corresponds to a decrease of 1.75 pF. $\longleftarrow$

(6-23) For this case Eq. (6-47) is valid, i.e. $W^2 = \frac{2\epsilon V_B}{eN_A}$. But $\sigma_p = eN_A\mu_p$

$$\left.\begin{array}{l} \text{or } eN_A = \frac{\sigma_p}{\mu_p} \\ \text{and } V_B = V_o \end{array}\right\} \text{ Substituting we get } W = \left(\frac{2\epsilon\mu_p V_o}{\sigma_p}\right)^{1/2} \quad \text{QED}$$

(6-24) a) From Eq. (6-49), $C_T = \frac{\epsilon A}{W}$ where W is given by Eq. (6-47), $W = \left(\frac{2\epsilon V_B}{eN_A}\right)^{1/2}$

or $\frac{C_T}{A} = \frac{\epsilon}{W} = \frac{\epsilon}{\sqrt{2\epsilon V_B}} \cdot \sqrt{eN_A} = \sqrt{\frac{e\epsilon}{2}}\sqrt{\frac{N_A}{V_B}}$. For Si $\epsilon = \frac{12}{36\pi \times 10^{11}}$ F/cm

Thus $\left(\frac{e\epsilon}{2}\right)^{1/2} = \left(\frac{1}{2} \times 1.6 \times 10^{-19} \times \frac{12}{36\pi} \times 10^{-11}\right)^{1/2} = 2.9 \times 10^{-16}$ and $C_T = 2.9 \times 10^{-16}\left(\frac{N_A}{V_B}\right)^{1/2}$ F/cm²

or $C_T = 2.9 \times 10^{-4}\left(\frac{N_A}{V_B}\right)^{1/2}$ pF/cm² QED

b) $A = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(40 \times 10^{-3}\text{in} \times 2.54 \frac{\text{cm}}{\text{in}}\right)^2 = 8.14 \times 10^{-3}\ \text{cm}^2$; $\frac{1}{N_A \mu_p e} = 3.5$ or

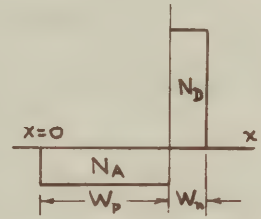
$N_A = \frac{1}{3.5 \times 1.6 \times 10^{-19} \times 500} = 3.58 \times 10^{15}/\text{cm}^3$; $V_B = 5 + 0.35 = 5.35\ \text{V}$

Hence, $C_T = 2.9 \times 10^{-4}\left(\frac{3.58 \times 10^{15}}{5.35}\right)^{1/2}(8.14 \times 10^{-3}) = 61\ \text{pF}$ ◁——

(6-25) From Eq. (6-49), $C_T = \frac{\epsilon A}{W}$ where for Ge $\epsilon = \frac{16}{36\pi \times 10^{11}}$ F/cm

$\therefore C_T = \left(\frac{16}{36\pi \times 10^{11}}\right)\frac{10^{-1} \times 10^{-1}}{2 \times 10^{-4}} = 70.8\ \text{pF}$ ◁——

(6-26) From Eq. (6-45) for $0 \le x \le W_p$, $\frac{d^2V}{dx^2} = \frac{eN_A}{\epsilon}$.



Since $\frac{dV}{dx} = 0 = V$ @ $x = 0$, then integrating we get:

$\frac{dV}{dx} = \frac{eN_A}{\epsilon}x$ and $V = \frac{eN_A}{2\epsilon}x^2$. At the junction

$(x = W_p)$, $\frac{dV}{dx} = \frac{eN_A}{\epsilon}W_p$ and $V = \frac{eN_A}{2\epsilon}W_p^2$

Let us now, for simplicity, shift the origin x=0 to the junction. Then for $0 \le x \le W_n$, $\frac{d^2V}{dx^2} = -\frac{eN_D}{\epsilon}$. Integration subject to the boundary conditions found above, namely, at $x = 0$, $\frac{dV}{dx} = \frac{eN_A W_p}{\epsilon}$ and $V = \frac{eN_A W_p^2}{2\epsilon}$ gives:

$\frac{dV}{dx} = -\frac{eN_D}{\epsilon}x + \frac{eN_A W_p}{\epsilon}$ and $V = -\frac{eN_D}{2\epsilon}x^2 + \frac{eN_A W_p}{\epsilon}x + \frac{eN_A W_p^2}{2\epsilon}$

At $x = W_n$, $V = V_B$ so that $\frac{2\epsilon}{e}V_B = -N_D W_n^2 + 2N_A W_p W_n + N_A W_p^2$

From Eq. (6-44) $N_A W_p = N_D W_n$. Also $W = W_n + W_p$ so that $W = \frac{N_A}{N_D}W_p + W_p$

or $W_p = \frac{N_D}{N_A + N_D}W$ and $W_n = \frac{N_A}{N_A + N_D}W$

Using these expressions for W_p and W_n in the equation for V_B, we have:

$$\frac{2\epsilon}{e}V_B = \frac{W^2}{(N_A + N_D)^2}\left[-N_D N_A^2 + 2N_A^2 N_D + N_A N_D^2\right] = \frac{W^2}{(N_A + N_D)^2}\left[N_A N_D(N_A + N_D)\right]$$

$$\therefore V_B = \frac{e}{2\epsilon}W^2\frac{N_A N_D}{N_A + N_D} = \frac{eW^2}{2\epsilon}\left(\frac{1}{\frac{1}{N_A} + \frac{1}{N_D}}\right) \quad \text{QED}$$

(6-27) From Eq. (6-47), $V_B = V_o - V = \frac{e}{2\epsilon}N_A W^2$ where $V_o = 0.2\text{V}$ and $N_A = 3 \times 10^{20}$ atoms/m³

or $W^2 = \frac{2 \times 16}{36\pi \times 10^9} \times \frac{1}{1.6 \times 10^{-19}} \times \frac{1}{3 \times 10^{20}}(0.2 - V) = 5.9 \times 10^{-12}(0.2 - V)$

(6-27) (Cont'd) a) $V=-10\,V$, $W^2 = 60.2\times10^{-12}$ or $W = 7.75\times10^{-6}\,m = 7.75\,\mu$ ←

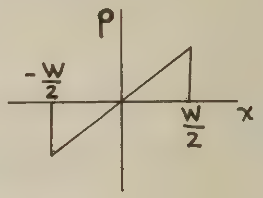
b) $V=-0.1\,V$, $W^2 = 1.77\times10^{-12}$ or $W = 1.33\,\mu$ ←

c) $V=+0.1\,V$, $W^2 = 0.59\times10^{-10}$ or $W = 0.77\,\mu$ ←

d) $C_T = \frac{\epsilon A}{W} = \frac{16}{36\pi\times10^9}\left(\frac{10^{-6}}{W}\right) = \frac{1.42\times10^{-16}}{W}$. For $V=-10$, $C_T = 18.3\,pF$ ←

$V=-0.1$, $C_T = 107\,pF$ and for $V=+0.1$, $C_T = 185\,pF$ ←

(6-28) a) From Poisson's equation we have, $\frac{d^2V}{dx^2} = -\frac{\rho}{\epsilon} = -\frac{ax}{\epsilon}$



$\frac{dV}{dx} = -\frac{ax^2}{2\epsilon} + C_1$. Since @ $x=-\frac{W}{2}$, $\frac{dV}{dx} = 0$, $C_1 = \frac{aW^2}{8\epsilon}$

or $\frac{dV}{dx} = -\frac{ax^2}{2\epsilon} + \frac{aW^2}{8\epsilon}$. $V = -\frac{ax^3}{6\epsilon} + \frac{aW^2}{8\epsilon}x + C_2$.

Since @ $x=-\frac{W}{2}$, $V=0$, $C_2 = -\frac{aW^3}{48\epsilon} + \frac{aW^3}{16\epsilon} = \frac{aW^3}{24\epsilon}$

@ $x=+\frac{W}{2}$, $V = V_B = -\frac{aW^3}{48\epsilon} + \frac{aW^3}{16\epsilon} + \frac{aW^3}{24\epsilon} = \frac{aW^3}{12\epsilon}$

If $V_B = V_o$, Then $V_o = \frac{aW^3}{12\epsilon}$ QED

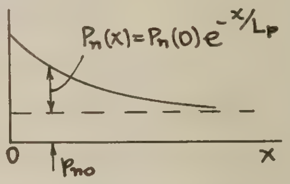
b) $Q = \int_0^{\frac{W}{2}} A\rho\,dx = \int_0^{\frac{W}{2}} Aax\,dx = \frac{Aa}{2}\left(\frac{W}{2}\right)^2 = \frac{AaW^2}{8}$; $C_T = \frac{dQ}{dV} = \frac{AaW}{4}\frac{dW}{dV}$

From part (a), $dV = \frac{aW^2}{4\epsilon}dW$ $\therefore C_T = \frac{AaW}{4}\frac{4\epsilon}{aW^2} = \frac{\epsilon A}{W}$ QED

(6-29) From Eqs. (6-54) and (6-40), $C_D = \frac{L_p^2}{D_p}\frac{I}{\eta V_T}$ or $L_p^2 = \frac{C_D D_p \eta V_T}{I}$

or $L_p^2 = \frac{1\times10^{-6}\times13\times2\times0.026}{1\times10^{-3}} = (26)^2\times10^{-6}$ $\therefore L_p = 26\times10^{-3}$ cm ←

(6-30) $C_{D_p} = \frac{dQ_P}{dV} = \frac{d}{dV}\int_0^\infty eA\,P_n(0)\,e^{-x/L_p}dx = eAL_p\frac{dP_n(0)}{dV}$



From Eq. (6-52), $I_{pn}(0) = \frac{eAD_p P_n(0)}{L_p}$; $\frac{dP_n(0)}{dV} = \frac{dP_n(0)}{dI_{pn}(0)}\frac{dI_{pn}(0)}{dV}$

where $\frac{dP_n(0)}{dI_{pn}(0)} = \frac{L_p}{eAD_p}$ and $\frac{dI_{pn}(0)}{dV} = g_p$

Hence, $C_{Dp} = eAL_p\cdot\frac{L_p}{eAD_p}\cdot g_p = \frac{L_p^2 g_p}{D_p}$. Similarly we obtain for C_{Dn} ,

$C_{Dn} = \frac{L_n^2 g_n}{D_n}$

$\therefore C_D = C_{Dp} + C_{Dn} = \frac{L_p^2 g_p}{D_p} + \frac{L_n^2 g_n}{D_n} = \tau_p g_p + \tau_n g_n$ ←

(6-31) a) From Eq. (5-46) with $\mathcal{E}=0$, $\frac{\partial p_n}{\partial t} = -\frac{p_n - p_{no}}{\tau_p} + D_p\frac{\partial^2 p_n}{\partial x^2}$ (1)

If $p_n - p_{no} = p_{no}(e^{V_1/V_T}-1)e^{-x/L_p} + F(x)V_m e^{j\omega t}$ (2) Then $\frac{\partial p_n}{\partial t} = F(x)V_m(j\omega)e^{j\omega t}$ (3)

and eq. (1) becomes $D_p\frac{\partial^2 p_n}{\partial x^2} = F(x)V_m(j\omega)e^{j\omega t} + \frac{p_{no}}{\tau_p}(e^{V_1/V_T}-1)e^{-x/L_p} + \frac{F(x)}{\tau_p}V_m e^{j\omega t}$

Differentiating eq. (2) twice with respect to x we get:

$D_p\frac{\partial^2 p_n}{\partial x^2} = \frac{D_p p_{no}}{L_p^2}(e^{V_1/V_T}-1)e^{-x/L_p} + D_p\frac{\partial^2 F(x)}{\partial x^2}V_m e^{j\omega t}$ (4)

From Eq. (5-51) $\tau_p = \frac{L_p^2}{D_p}$, thus $D_p\frac{\partial^2 p_n}{\partial x^2} = \frac{p_{no}}{\tau_p}(e^{V_1/V_T}-1)e^{-x/L_p} + D_p\frac{\partial^2 F(x)}{\partial x^2}V_m e^{j\omega t}$

(6-31) (cont'd) Equating the two values of $D_p \frac{\partial^2 p_n}{\partial x^2}$ and simplifying, we get

$$D_p \frac{d^2F(x)}{dx^2} = \left(j\omega + \frac{1}{\tau_p}\right) F(x) = \left(\frac{1+j\omega\tau_p}{\tau_p}\right) F(x) \quad \text{or} \quad \frac{d^2F}{dx^2} - \frac{1+j\omega\tau_p}{L_p^2} F(x) = 0 \qquad (5)$$

The solution of eq. (5) is $F(x) = K_1 e^{-\frac{(1+j\omega\tau_p)^{1/2}}{L_p}x} + K_2 e^{+\frac{(1+j\omega\tau_p)^{1/2}}{L_p}x}$ (6)

At $x \to \infty$, $F(x)$ must be finite and therefore $K_2 = 0$

$$\therefore \; F(x) = K e^{-(1+j\omega\tau_p)^{1/2} x/L_p} \qquad \text{QED}$$

b) From eqs. (2) and (6)

$$p_n - p_{no} = p_{no}(e^{V_1/V_T} - 1) e^{-x/L_p} + K e^{-(1+j\omega\tau_p)^{1/2} x/L_p} V_m e^{j\omega t} \qquad (7)$$

At $x = 0$, $p_n - p_{no} = p_{no}\left(e^{\frac{V_1 + V_m e^{j\omega t}}{V_T}} - 1\right) \approx p_{no} e^{V_1/V_T}\left(1 + \frac{V_m}{V_T} e^{j\omega t} + \dots\right) - p_{no}$

$$= p_{no}(e^{V_1/V_T} - 1) + p_{no} \frac{V_m}{V_T} e^{V_1/V_T} e^{j\omega t} \qquad (8)$$

Comparing eqs. (7) @ $x=0$ and (8) we get $K = \frac{p_{no}}{V_T} e^{V_1/V_T}$ $\longleftarrow$

c) Eq. (7) becomes: $p_n - p_{no} = p_{no}(e^{V_1/V_T} - 1) e^{-x/L_p} + \frac{p_{no} V_m}{V_T} e^{V_1/V_T} e^{-(1+j\omega\tau_p)^{1/2}\frac{x}{L_p}} e^{j\omega t}$ (9)

From Eq. (6-18), $I_{pn}(0) = -eD_pA \frac{dp_n}{dx}\Big|_{x=0} = \frac{eD_pAp_{no}}{L_p}\left[(e^{V_1/V_T} - 1) + \frac{V_m}{V_T} e^{V_1/V_T}(1+j\omega\tau_p)^{1/2} e^{j\omega t}\right]$

$$= I_1 + eD_pA \frac{p_{no}}{V_T L_p} e^{V_1/V_T}(1+j\omega\tau_p)^{1/2} V_m e^{j\omega t} \qquad (10)$$

where I_1 from Eqs. (6-19) @ $x=0$ and (6-23) is the direct current corresponding to V_1.

d) For $\omega\tau_p \ll 1$, $(1+j\omega\tau_p)^{1/2} \approx 1 + \frac{1}{2}(j\omega\tau_p) + \dots$. Also $g_p = \frac{\partial I_{pn}}{\partial V}\Big|_{V=V_1} = \frac{eD_pAp_{no}}{L_pV_T} e^{V_1/V_T}$

Thus, eq. (10) becomes: $I_{pn}(0) = I_1 + g_p V_m e^{j\omega t} + j\omega C_{Dp} V_m e^{j\omega t}$ QED

where $C_{Dp} = \frac{\tau_p}{2} g_p$

e) For $\omega\tau_p \gg 1$, $(1+j\omega\tau_p)^{1/2} \approx (j\omega\tau_p)^{1/2} = (\omega\tau_p)^{1/2} e^{j\pi/4} = (\omega\tau_p)^{1/2}\left(\frac{1+j}{\sqrt{2}}\right) = \left(\frac{\omega\tau_p}{2}\right)^{1/2}(1+j)$

Then eq (10) becomes: $I_{pn}(0) = I_1 + g_p\left(\frac{\omega\tau_p}{2}\right)^{1/2} V_m e^{j\omega t} + j\omega C_{Dp} V_m e^{j\omega t}$

where $C_{Dp} = \left(\frac{\tau_p}{2\omega}\right)^{1/2} g_p$ QED

(6-32) a) Assuming that $N_A \ll N_D$, we have from Eq. (6-46), $V = \frac{eN_A}{2\epsilon} x^2$. Then $|\mathcal{E}| = \frac{dV}{dx} = \frac{eN_A}{\epsilon} x$. The electric field is maximum at the junction, i.e. at $x = W_p \approx W$. $\therefore \; \mathcal{E}_m = \frac{eN_A}{\epsilon} W = \frac{2V_B}{W}$ where V_B is given by Eq. (6-47)

b) From Eq. (6-47), $W = \left(\frac{2\epsilon V_B}{eN_A}\right)^{1/2}$; $\mathcal{E}_z = \frac{2V_B}{W} = \frac{2V_B}{\left(\frac{2\epsilon V_B}{eN_A}\right)^{1/2}} = \frac{(2eN_AV_B)^{1/2}}{\epsilon}$

or $V_B = V_z = \frac{\epsilon \mathcal{E}_z^2}{2eN_A}$ QED

(6-33) a) From Prob. 6-32, $V_z = \frac{\epsilon \mathcal{E}_z^2}{2eN_A}$; $\sigma_p = N_A e \mu_p$ or $eN_A = \frac{\sigma_p}{\mu_p}$. Substituting,

$$V_z = \frac{\epsilon \mathcal{E}_z^2 \mu_p}{2\sigma_p} = \left(\frac{16}{36\pi \times 10^9} \, F/m\right) \times \left(\frac{4 \times 10^{14}}{2} \, V^2/m^2\right) \times \left(1800 \, \frac{cm^2}{V\text{-}sec}\right) \times \left(10^{-6} \, \frac{m^3}{cm^3}\right) \times \frac{1}{\sigma_p} \, (\Omega\text{-cm})$$

$V_z = \frac{51}{\sigma_p}$ where σ_p is in $(\Omega\text{-cm})^{-1}$. QED

b) From Table 5-1, $\sigma_i = \frac{1}{45}$ $\therefore \; V_z = 51 \times 45 = 2300\,V$ $\longleftarrow$

(6-33) (Cont'd) c) Now $\sigma_p = \frac{1}{3.7} (\Omega\text{-cm})^{-1}$ $\therefore V_z = 51 \times 3.7 = 188$ V $\leftarrow$

d) From part (a), $\frac{1}{\sigma_p} = \rho_p = \frac{V_z}{51} = \frac{1}{51} \approx 0.02\ \Omega\text{-cm}$ $\leftarrow$

(6-34) a) The current is the same in each diode but in one it is in the reverse direction. To obtain a forward current equal to the reverse saturation current requires a very small voltage. From Eq. (6-33), $I = I_o(e^{V/\eta V_T} - 1) = I_o$ For Ge $\eta = 1$ and @ $T = 300°K$, $V_T = 0.026$.

Thus, $e^{V/V_T} = 2$ or $V = 0.693 \times 0.026 = 0.018$ V

Hence, the voltage across the second diode is $5 - 0.018 \approx 4.98$ V

Since this voltage is in the reverse direction, then the current is

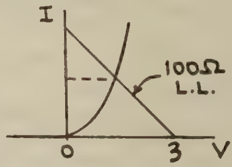
$I = I_o(e^{-4.98/0.026} - 1) \approx -I_o$. This confirms the above assumption that saturation current flows.

Note that the magnitude of the voltages depends upon the temperature, since $V_T = T/11{,}600$.

b) If the Zener voltage is 4.9V then the voltage across the reverse-biased diode is 4.9V. This leaves 0.1V across the forward-biased diode. If $I_o = 5\ \mu A$, then $I = 5 \times 10^{-6}(e^{100/26} - 1) = 5 \times 10^{-6}(46.5 - 1) = 228\ \mu A$ $\leftarrow$

(6-35) a) The voltage across the forward-biased diode is 3 V. With $I_o = 5\ \mu A$, $I = 5 \times 10^{-6}(e^{\frac{3}{0.026}} - 1) = 5 \times 10^{-6} e^{115}$. Taking the logarithm to the base 10, $\log I = -6 + \log 5 + 115 \times 0.434 = 44 + \log 5$ or $I = 5 \times 10^{44}$ A !

b) There are 2V across the Zener diode. Hence, the rest of the circuit consists of a 3-V battery, a forward-biased diode and a 100Ω resistor in series. The intersection of the load line with the diode characteristic gives the solution graphically. Analytically we can obtain the solution by a method of successive approximations.



If we neglect the diode voltage, V, then the current is $\frac{3}{100} = 0.03$ A. The next approximation to V is $0.03 = 5 \times 10^{-6}(e^{\frac{V}{0.026}} - 1)$ or $e^{38.4V} = 1 + \frac{3 \times 10^{-2}}{5 \times 10^{-6}} = 6 \times 10^3$; $(0.434)(38.4V) = 3.778$ or $V = 0.227$ V

2nd approx. : $I = \frac{3 - 0.227}{100} \approx 2.77 \times 10^{-2}$ A ; $e^{38.4V} \approx \frac{2.77 \times 10^{-2}}{5 \times 10^{-6}} = 5.54 \times 10^3$

$0.434 \times 38.4V = 3.743$ or $V = 0.225$V

3rd approx. : $I = \frac{3 - 0.225}{100} \approx 2.77 \times 10^{-2}$ A. Since this current agrees with the previous approximation, then this is the correct value.

Hence, $I = 27.7$ mA $\leftarrow$

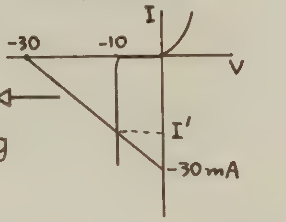
(6-36) a) The solution can be found graphically by plotting the diode characteristic and drawing the load line as shown in Prob. 6-35.

Using the method of successive approximations, we have: With diode drop equal to zero, $I = \frac{30}{1K} = 30$ mA. For this current V is given by $30 \times 10^{-3} = 10 \times 10^{-6}(e^{38.4V} - 1)$ or $e^{38.4V} = 3000$ and $V = 0.208$V

Hence, $I = \frac{3 - 0.208}{1K} \approx 29.8$ mA. For this current, $V = 0.2$V $\therefore I = 29.8$ mA $\leftarrow$

(6-36) (Cont'd) b) The diode voltage is $-30V$ and $I = -I_o = -10\ \mu A$. ←
The voltage drop across the 1K resistor is only 10 mV and may be neglected.

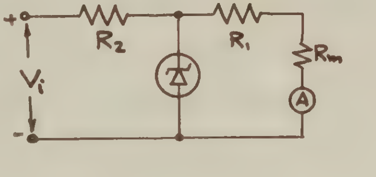
c) In the forward direction, the answer is the same as in part (a), namely, $I = 29.8\ mA$ ←



In the reverse direction, we draw a load line from $V = -30V$ to $I = -30\ mA$ as shown. Then $I' = -30 \times \frac{20}{30} = -20\ mA$ ←

Alternately, there are 10V across the diode, leaving 20V across the 1K resistor ∴ current = 20 mA.

(6-37) With $V_i = 20V$ the zener diode does not conduct.



Hence, $20 = (R_1 + R_2 + R_m) I = (99.5 + 0.56) \times 10^3 I$

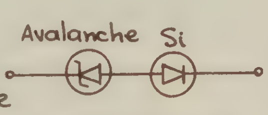
or $I \approx \frac{20}{100K} = 200\ \mu A$ (Full scale)

With $V_i > 20V$, the voltage across R_1 and R_m (meter) must be equal to the Zener voltage, $V_z = 16V$

or $R_1 + R_m = \frac{V_z}{I} = \frac{16}{2 \times 10^{-4}} = 80K$ ∴ $R_1 \approx 79.5\ K$ and $R_2 = 20K$ ←

(6-38) From Fig. 6-19a, the temperature coefficient of a 5.7V Zener diode is zero when $I_z \approx 5.1\ mA$. Hence, $R = \frac{100 - 5.7}{5.1 \times 10^{-3}} = 18.5\ K$ ←

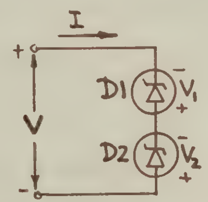
(6-39) For the Si diode the temperature coeff. $= -1.7\ mV/°C$



If the series combination is to have a zero-temperature coefficient, then the temp. coeff. of the avalanche diode must be at the biasing current $+1.7\ mV/°C$.

In percent, temp. coeff. $= \frac{1.7 \times 10^{-3} V}{15 V} (100\%) = 0.0113\ \% \ /°C$ ←

(6-40) $I_{o1} = 1\ \mu A$, $I_{o2} = 2\ \mu A$, $V_{z_1} = V_{z_2} = 100V$. From Eq. (6-33),



$I' = I_o (e^{V/\eta V_T} - 1)$ we get $V' = \eta V_T \ln (1 + \frac{I'}{I_o})$ (1)

Assume that the diodes are Si so that $\eta = 2$.

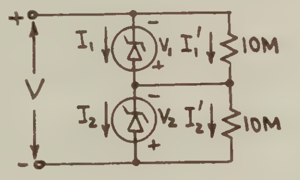
a) (i) $V = 90V$: None of the diodes will break down and I is determined by the diode with the smallest I_o, i.e. D1.

Thus, $I = 1\ \mu A$ and for D2 $I' = -I = -1\ \mu A$. Substituting in (1) we get

$V_2 = 2 \times 0.026 \ln (1 - \frac{1}{2}) = -36\ mV$ and $V_1 = -89.964\ V$ ←

(ii) $V = 110V$: As the applied voltage V is increased to 110V, D1 will break down, while D2 will be reverse biased.

Thus, $I = I_{o2} = 2\ \mu A$, $V_1 = -100V$ and $V_2 = -10V$



b) (i) $V = 90V$. The 10M resistors tend to equalize the voltages across the diodes, thus both diodes are reverse biased

Hence, $I_1 = I_{o1} = 1\ \mu A$, $I_2 = I_{o2} = 2\ \mu A$ ←

(6-40) (Cont'd) From the circuit we note that

$$I_1 + I_1' = I_2 + I_2' \quad \text{or} \quad I_1' = I_2 - I_1 + I_2' = 1 + I_2' \ \mu A \qquad (2)$$

$$I_1' = -\frac{V_1}{10} \ , \quad I_2' = \frac{V+V_1}{10} \quad \mu A \qquad (3)$$

Substituting (3) into (2) we get : $\frac{V_1}{10} = 1 + \frac{V-V_1}{10}$

or $V_1 = -\frac{10+V}{2}$. This is valid for $V_1 < 100$

Hence for $V=90$, $V_1=-50$ and $V_2=-40$ $\leftarrow$

(ii) for $V=110$, $V_1=-60$ and $V_2=-50$ $\leftarrow$

(6-41) a) If $I_L=0$, $R = \frac{200-50}{40\times10^{-3}} = 3750\ \Omega$ $\leftarrow$

I_{max} occurs when $I_D = 5$ mA $\therefore$ $I_{max} = 35$ mA $\leftarrow$

b) For V_{min} : $I_L + I_D = 25+5 = 30$ mA . $\therefore$ $V_{min} = 50 + (30\times10^{-3})(3750) = 162.5\,V$ $\leftarrow$

For V_{max} : $I_L + I_D = 25+40 = 65$ mA . $\therefore$ $V_{max} = 50 + (65\times10^{-3})(3750) = 293.8\,V$ $\leftarrow$

(6-42) a) $N = 6.02\times10^{23}\,\frac{\text{atoms}}{\text{mole}} \times \frac{1\,\text{mole}}{72.6\,g} \times 5.32\,\frac{g}{cm^3} = 4.41\times10^{22}$ atoms/cm³

$N_A = N_D = 10^{-3}N = 4.41\times10^{19}\ /cm^3$

From Eq. (6-8) and Table 5-1 , $E_o = kT\ln\frac{N_A N_D}{n_i^2} = (0.026)(2.3)(2)\log_{10}\frac{4.41\times10^{19}}{2.5\times10^{13}}$

or $E_o = 0.12\times6.245 = 0.75$ eV $\leftarrow$

b) From Eq. (6-47) with $\frac{1}{N_A}$ replaced by $\frac{1}{N_A}+\frac{1}{N_D} = \frac{2}{N_A}$, we have :

$$W^2 = \frac{4\epsilon V_o}{eN_A} = \frac{4\times16\times0.75}{36\pi\times10^9\times4.41\times10^{19}\times10^6\times1.6\times10^{-19}} = 60.2\times10^{-18}\ m^2$$

or $W = 77.7\times10^{-10}\,m = 77.7$ Å $\leftarrow$

(6-43) a) $N = 6.02\times10^{23}\,\frac{\text{atoms}}{\text{mole}} \times \frac{1\,\text{mole}}{28.1\,g} \times 2.33\,\frac{g}{cm^3} = 5\times10^{22}$ atoms/cm³

$N_A = N_D = 10^{-3}N = 5\times10^{19}/cm^3$; From Eq. (6-8) and Table 5-1 ,

$E_o = kT\ln\frac{N_A N_D}{n_i^2} = 0.026\times2.3\times2\log_{10}\frac{5\times10^{19}}{1.5\times10^{10}} = 1.09$ eV $\leftarrow$

b) From Eq. (6-47) with $\frac{1}{N_A}$ replaced by $\frac{1}{N_A}+\frac{1}{N_D} = \frac{2}{N_A}$, we have :

$$W^2 = \frac{4\epsilon V_o}{eN_A} = \frac{4\times12\times1.09}{36\pi\times10^9\times5\times10^{19}\times10^6\times1.6\times10^{-19}} = 57.7\times10^{-18}\ m^2$$

or $W = 76\times10^{-10}\,m = 76$ Å $\leftarrow$

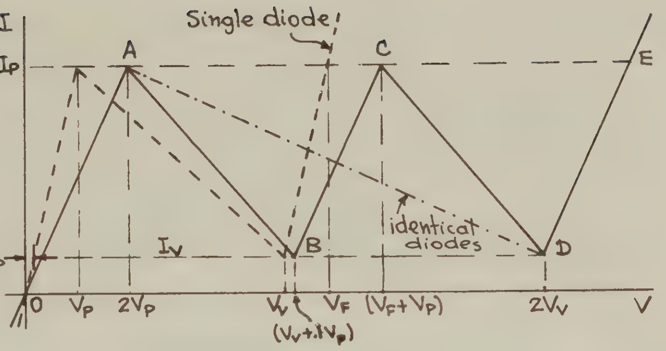
(6-44) a) From Eq. (6-6) for n material, $E_F = E_c - kT\ln\frac{N_c}{N_D}$. If $E_F = E_c$, then $N_c = N_D$. From Eq. (5-10), we have assuming that $m_n = m$,

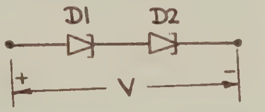
$$N_c = 2\left(\frac{2\pi m_n \bar{k} T}{h^2}\right)^{3/2} = 2\left(\frac{2\pi\times9.1\times10^{-31}\times1.38\times10^{-23}\times300}{44\times10^{-68}}\right)^{3/2} = 2.5\times10^{19}/cm^3$$

$\therefore N_D = N_A = 2.5\times10^{19}/cm^3$ $\leftarrow$

b) For Ge , $N_D = N_A = 2.5\times10^{19}/cm^3$, same as for Si. $\leftarrow$

(6-45) a) As we increase V starting from 0, for $V < 2V_P$, the voltages across the diodes are equal and I increases linearly with V. This is segment OA with a slope $\frac{1}{2}\frac{I_P}{V_P}$.



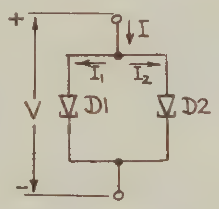


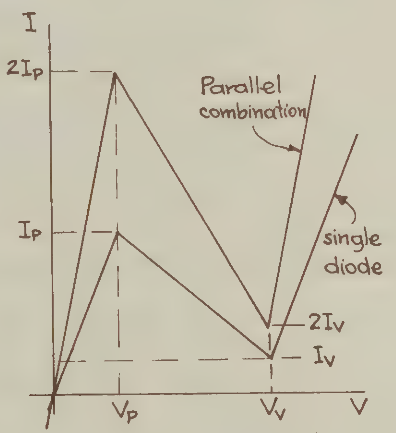
Now assume that the peak current of one diode is slightly different from that of the other (a realistic assumption). Then as V exceeds $2V_P$ one of the diodes, say D1, goes over its peak, while the other, D2, "retreats", thus causing the total current to decrease as V increases. This corresponds to segment AB. When the voltage across D1 reaches V_V, the voltage across D2 is $.1V_P$. Hence, at the first valley point the current is I_V and the voltage across the diodes is $V_V+.1V_P$.

As V increases further, the current and voltages across the diodes increase (segment BC) until diode D2 goes over its peak. At this point, $V_{D1}=V_F$ and $V_{D2}=V_P$ i.e. the second peak of the composite characteristic occurs at $V=V_F+V_P$ and $I=I_P$. Once more the total current decreases as V increases (segment CD) until the second valley point is reached at $V=2V_V$ and $I=I_V$. Beyond this point, the current increases linearly with V (segment DE) with a slope equal to $\frac{1}{2}$ that of the single diode characteristic corresponding to this region.

If the diodes are identical, segment AD is traced.

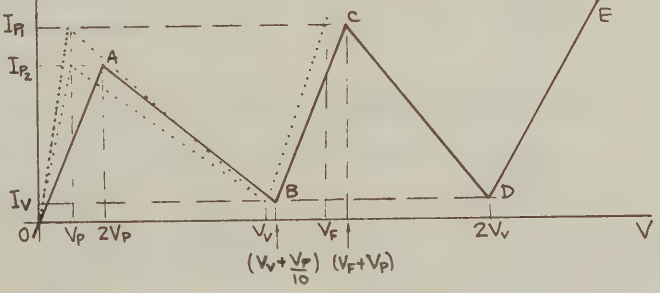
b) With the diodes in parallel we note that for any applied voltage, V, the total current $I = I_1 + I_2 = 2I_1$. Thus, the composite characteristic is as shown, with the current at a given voltage twice as large as that for a single diode.



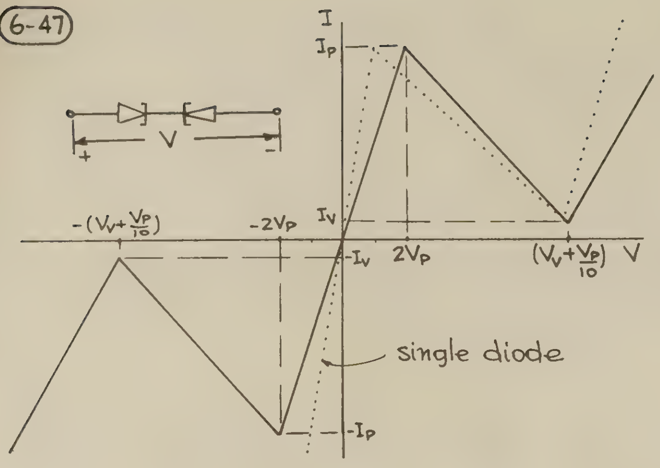


(6-46) a) The composite V-I characteristic of two TD's in series with $I_{P_2}=0.8I_{P_1}$ is shown in the plot. See also Prob. 6-45 a.

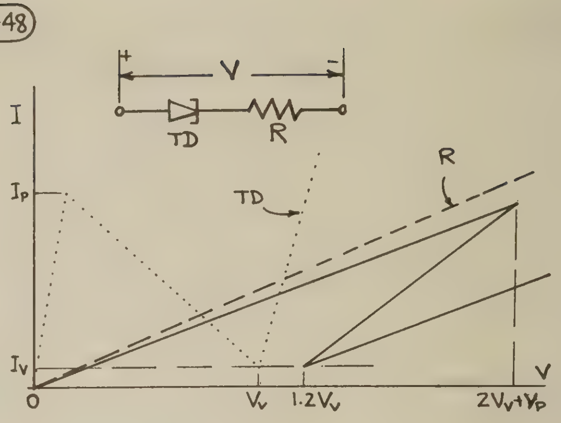
b) If n diodes are operated in series, there will be n peaks and n valleys of various values.



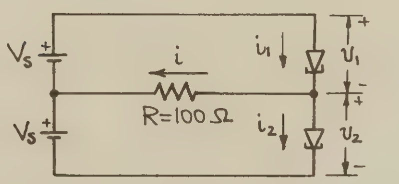
(6-47)



(6-48)

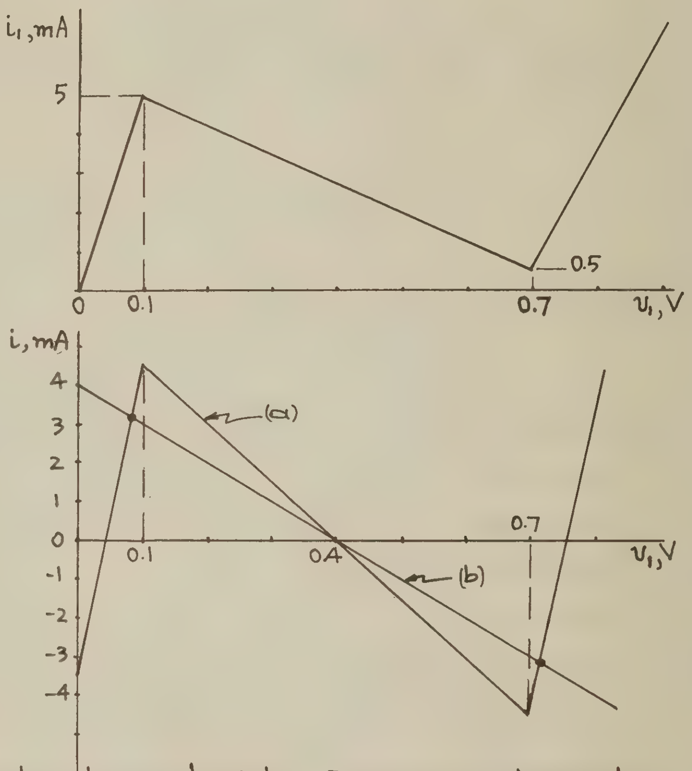


(6-49) a)



$i = i_1 - i_2$, $v_2 = 2V_s - v_1$; $V_s = 0.4V$

v_1	v_2	i_1	i_2	i
0	.8	0	3.4	-3.4
.1	.7	5	.5	4.5
.4	.4	2.7	2.7	0
.7	.1	.5	5	-4.5
.8	0	3.4	0	3.4



b) Load line equation: $v_1 + iR = V_s$

or $v_1 + 0.1i = 0.4$

This is plotted on the composite characteristic. From the intersection we get: $v_1 = 0.08\,V$, $i = 3.2$ mA.

Hence, $i_1 = 4$ mA (from the single TD curve); $i_2 = i_1 - i = 4 - 3.2 = 0.8$ mA

and $v_2 = 2V_s - v_1 = 2\times 0.4 - 0.08 = 0.72\,V$ ⟵

c) If the current in one diode is to be zero, say $i_2 = 0$, (but the diode is not open-circuited), then $v_2 = 0$ and the lower V_s battery in the circuit appears across R.

Then we must have: $v_1 = 2V_s$ (1)

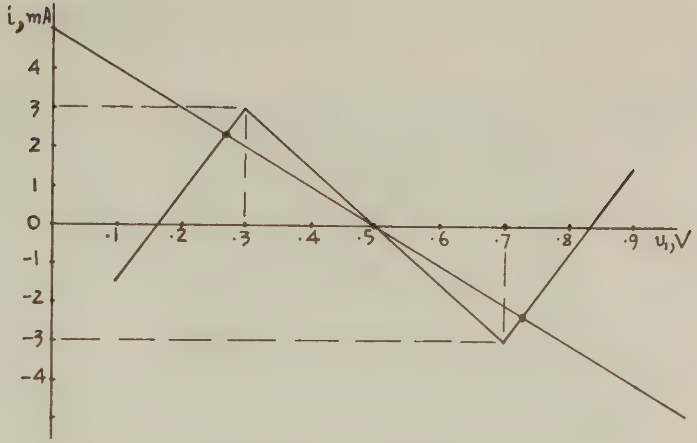
and $-i_1R = V_s$ (2)

Eqs. (1) & (2) are satisfied only if $i_1 = 0$, $v_1 = 0$ and $V_s = 0$ ⟵

(6-49) (Cont'd)

d)

v_1	v_2	i_1	i_2	i
.1	.9	5	6.5	-1.5
.3	.7	3.5	.5	3
.5	.5	2	2	0
.7	.3	.5	3.5	-3
.8	.2	3.5	4.25	-.75
.9	.1	6.5	5	1.5



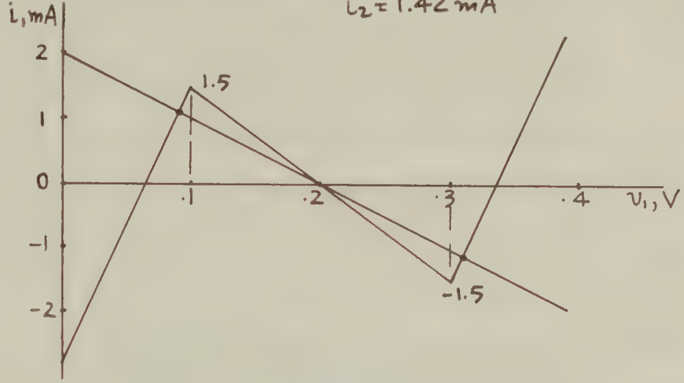
Load line eq. : $v_1 + 0.1i = 0.5$

or $i = 5 - 10v_1$

The intersection gives: $v_1 = 0.27\text{V}$, $i = 2.3$ mA. Then, $i_1 = 3.72$ mA, $v_2 = 0.73$V
$i_2 = 1.42$ mA

e)

v_1	v_2	i_1	i_2	i
0	.4	0	2.75	-2.75
.1	.3	5	3.5	1.5
.2	.2	4.25	4.25	0
.3	.1	3.5	5	-1.5
.4	0	2.75	0	-2.75
V	V	mA	mA	mA



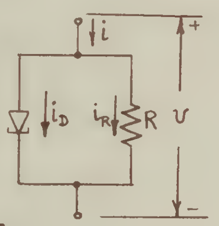
Load line eq. : $v_1 + 0.1i = 0.2$

or $i = 2 - 10v_1$

The intersection gives: $v_1 = 0.0905$V, $i = 1.1$ mA. Then, $i_1 = 4.5$ mA
$i_2 = 3.4$ mA, $v_2 = 0.31$ V

(6-50) From the circuit, $i = i_D + i_R = i_D + \frac{v}{R}$. If the composite characteristic (i vs. v) is to have no negative resistance region, $\frac{di}{dv} \geq 0$ for all v.

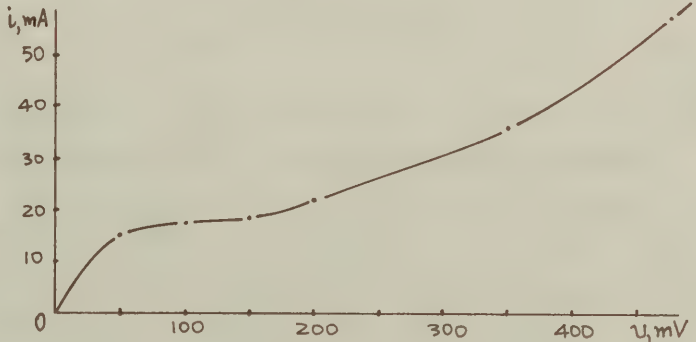
Thus, $\frac{di}{dv} = \frac{di_D}{dv} + \frac{1}{R} \geq 0$ or $\frac{1}{R} \geq -\frac{di_D}{dv}$.



From the given TD characteristic we find $\left|\frac{di_D}{dv}\right|_{max} \approx \frac{1}{10}$ ℧

$\therefore R_{max} = 10\ \Omega$

v	0	50	100	150	200	350	475	mV
i_D	0	10	7.5	3.5	2.2	1	10	mA
i_R	0	5	10	15	20	35	47.5	mA
i	0	15	17.5	18.5	22.2	36	57.5	mA

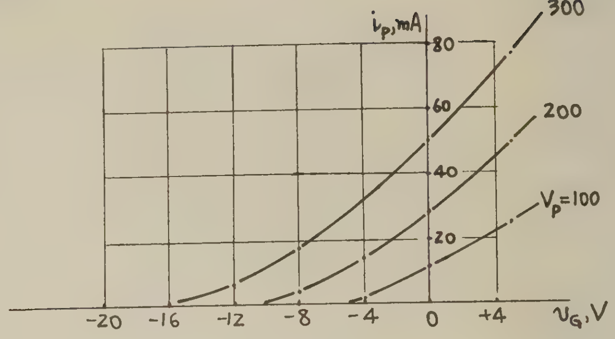


CHAPTER 7

(7-1) a)

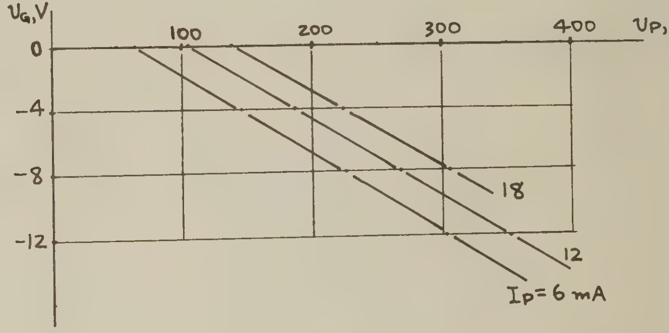
v_G	5	0	-2	-4	-6	-8
i_p @ $V_p=100$	25	12	5.5	1.6	0	0
$=200$	50	29	21.7	14	8	3.5
$=300$	77	51	–	–	–	17

v_G	-10	-12	-14	-16	-18	-20
i_p @ $V_p=100$	0	0	0	0	0	0
$=200$	1	0	0	0	0	0
$=300$	10.2	5.6	2.6	0.8	0.2	0



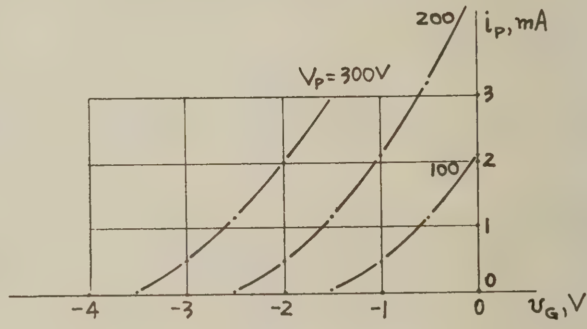
b)

v_G	v_p@$I_p=6$	v_p@$I_p=12$	v_p@$I_p=18$
0	62	103	137
−4	146	187	223
−8	227	270	306
−12	304	354	—

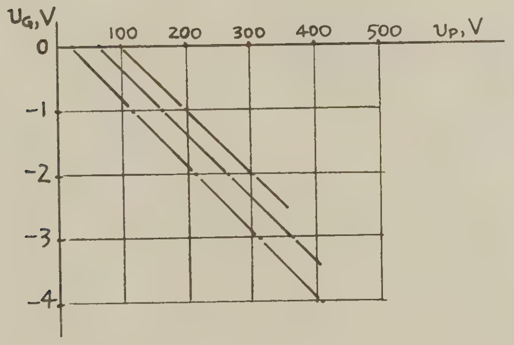


(7-2) a)

v_G	0	-0.5	-1.0	-1.5	-2.0	-2.5	-3.0	-3.5
i_p@ $V_p=100$	2.12	1.2	.48	.07	0	0	0	0
$=200$	—	3.25	2.13	1.2	.52	.09	0	0
$=300$	—	–	—	—	2.07	1.19	.53	.15



b)



v_G	0	-.5	-1	-1.5	-2	-2.5	-3	-3.5	-4
v_p@$I_p=.7$	23	67	118	167	216	267	315	364	410
$=1.4$	63	111	163	212	263	313	365	—	—
$=2.1$	99	147	199	249	301	—	–	—	–

(7-3) From Eq. (7-1), $i_p = G\left(v_G + \frac{v_P}{\mu}\right)^n$; $g_p = \left.\frac{\partial i_p}{\partial v_P}\right|_{V_G} = \frac{nG}{\mu}\left(v_G+\frac{v_P}{\mu}\right)^{n-1} = \frac{nG}{\mu}\left(\frac{i_p}{G}\right)^{\frac{n-1}{n}}$

Hence, $r_p = \frac{1}{g_p}$ varies as i_p^{-m} where $m = \frac{n-1}{n}$ QED

$g_m = \frac{\partial i_p}{\partial v_G} = nG\left(v_G+\frac{v_P}{\mu}\right)^{n-1} = nG\left(\frac{i_p}{G}\right)^{\frac{n-1}{n}}$. Thus, g_m varies as i_p^m. If $n=\frac{3}{2}$, $m = \frac{\frac{3}{2}-1}{3/2} = \frac{1}{3}$

(7-4) a) $V_p = 250V = \text{const.}$, $\Delta i_p = 12-8 = 4\text{mA}$. From p. 166, $g_m = \left.\frac{\Delta i_p}{\Delta v_G}\right|_{V_p} = \frac{\mu}{r_p} = \frac{20}{10K} = 2\times10^{-3}\,℧$

$\therefore\ \Delta v_G = \frac{\Delta i_p}{2\times10^{-3}} = \frac{4\times10^{-3}}{2\times10^{-3}} = 2V$ $\quad\therefore\ V_G = -8+2 = -6V$ ⟵

b) $V_G = -6V = \text{const.}$ From p. 166, $\Delta v_P = r_p\Delta i_p = 10\times(-4) = -40V$ $\quad\therefore\ V_p = 250-40 = 210V$ ⟵

(7-5) a) From p. 166, $r_p = \frac{\Delta v_P}{\Delta i_p}\Big|_{V_G} = \frac{375-200}{26-2} = \frac{175}{24} = 7.3\text{K}$

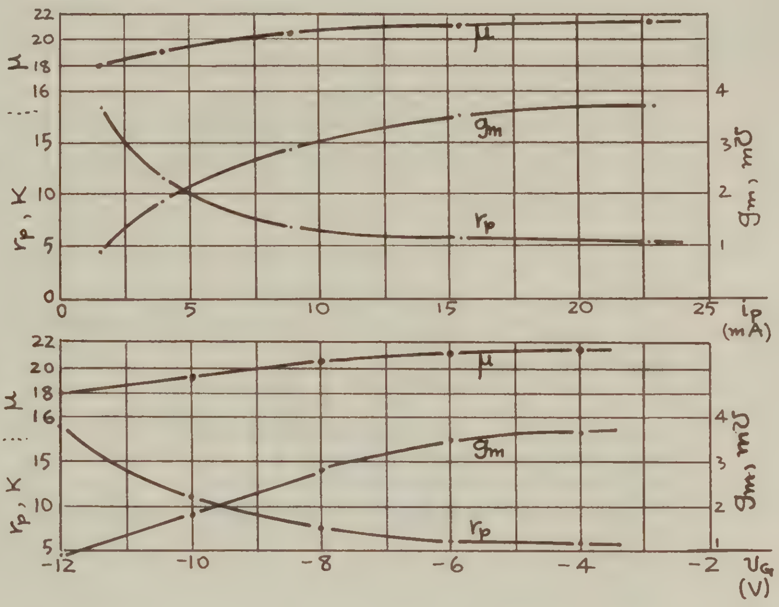
$$\mu = -\frac{\Delta v_P}{\Delta v_G}\Big|_{I_P} = \begin{cases} \text{above} & -\frac{250-208}{-8-(-6)} = \frac{42}{2} = 21 \\ \text{below} & -\frac{290-250}{-10-(-8)} = \frac{40}{2} = 20 \end{cases} \Bigg\} \text{ average } \quad \mu = 20.5$$

$$g_m = \frac{\Delta i_P}{\Delta v_G}\Big|_{V_P} = \begin{cases} \text{above} & \frac{15.6-8.8}{-6-(-8)} = \frac{6.8}{3} = 3.4 \\ \text{below} & \frac{8.8-4.4}{-8-(-10)} = \frac{4.4}{2} = 2.2 \end{cases} \Bigg\} \text{ average } \quad g_m = 2.8 \text{ m℧}$$

Check : $r_p g_m = \mu$ or $7.3 \times 2.8 = 20.5 = \mu$ OK

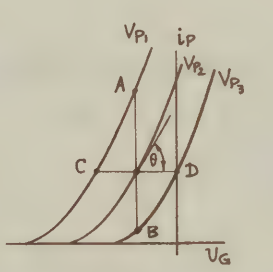
b) & c)

V_G, V	-12	-10	-8	-6	-4
I_P, mA	1.8	4.2	8.8	15.6	22.8
$\mu = \left\lvert\frac{\Delta V_P}{\Delta V_G}\right\rvert$	18	18.8	20.5	21.2	21.5
$r_p = \frac{\Delta V_P}{\Delta I_P}$	18K	11.3	7.3	5.96	5.62
$g_m = \left\lvert\frac{\Delta I_P}{\Delta V_G}\right\rvert$	.95	1.75	2.85	3.5	3.6
$\mu = g_m r_p$	17.1	19.8	20.8	20.8	20.2



(7-6) From p. 166, $r_p = \frac{\Delta v_P}{\Delta i_p}\Big|_{V_G} = \frac{V_{P_1} - V_{P_3}}{AB}$,

$\mu = -\frac{\Delta v_P}{\Delta v_G}\Big|_{I_P} = \frac{V_{P_1} - V_{P_3}}{CD}$, $g_m = \frac{\Delta i_P}{\Delta v_G}\Big|_{V_P} = \tan\theta$



(7-7) We have that $i_P = Av_P + Bv_G + C$. From Eqs. (7-4),
$\left(\frac{\partial i_P}{\partial v_P}\right)_{V_G} = \frac{1}{r_p} = A$, $\left(\frac{\partial i_P}{\partial v_G}\right)_{V_P} = g_m = B$. When $i_P = 0$ and $v_G = 0$, then $v_P = v_0$
or $0 = Av_0 + C$ and $C = -Av_0 = -\frac{v_0}{r_p}$.
Hence, $i_P = \frac{v_P}{r_p} + g_m v_G - \frac{v_0}{r_p} = \frac{1}{r_p}(v_P + \mu v_G - v_0)$ since $\mu = g_m r_p$. QED.

(7-8) a) If two tubes are in parallel, then the current change is double that of a single tube for a given change in grid voltage. Hence,

$g_m = \frac{\Delta i_P}{\Delta v_G}\Big|_{V_P}$ is doubled.

If V_G is constant and the plate voltage is changed, then the plate current is doubled for a given change in plate voltage. Hence, r_p is halved.

(7-8) (Cont'd) b) $g_{m_1} = \frac{\Delta i_{p_1}}{\Delta v_G}\Big|_{V_p}$, $g_{m_2} = \frac{\Delta i_{p_2}}{\Delta v_G}\Big|_{V_p}$, $g_m = \frac{\Delta i_p}{\Delta v_G}\Big|_{V_p}$ where $\Delta i_p = \Delta i_{p_1} + \Delta i_{p_2}$

Hence, $g_m = g_{m_1} + g_{m_2}$

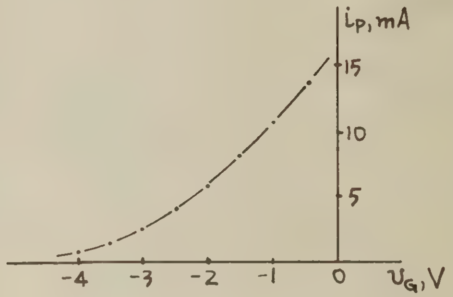
In a similar manner we can show that $g_p = \frac{\Delta i_p}{\Delta v_p}\Big|_{V_G} = g_{p_1} + g_{p_2}$

Since $g_p = \frac{1}{r_p}$, then $\frac{1}{r_p} = \frac{1}{r_{p_1}} + \frac{1}{r_{p_2}} = \frac{r_{p_1} + r_{p_2}}{r_{p_1} r_{p_2}}$

$\mu = g_m r_p = (g_{m_1} + g_{m_2}) \frac{r_{p_1} r_{p_2}}{r_{p_1} + r_{p_2}} = \frac{\mu_1 r_{p_2} + \mu_2 r_{p_1}}{r_{p_1} + r_{p_2}}$ since $\mu = g_m r_p$.

(7-9)

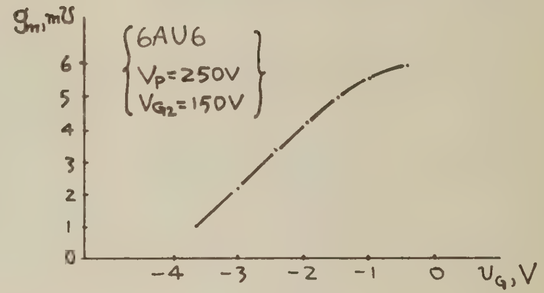
v_{G1}	-.5	-1	-1.5	-2	-2.5	-3	-4
i_p	13.8	10.8	8.3	5.9	3.9	2.5	0.8



(7-10)

$$g_m = \left(\frac{\partial i_p}{\partial v_G}\right)_{V_p} \approx \frac{\Delta i_p}{\Delta v_G}\Big|_{V_p}$$

v_{G1}	-.5	-1	-1.5	-2	-2.5	-3	V
g_m	6	5.5	4.9	4.4	3.4	2.07	m℧



(7-11) a) The slope of the plate characteristic is 0.4 mA in 400V or

$r_p = \frac{400}{0.4} 10^3 = 1\text{ M}$ ←

b)

V_G	-2	-2.5	-3
I_p	5.8	3.8	2.4

$g_m = \frac{\Delta I_p}{\Delta V_G}\Big|_{V_p=200} = \frac{5.8 - 3.8}{0.5} = 4$ m℧ ←

c) $g_m = \frac{3.8 - 2.4}{0.5} = \frac{1.4}{0.5} = 2.8$ m℧ ←

d) $(g_m)_{av} = \frac{4 + 2.8}{2} = 3.4$ m℧ ←

e) $\mu = g_m r_p = (3.4 \times 10^{-3})(10^6) = 3400$ ←

$\mu = \frac{\Delta V_P}{\Delta V_G}\Big|_{I_p}$ ∴ For $\Delta V_G = 0.5$V , $\Delta V_P = 3400 \times 0.5 = 1700$V which is way off the curve.

(7-12) a) From the plate characteristics of Fig. 7-12 , we obtain at the intersection of the load line with the −2 V bias voltage curve:

$I_P = 5.9$ mA and $V_P = 185$V ←

b) If the peak-to-peak swing is 1V around the quiescent point of part (a) the output peak-to-peak swing is 225−140 = 85 V ←

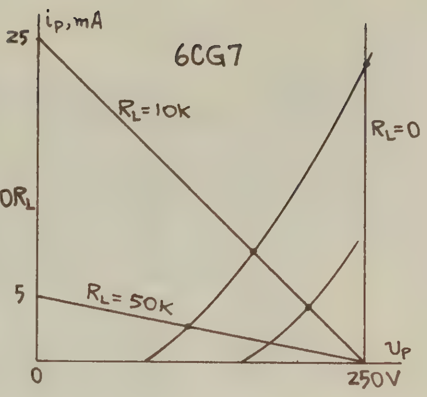
(7-13) a) From the load lines for $R_L = 0$, 10K and 50K we find the intersection with $V_G = -4V$ to be, respectively, 23, 8.5 and 2.7mA

b) 250, 165 and 115V.

c) For $I_P = 10$ mA and $V_G = -4V$ we find $V_P = 175 = 250 - 10R_L$ or $R_L = 7.5K$

d) For $V_G = -8V$ and $R_L = 10K$, $V_P = 210V$

Hence, $I_P R_L = 250 - 210 = 40V$



(7-14) a) For $V_P = 250V$ and $V_G = -2V$ we find that $I_P = 1.2$ mA.

$V_{PP} = V_P + I_P R_L$. For $R_L = 50K$, $V_{PP} = 250 + 1.2 \times 50 = 310V$

For $R_L = 100K$, $V_{PP} = 250 + 1.2 \times 100 = 370V$; For $R_L = 200K$, $V_{PP} = 490V$

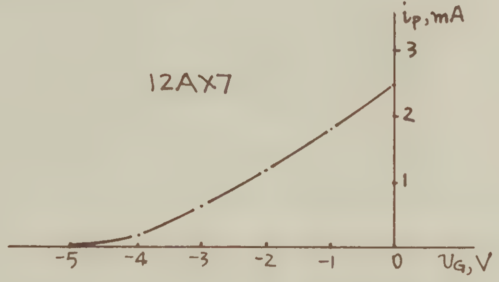
b) From load lines drawn through the Q-point: $I_P = 1.2$mA, $V_P = 250$, we find:

R_L	V_G	$I_P = I_{max}$	R_L	V_G	$I_P = I_{min}$
50K	0	3.2 mA	50K	-4V	0 mA
100K	0	2.53	100K	-4	0.2
200K	0	2.0	200K	-4	0.5

c)

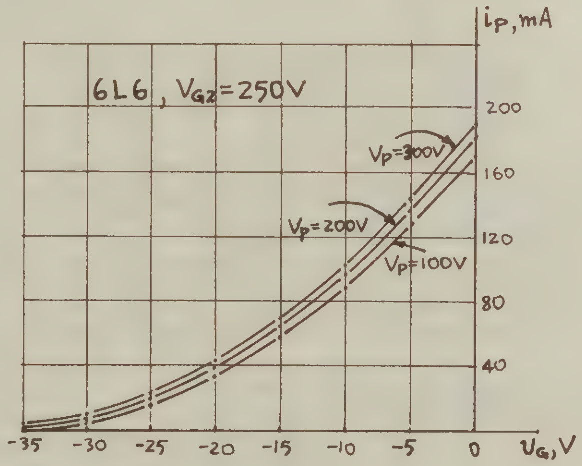
Dynamic Transfer Characteristic
with $R_L = 100K$, $V_{PP} = 250V$

v_G	0	-1	-2	-3	-4	-5
i_P	2.51	1.84	1.2	0.62	0.2	0



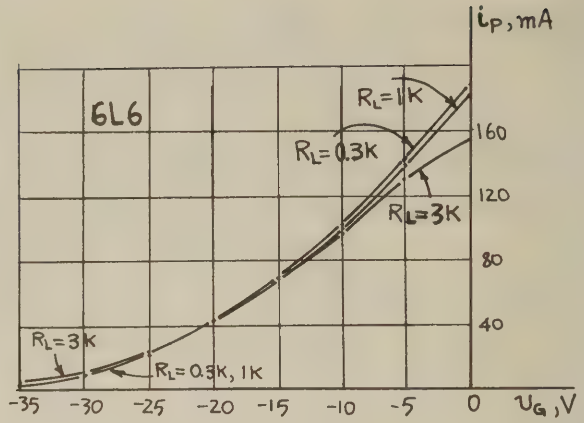
(7-15)

v_{G1}, V	i_P @ V_P=100V	i_P @ V_P=200V	i_P @ V_P=300V
0	171mA	183mA	190mA
-5	130	137	144
-10	90	98	103
-15	56	65	69
-20	33	40	43
-25	14	20	22
-30	5	7	9



(7-16)

v_G, V	i_P (0.3K)	i_P (1K)	i_P (3K)
0	188	182	155
-5	142	140	130
-10	102	101	99
-15	69	69	69
-20	43	44	45
-25	22	23	25
-30	10	10	11
-35	4	4	5



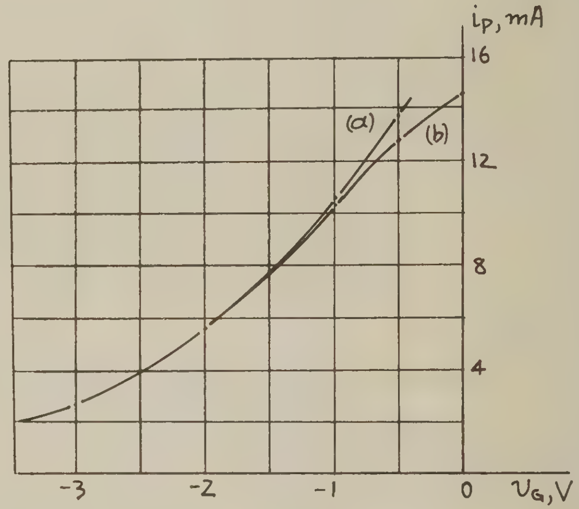
(7-17)

a)

v_G	-0.5	-1	-2	-3
i_P	13.7	10.7	5.8	2.5

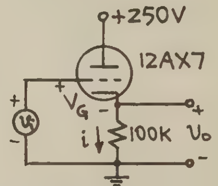
b)

v_G	0	-0.5	-1	-2
i_P	14.4	12.9	10.6	5.8



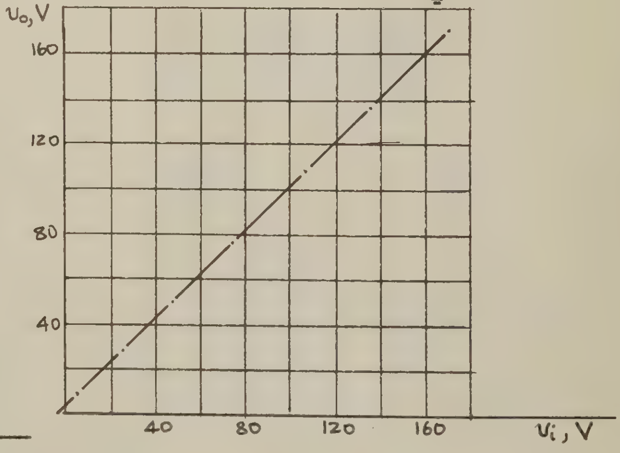
(7-18) a) If the tube is cutoff, $i = 0$. Thus $v_o \approx 0$ and $v_i = V_G$ (cutoff) $= -3.2$ V (From Fig. D-6) ←

b) From a 100K load line and $V_G = 0$ we find $i = 1.7$ mA. Hence, $v_i = v_o = 170$ V ←



c)

v_G	i_P (from L.L.)	$v_i = v_G + 100 i_P$	$v_o = 100 i_P$
-3.2	≈0 mA	-3.2 V	≈0 V
-3	0.08	5	8
-2.5	0.26	23.5	26
-2	0.50	48.0	50
-1.5	0.775	76	77.5
-1	1.075	106.5	107.5
-0.5	1.38	137.5	138
0	1.70	170	170

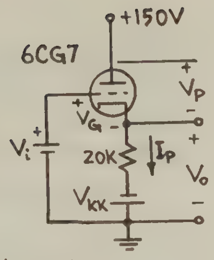


d) From the plot we have:

$$A = \frac{\Delta v_o}{\Delta v_i} = \frac{170-26}{170-23.5} = 0.983 \leftarrow$$

(7-19) From the circuit, KVL in the plate loop gives:

$$150 + V_{KK} = V_P + 20 I_P$$

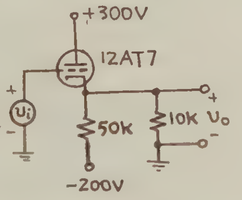


Draw a load line through the points $V_P = 150 + V_{KK}$, $I_P = 0$ and $V_P = 0$, $I_P = \frac{150+V_{KK}}{20}$ for $V_{KK} = 0, 50, 100$ and $150V$. KVL in the grid loop gives: $-V_i + V_G + 20I_P - V_{KK} = 0$. Thus, the bias curve is defined by $I_P = \frac{V_i + V_{KK} - V_G}{20}$ and the intersection of this curve and the load line gives I_P and V_G; Then, $V_o = 20I_P - V_{KK}$ or $V_o = V_i - V_G$. We find in tabulated form:

a) $V_i = 0$	V_{KK}	I_P	V_G	V_o
	0	0.4	−8	8
	50	2.8	−6	6
	100	5.2	−4	4
	150	7.65	−3	3

b) $V_i = 50$	V_{KK}	I_P	V_G	V_o
	0	2.65	−3	53
	50	5.1	−2	52
	100	7.55	−1	51
	150	10	0	50

(7-20) a) If $v_o = 0$, then there are 200V across 50K or 4 mA tube current. For $V_P = 300V$ and $I_P = 4\,mA$, we get from Fig. D-4, $V_G = -4.5V = v_i$ ←



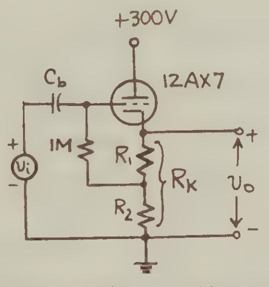
b) If $v_i = -100V$, the voltage from cathode to ground is slightly larger, say, −95 V. Hence, the tube current is $\frac{200-95}{50} - \frac{95}{10} \approx 2 - 9.5 = -7.5\,mA$. Since a negative current is impossible, then the tube must be cut off. Then,

$$v_o = (-200 \times 10)/(10+50) = -33.3V \leftarrow$$

As a check: $v_{gK} = -100 - (-33.3) \doteq -67V$ which is well below cut off.

c) If $v_o = 50V$, then $I_P = (50+200)/50 + \frac{50}{10} = 10\,mA$ and $V_P = 250V$. Then, from Fig. D-4 we find $V_G = -2\,V$ ←

(7-21) We must choose R_K so that the output swing from cutoff to $V_G = 0$ is at least 150V. If we take $R_K = 100K$, then at $V_G = 0$, $I_P = 2\,mA$ and $I_P R_K = 200V$. Hence, this is a good choice, $R_K = 100\,K$.



We choose the quiescent point in the middle of the load line, between $V_G = 0$ and cutoff. Hence, if we take $V_G = -1.5V$, we find $I_P = 1.06\,mA$ and $V_P = 194V$. Since the gain of the cathode follower is approximately unity, the voltage across the tube for ±75V input is 194 ± 75 or from 119 to 269 V. Since these extreme points are within the grid base of the tube ($V_G = 0$, $V_P = 95V$ and $V_G = -4$ $V_P = 300V$), then this is a satisfactory design.

$$\text{Hence,}\quad R_1 = |V_G|/I_P = (1.5)/(1.06 \times 10^{-3}) = 1.41\,K \leftarrow$$

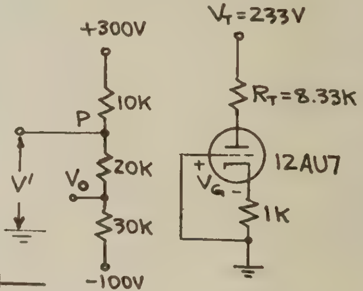
With $R_1 = 1.41K \ll R_K = 100K$, the above results will not change appreciably, if R_2 is chosen equal to 100K. ←

(7-22) Draw a load line for $R = 10K$ and $V_{PP} = 200V$. Then we find that the current at $V_G = 0$ is 10.7 mA. Cutoff is at $V_G = -12V$. In order to avoid distortion near cutoff, let us allow the grid to swing no more negative than $-10V$. At $V_G = -10$, $I_P = 0.7$ mA. Hence, the total swing is $\Delta I_P = 10.7 - 0.7 = 10$ mA

The quiescent current is $10.7 - \frac{10}{2} = 5.7$ mA. The corresponding bias is $V_G = -4V$. Hence, $R_1 = \frac{4}{5.7} = 0.7K = 700\ \Omega$ ◁——

At $V_G = 0$, $I_P = 10.7$ and $v_i = v_o = 10.7 \times 10 = 107$ V. At $V_G = -10$, $I_P = 0.7$ mA and $v_o = 0.7 \times 10 = 7$ V. Thus $v_i = -10 + 7 = -3V$. Hence, the total peak-to-peak input signal is $107 - (-3) = 110V$ ◁——

Incidentally, the total output swing is $107 - 7 = 100V$ which means that the voltage gain is $\frac{100}{110} = 0.91$



(7-23) a) From Thévenin's Theorem, $V_T = 300 \times \frac{50}{60} - 100 \times \frac{10}{60} = 233V$

and $R_T = \frac{50 \times 10}{50 + 10} = 8.33K$

Draw a load line through $v_P = 233V$ and $i_P = \frac{233}{9.33} = 25$ mA

The bias curve is given by $i_P = -\frac{v_G}{1K} = -v_G$ mA

The intersection is at $V_P = 175V$ and $I_P = 6.2$ mA ◁——

b) The voltage from plate to ground, V', is $V' = I_P R_K + V_P = 6.2 + 175 = 181.2V$

As a check: $V' = 233 - (8.33)(6.2) = 181.3$ V.

Using superposition, we have: $V_o = \frac{30}{50} V' - \frac{20}{50}(100) = 108.5 - 40 = 68.5$ V ◁——

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CHAPTER 8

(8-1) a) $i_P = 4 \times 10^{-4} (v_G + 0.1 v_P)^{1.5}$ A. $g_P = \left.\frac{\partial i_P}{\partial v_P}\right|_{V_G} = 6 \times 10^{-5} (v_G + 0.1 v_P)^{0.5} = 6 \times 10^{-5} (-20 + 45)^{0.5}$

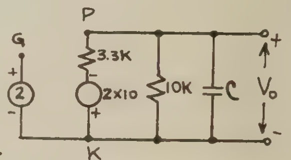
or $g_P = 3 \times 10^{-4}$ and $r_P = \frac{1}{g_P} = \frac{1}{3} \times 10^4 = 3.33K$; $\mu = -\left.\frac{\partial v_P}{\partial v_G}\right|_{I_P} = 10$ ◁——

b) $I_P = 4 \times 10^{-4} (-20 + 45)^{1.5} = 50 \times 10^{-3}$ A $= 50$ mA

Then, $V_{PP} = V_P + I_P R_L = 450 + 50 \times 10 = 950$ V ◁——

c) $X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 10^3 \times 0.016} = 10K$, $Z_L = \frac{10 \times (-j10)}{10 - j10} = (5 - j5)$ K

From the equivalent circuit shown, we find using voltage division, $V_o = \frac{-20(5 - j5)}{5 - j5 + 3.3} = 14.5 \angle 166°$ V ◁——

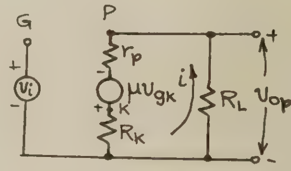


(8-2) a) From the equivalent circuit shown, we have,

$-\mu v_{gk} + i(R_K + R_L + r_p) = 0$ where $v_{gk} = v_i - iR_K$.

or $-\mu(v_i - iR_K) + i(R_K + R_L + r_p) = 0$ $\therefore\ i = \frac{\mu v_i}{R_L + [r_p + (\mu+1)R_K]}$

Hence, $v_{op} = -iR_L = -\frac{\mu R_L v_i}{R_L + [r_p + (\mu+1)R_K]}$ ◁——



b) Considering R_L external to the amplifier, we have $V_{Th} = V_{oc} = \lim_{R_L \to \infty} (-iR_L)$

or $V_{Th} = \lim_{R_L \to \infty} \frac{-\mu R_L v_i}{R_L + [r_p + (\mu+1)R_K]} = -\mu v_i$; $I_{sc} = \lim_{R_L \to 0} (i) = \frac{\mu v_i}{r_p + (\mu+1)R_K}$

Hence, $Z_{out} = \frac{V_{oc}}{-I_{sc}} = r_p + (\mu+1)R_K = R_o$ QED

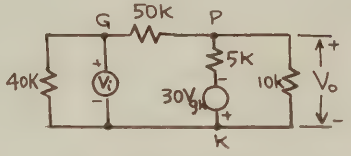
(8-3) a) The same equivalent circuit and expression for the current is used as in Prob. 8-2.

$$\text{Then,}\quad v_{oK} = iR_K = \frac{\mu R_K v_i}{R_L + r_p + (\mu+1)R_K}$$

b) $$V_{Th} = V_{oc} = \lim_{R_K \to \infty} (iR_K) = \lim_{R_K \to \infty} \frac{\mu R_K v_i}{R_L + r_p + (\mu+1)R_K} = \frac{\mu v_i}{\mu+1}$$

$$I_{sc} = \lim_{R_K \to 0} (i) = \lim_{R_K \to 0} \frac{\mu v_i}{R_L + r_p + (\mu+1)R_K} = \frac{\mu v_i}{R_L + r_p} \ ; \quad R_o = \frac{V_{oc}}{I_{sc}} = \frac{R_L + r_p}{\mu+1} \qquad \text{QED}$$

(8-4) From the equivalent circuit, we see that $V_{gk} = V_i$

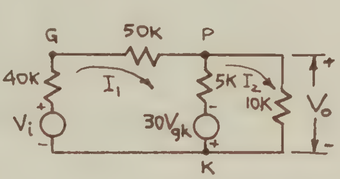


If the output were short-circuited, the short-circuit current would be, $I_{sc} = \frac{V_i}{50} - \frac{30V_i}{5} = -\frac{299}{50}V_i$

If the input generator is shorted, the admittance seen looking back from the output is $Y = \frac{1}{50} + \frac{1}{5} + \frac{1}{10} = \frac{16}{50}$

Then $V_o = \frac{I_{sc}}{Y} = \frac{-299V_i}{50 \times 16/50} = -18.7V_i \qquad \therefore A = \frac{V_o}{V_i} = -18.7$ ←

(8-5) KVL in the two loops gives:



$-V_i - 30V_{gk} + 95I_1 - 5I_2 = 0 \qquad (1)$

$30V_{gk} - 5I_1 + 15I_2 = 0 \qquad (2) \quad$ where $V_{gk} = V_i - 40I_1 \quad (3)$

Substituting (3) into (1) and (2) we get:

$-31V_i + 1295I_1 - 5I_2 = 0 \qquad (4)$

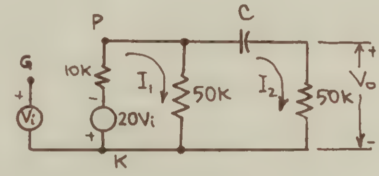
$30V_i - 1205I_1 + 15I_2 = 0 \qquad (5)$. Adding (4) and (5): $-V_i + 90I_1 + 10I_2 = 0 \qquad (6)$

From (6) we have: $I_1 = \frac{V_i}{90} - \frac{I_2}{9}$. (7). From (5) and (7) then we get:

$30V_i - \frac{1205}{90}V_i + \frac{1205}{9}I_2 + 15I_2 = 0 \qquad$ or $\quad I_2 = -0.111V_i$

But $A = \frac{V_o}{V_i} = \frac{10I_2}{V_i} = -1.11$ ←

(8-6) $X_c = \frac{1}{2\pi \times 10^3 \times 3 \times 10^{-9}} = 53K$



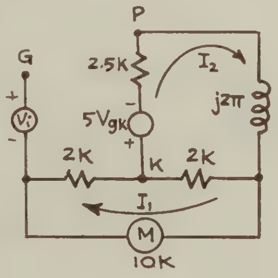
Meshes 1 & 2 give: $\quad 20V_i + 60I_1 - 50I_2 = 0$

$\qquad -50I_1 + (100 - j53)I_2 = 0$

or $I_1 = (2 - j1.06)I_2 \quad$ and $-20V_i = 60(2 - j1.06)I_2 - 50I_2 \ ; \quad V_o = 50I_2 = \frac{-50 \times 20V_i}{70 - j63.6}$

or $V_o = (-7.8 + j7.1)V_i \quad$ and $\quad A = \frac{V_o}{V_i} = -7.8 - j7.1 = 10.6\,\underline{/-137.7^\circ}$ ←

(8-7) Meshes 1 & 2 give: $\quad 14I_1 - 2I_2 = 0 \quad$ or $\quad I_2 = 7I_1$



$\qquad 5V_{gk} - 2I_1 + (4.5 + j2\pi)I_2 = 0$

where $V_{gk} = V_i + 2I_1 = 12 + 2I_1$

Substituting, we have: $5(12 + 2I_1) - 2I_1 + (4.5 + j2\pi)(7I_1) = 0$

or $I_1 = \frac{-60}{39.5 + j43.96} \ ; \quad |I_1| = 1.015\ mA$

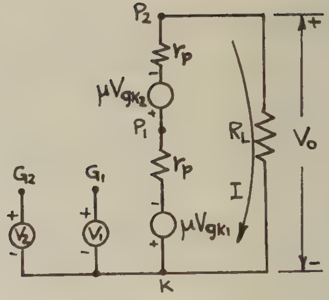
Voltmeter thus reads $10 \times |I_1| = 10.15\ V$ ←

(8-8) $I(R_L+2r_p)+\mu V_{gk1}+\mu V_{gk2}=0$ where $V_{gk1}=V_1$

and $V_{gk2}=V_2+\mu V_{gk1}+Ir_p = V_2+\mu V_1+Ir_p$

$$I(R_L+2r_p)+\mu V_1+\mu V_2+\mu^2V_1+\mu I r_p=0$$

$$\text{Hence,}\quad V_o=IR_L=\frac{-\mu(\mu+1)V_1-\mu V_2}{R_L+(\mu+2)r_p}R_L \quad\longleftarrow$$



(8-9) $21.5I+20V_{gk1}+20V_{gk2}=0$ where $V_{gk1}=V_1+0.5I$

and $V_{gk2}=V_2+10.5I+20V_{gk1}=V_2+20.5I+20V_1$

Substituting, $21.5I+20V_1+10I+20V_2+410I+400V_1=0$

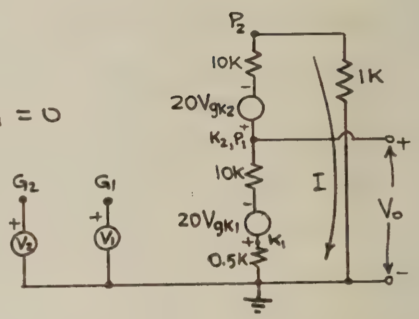
a) If $V_2=0$, $I=-\dfrac{420V_1}{441.5}=-0.951V_1$

$V_o=-10.5I-20V_{gk1}=-20.5I-20V_1$

$$\frac{V_o}{V_1}=20.5\times0.951-20=-0.50 \quad\longleftarrow$$

b) If $V_1=0$, $I=-\dfrac{20V_2}{441.5}=-0.0453V_2$; $V_o=-10.5I-20V_{gk1}=-20.5I$

$$\frac{V_o}{V_2}=(20.5)(0.0453)=0.928 \quad\longleftarrow$$

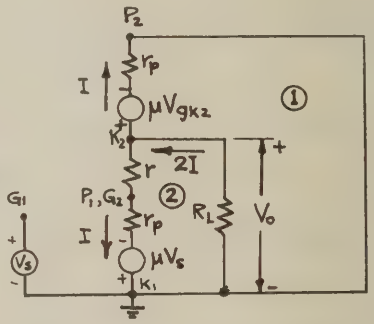


(8-10) a) The equivalent circuit under the condition that the signal current is the same in both tubes is indicated.

KVL in loop 1 gives: $2IR_L+\mu V_{gk2}+Ir_p=0$

where $V_{gk2}=-Ir_p$. Thus $I(2R_L+r_p)=I\mu r$

$$\therefore\ r=\frac{2R_L+r_p}{\mu}=\frac{2R_L}{\mu}+\frac{1}{g_m} \qquad \text{QED}$$



b) KVL in loop 2 gives:

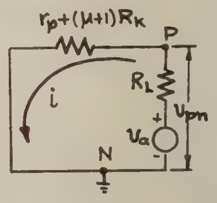
$$(2R_L+r+r_p)I=\mu V_s\ ;\quad A=\frac{V_o}{V_s}=\frac{-2IR_L}{V_s}=-2\left(\frac{\mu}{2R_L+r+r_p}\right)R_L=\frac{-2R_L\mu}{(\mu+1)r}$$

$$\text{or}\ A=\frac{-2R_L\mu^2}{(\mu+1)(2R_L+r_p)}=-\frac{\mu^2}{\mu+1}\ \frac{R_L}{R_L+r_p/2} \qquad \text{QED}$$

(8-11) a) Looking into the plate, we see the equivalent circuit:

$i[r_p+(\mu+1)R_K+R_L]=v_a$ or $i=\dfrac{v_a}{r_p+(\mu+1)R_K+R_L}$

$$v_{pn}=i[r_p+(\mu+1)R_K]=\frac{r_p+(\mu+1)R_K}{r_p+(\mu+1)R_K+R_L}v_a \quad\longleftarrow$$

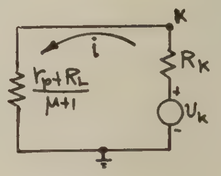


b) $$v_{kn}=iR_k=\frac{v_aR_K}{r_p+(\mu+1)R_K+R_L} \quad\longleftarrow$$

c) Looking into the cathode, we see the equivalent circuit:

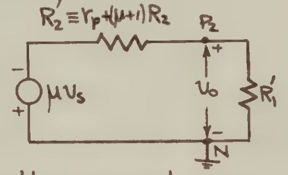
$$i\left(R_K+\frac{r_p+R_L}{\mu+1}\right)=v_K.\ \text{Thus}\ v_{pn}=-iR_L=\frac{-(\mu+1)R_Lv_K}{r_p+R_L+(\mu+1)R_K} \quad\longleftarrow$$

$$\text{and}\ v_{kn}=i\left(\frac{r_p+R_L}{\mu+1}\right)=\frac{(r_p+R_L)v_K}{r_p+R_L+(\mu+1)R_K} \quad\longleftarrow$$



(8-11) (Cont'd) d) If $R_L = 0$ and $(\mu+1)R_K >> r_p$, then from (b) $v_{kn} \approx \frac{v_a R_K}{(\mu+1)R_K} = \frac{v_a}{\mu+1}$.
Thus, any change in V_{PP} is divided by the factor $(\mu+1) >> 1$.
From (c), $v_{kn} \approx \frac{r_p v_K}{(\mu+1)R_K}$ and any change in V_{KK} is divided by $(\mu+1)\frac{R_K}{r_p} >> 1$.

Hence, the output voltage of a cathode follower is insensitive to ripple in either the positive or the negative supply voltages.

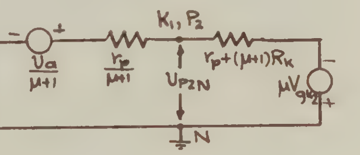


(8-12) Between the plate of V1 and the bottom of R_1 the circuit behaves as a resistor whose value is $r_p + (\mu+1)R_1 \equiv R_1'$. Since the plate of V1 is at signal ground N, and the bottom of R_1 is at the plate P_2 of V2, then this R_1' is the load on V2. Looking into the plate of V2, we see the circuit shown.
If $R_1 = R_2 = R$, then $R_1' = R_2'$ and $v_o = -\frac{\mu v_s}{2}$ or $\frac{v_o}{v_s} = -\frac{\mu}{2}$
The impedance between P_2 and N is $R_1' \parallel R_2'$ or $\frac{1}{2}R_1' = \frac{1}{2}[r_p + (\mu+1)R]$

(8-13) The analysis is given in Prob. 8-12. $R_1' = r_p + (\mu+1)R_1 = 62 + 101 \times 2 = 264K$
$R_2' = 62 + 101 \times 1 = 163K$. From the equivalent circuit, we have:

$$v_o = \frac{-\mu v_s R_1'}{R_1' + R_2'} = \frac{-100 \times 5 \times 264}{264 + 163} = -309 \text{ V}$$

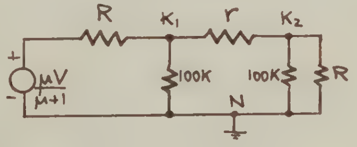
$$R_o = \frac{R_1' R_2'}{R_1' + R_2'} = \frac{264 \times 163}{264 + 163} = 101 \text{ K}$$



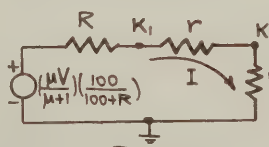
(8-14) The signal voltage from G_2 to ground is $\frac{100}{150}v_a = .667$ mV
The signal voltage from G_1 to ground is zero. Looking into K_1 and also P_2 we see the equivalent circuit shown,
where $\frac{r_p}{\mu+1} = \frac{7.2K}{48} = 0.15$ K, $r_p + (\mu+1)R_K = 7.2K + 48 \times 0.1K = 12K$, $\mu V_{gk_2} = 47 \times 0.667 = 31.4$ mV
$\frac{v_a}{\mu+1} = \frac{1}{48} = 0.0211$ mV.

Hence, $v_{P2N} = \frac{12}{12.15}(0.0211) - \frac{0.15}{12.15}(31.4) = -0.367$ mV

(8-15) a) If ided cathode followers are assumed, then K_2 is at ground potential and K_1 is at a potential V above ground. Hence, V is impressed directly across r and $r = V/I = 10/0.2 = 50$ K



The equivalent circuit looking into each cathode is as shown with $R = \frac{r_p}{\mu+1} = \frac{7.7K}{21} = 0.367$ K
We may approximate R in parallel with 100K by R.
Using Thévenin's theorem to the left of K_1 and N, we obtain the one-loop circuit indicated.


R
K_1
r
K_2
$\left(\frac{\mu V}{\mu+1}\right)\left(\frac{100}{100+R}\right)$
I
R

$$I = \left(\frac{200}{21} \times \frac{100}{100.367}\right)\left(\frac{1}{r + 2 \times 0.367}\right) = 0.2 \text{ mA} \quad \text{or} \quad r = 46.7K$$

b) The parallel combination of 100K and R is $\frac{100R}{100+R} = \frac{R}{1 + R/100} \approx R\left(1 - \frac{R}{100}\right) \approx R$
In the approximation used in (a), the second order term, $R^2/100$ was neglected.

(8-15) (Cont'd) The Thévenin's generator is $\frac{200}{21}\cdot\frac{100}{100+R} = 9.52\,\frac{1}{1+R/100} \approx 9.52(1-\frac{R}{100})$

Then, $I = \frac{9.52\,(1-R/100)}{r+2R} = \left(\frac{9.52}{r}\right)\left(\frac{1-R/100}{1+2R/r}\right) \approx \frac{9.52}{r}\left(1-\frac{R}{100}\right)\left(1-\frac{2R}{r}\right)$

$\approx \left(\frac{9.52}{r}\right)\left(1-\frac{R}{100}-\frac{2R}{r}\right) = \left(\frac{9.52}{r}\right)(1-0.0527R)$ where higher order terms have been neglected.

$0.0527R = 0.0527\left(\frac{r_p}{\mu+1}\right) = 0.0527\times0.367 = 0.019$. If r_p decreases by 10% and μ stays constant, then R decreases by 10% or $0.0527R$ by 0.0019. Thus, the current increases by 0.19% and the voltmeter readings should be decreased by 0.19% in order to be correct.

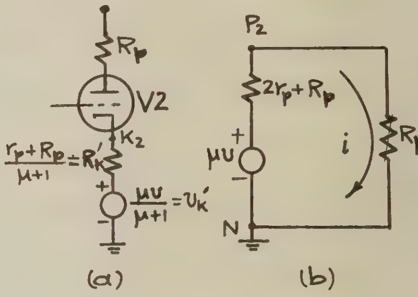
(8-16) From Eq. (8-30), $A = \frac{\mu R_K}{r_p+(\mu+1)R_K} = \frac{20\times25}{r_p+21\times25} = \frac{500}{r_p+525}$

For $r_p = 24K$, $A = \frac{500}{549}$ and for $r_p = 8K$, $A = \frac{500}{533}$

$\therefore$ The % change in gain is $\frac{\frac{500}{533}-\frac{500}{549}}{\frac{500}{549}}\times100\% = \frac{549-533}{533}\times100\%$

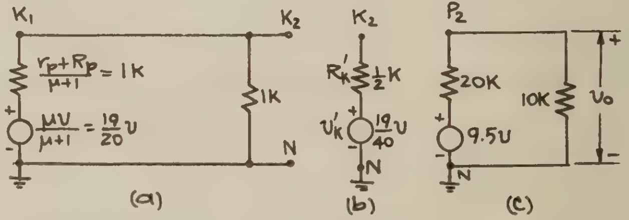
$= \frac{1600}{533}\% = 3.0\%$ QED.

(8-17) The equivalent circuit seen looking into the cathode of V1 is shown in (a). Looking into the plate of V2 we see a resistance $r_p+(\mu+1)R_K' = 2r_p+R_p$ and a generator $(\mu+1)v_K' = \mu v$. Hence, the equivalent circuit is as indicated in (b), from which the current is $i = \frac{\mu v}{2(r_p+R_p)}$ ←



(8-18) a) The equivalent circuit seen looking into the cathode K_1 is as in (a). The Thévenin's equivalent circuit at K_2 is indicated in (b).

Looking into the plate of V2 we see a resistance $r_p+(\mu+1)R_K' = 10+20\times\frac{1}{2} = 20K$ and a generator $(\mu+1)v_K' = 20(\frac{19}{40}v) = 9.5v$. The equivalent circuit between P_2 and ground is shown in (c); from this circuit the gain is $A = \frac{v_o}{v} = \frac{9.5\times10}{30} = 3.17$ ←



b) From (c) the impedance between P_2 and N is $20K \| 10K = \frac{20\times10}{30} = 6.67K$ ←

(8-19) a) From Fig. 8-16 b it is obvious that $i_1 > i_2$, since $i_2 = \frac{R_K i_1}{R_K + \frac{r_p+R_p}{\mu+1}}$

b) $\frac{i_1}{i_2} = 1 + \frac{r_p+R_p}{(\mu+1)R_K}$. If i_1 is to exceed i_2 by less than 10%, then $\frac{r_p+R_p}{(\mu+1)R_K} < 0.1$ or $R_K > 10\,\frac{r_p+R_p}{\mu+1}$ QED

(8-20) a) From Fig. 8-16b the short-circuit current, I_{sc} from the cathodes to ground is $I_{sc} = \left(\frac{\mu v}{\mu+1}\right) / \left(\frac{r_p+R_p}{\mu+1}\right) = \frac{\mu v}{r_p+R_p}$. The impedance, R between the cathodes and ground is R_K in parallel with $\frac{1}{2}\frac{r_p+R_p}{\mu+1}$ or

$$R = \frac{\frac{1}{2}\left(\frac{r_p+R_p}{\mu+1}\right)R_K}{\frac{1}{2}\left(\frac{r_p+R_p}{\mu+1}\right)+R_K} = \frac{(r_p+R_p)R_K}{r_p+R_p+2(\mu+1)R_K}$$

Hence, the voltage across R_K is $v_K = I_{sc}R = \frac{\mu v R_K}{r_p+R_p+2(\mu+1)R_K}$

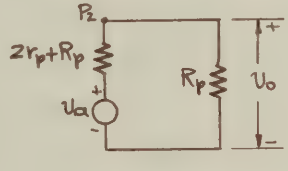
$$\text{or } v_K = \frac{1}{2}\left(\frac{\mu v}{\mu+1}\right)\frac{1}{1+\frac{r_p+R_p}{2(\mu+1)R_K}} \qquad \text{QED}$$

If $2(\mu+1)R_K \gg (r_p+R_p)$ and $\mu \gg 1$, then $v_K \approx \frac{v}{2}$

b) If $R_K \gg \frac{r_{p2}+R_{p2}}{\mu+1}$, then R_K may be omitted from Fig. 8-16b. Then the current in V1 is $i_1 \approx \frac{\frac{\mu v}{\mu+1}}{\frac{r_{p1}+R_{p1}}{\mu+1}+\frac{r_{p2}+R_{p2}}{\mu+1}}$; $|A| = \frac{v_o}{v} = \frac{i_1 R_{p1}}{v} \approx \frac{\mu R_{p1}}{R_{p1}+r_{p1}+r_{p2}+R_{p2}}$

Since $i_1 \approx i_2$, then as the current in V1 decreases from the quiescent value, the current in V2 increases by an equal amount. Hence, as r_{p1} increases, r_{p2} decreases by an almost equal amount and $r_{p1}+r_{p2}$ stays approximately constant. Thus, the gain, A is almost independent of current and therefore the amplitude distortion is low.

(8-21) a) The equivalent circuit seen looking into the cathode K_1 is a resistor $\frac{r_p+R_p}{\mu+1} \equiv R$ and a generator $\frac{v_a}{\mu+1} \equiv v_K$. The equivalent circuit seen looking into the plate P_2 of V2 is a resistor $r_p+(\mu+1)R = 2r_p+R_p$ and a generator $(\mu+1)v_K = v_a$ as shown:



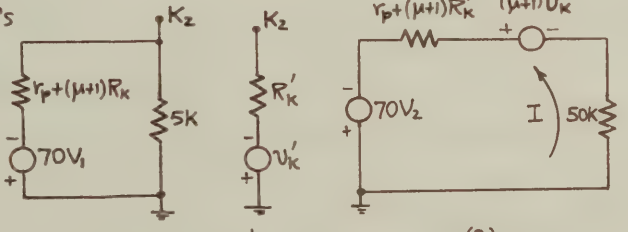
From this circuit it follows that $v_o = \frac{R_p v_a}{2(r_p+R_p)}$

b) The output impedance is R_p in parallel with $2r_p+R_p$

$$\text{or } Z_o = \frac{R_p(2r_p+R_p)}{2(R_p+r_p)} \qquad \text{QED}$$

(8-22) The equivalent circuit seen at the plate of V1 is shown in (a), where $r_p+(\mu+1)R_K = 44+71\times1 = 115K$. Its Thévenin's equivalent is indicated in (b) where $v_K' = (70V_1)\frac{5}{120} = 2.92V_1$ and $R_K' = 5 \| 115 = 4.8K$

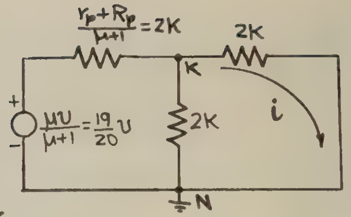
Looking at the plate of V2 we see the circuit shown in (c) where $r_p+(\mu+1)R_K' = 44+71\times4.8 = 385K$ and $(\mu+1)v_K' = 71\times2.92V_1 = 207V_1$



(a) (b) (c)

From the circuit in (c) we have: $I = \frac{207V_1+70V_2}{385+50} = 0.477V_1+0.161V_2 \leftarrow$

(8-23) a) From the equivalent circuit shown we have:



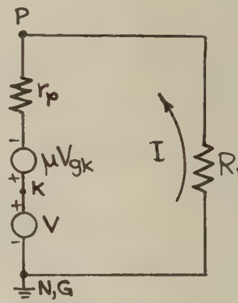
$\frac{r_p+R_p}{\mu+1}=2K$
2K
K
+
-
$\frac{\mu v}{\mu+1}=\frac{19}{20}v$
2K
i
N

$$i=\frac{19v}{20}\times\frac{1}{2+1}\times\frac{1}{2}=\frac{19v}{120}$$

$$A=\frac{v_{o2}}{v}=\frac{30i}{v}=\frac{19}{4}=4.75$$ ⟵

b) With $v=0$, the impedance to the left of KN is 1K. This is the equivalent cathode resistance, R_K, of V2. Looking at the plate of V2 we see a resistor $r_p+(\mu+1)R_K=10+20=30K$. Hence, the output impedance is $30K \| 30K = 15K$ ⟵

(8-24) The equivalent circuit of Fig. 8-10a is shown:



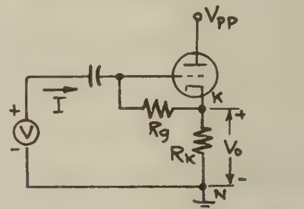
$$V_{gk}=-V \quad ; \quad I(r_p+R_p)+(\mu+1)V=0 \quad \text{or} \quad -I=\frac{(\mu+1)V}{r_p+R_p}$$

$$\text{Hence,} \quad Z_i=\frac{V}{-I}=\frac{r_p+R_p}{\mu+1} \qquad \text{QED}$$

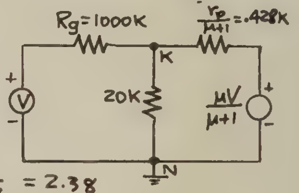
(8-25) a) If the voltage gain (with R_g in place) is A, then

$V_o=AV$ and $IR_g=V-V_o=V(1-A)$ or $I=\frac{V(1-A)}{R_g}$

Therefore, $Z_i=\frac{V}{I}=\frac{R_g}{1-A}$



b) With R_g connected between input and output, the equivalent circuit of the cathode follower becomes as indicated. From Table 7-1, p.166 $r_p=7.7K$ and $\mu=17$ for a 12AU7.



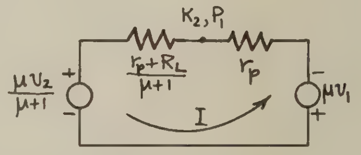
The voltage gain, $A=\frac{V_{KN}}{V}=\frac{I_{sc}Z_{KN}}{V}$ where I_{sc} is the short-circuit current between K and N.

$$I_{sc}=\frac{V}{1000}+\frac{0.944V}{0.428}=2.205V \;;\; Y_{KN}=\frac{1}{Z_{KN}}=\frac{1}{1000}+\frac{1}{20}+\frac{1}{0.428}=2.38$$

Thus, $A=\frac{2.205}{2.38}=0.926$. If R_g is considered infinite, the same result is obtained for the gain.

$$\text{Hence,} \quad Z_i=\frac{1M}{1-0.926}=13.5M$$ ⟵

(8-26) KVL gives: $\frac{\mu v_2}{\mu+1}+\mu v_1+I\left(r_p+\frac{r_p+R_L}{\mu+1}\right)=0$

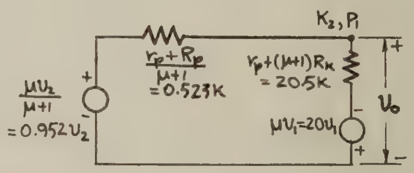


or $\mu(\mu+1)v_1+\mu v_2=-I[r_p+R_L+(\mu+1)r_p]$

$$\text{Hence,} \quad v_o=IR_L=-\frac{\mu(\mu+1)v_1+\mu v_2}{R_L+(\mu+2)r_p}R_L$$ ⟵

(8-27) a) If $v_2=0$, $\frac{v_o}{v_1}=\frac{-20\times0.523}{0.523+20.5}=-0.5$ ⟵

b) If $v_1=0$, $\frac{v_o}{v_2}=\frac{0.952\times20.5}{0.523+20.5}=0.928$ ⟵



K_2, P_1
$\frac{r_p+R_p}{\mu+1}=0.523K$
$r_p+(\mu+1)R_K=20.5K$
$\frac{\mu v_2}{\mu+1}=0.952v_2$
+
-
v_o
$\mu v_1=20v_1$
+
-

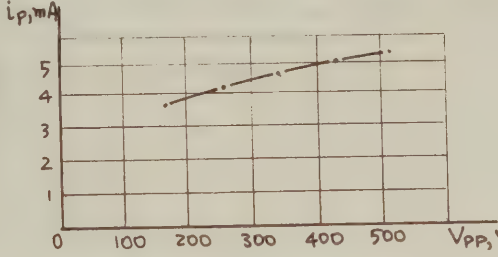
(8-28) As a first approximation, assume that $V_{gk2} = -2V$. This places K_2 or P_1 at $125+2 = 127V$. From Fig. D-4 we find for $V_{P1} = 127V$ and $V_{G1} = -2V$ that $I_{P1} = 1.9 mA$. Hence, $V_{P2K2} = 250 - 1.9 \times 10 - 127 = 104V = V_{P2}$. For $V_{P2} = 104V$ and $I_{P2} = 1.9 mA$ we find that $V_{G2} = -1.5V = V_{gk2}$ instead of -2 as assumed.

Now for $V_{P1} = 125+1.5 = 126.5V$ and $V_{G1} = -2V$ we find that $I_{P1} \approx 1.9 mA$. Hence, the quiescent current is 1.9 mA and $V_{P1} = 126.5 V$ $\longleftarrow$

(8-29) a) Grid current starts flowing when $V_{G1} \approx 0$. Then $V_{P2} = 125V$, $V_{G2} = -4V$ and $I_P = 3.6 mA$, from Fig. D-2. For $I_{P1} = I_{P2} = 3.6 mA$ and $V_{G1} = 0$, $V_{P1} = 42V$. Hence, $V_{PP} = V_{P1} + V_{P2} = 42 + 125 = 167V$

b) Since for a given V_{G2} the plate current is fairly constant with respect to plate voltage, this resembles the plate characteristic of a pentode.

V_{G1}	V_{P2}	I_P	V_{P1}	$V_{PP} = V_{P1} + V_{P2}$
-4	129	4.1	129	258
-8	133	4.5	211	344
-12	137	5.0	297	434
-16	141	5.4	380	521



(8-30) a) As a first approximation assume that $V_{gk1} = -5V$ and thus $V_{P2} = 100+5 = 105$. From Fig. D-5 we find that for $V_{P2} = 105V$ and $V_{G2} = 0$, $I_{P2} = 12.7 mA$. For $V_{P1} = 300 - 105 = 195V$ and $I_{P1} = I_{P2} = 12.7 mA$ we find that $V_{G1} \approx -4.7V$. The next approximation is $V_{P2} = 104.7V$ which gives $I_P \approx 12.7 mA$ $\longleftarrow$

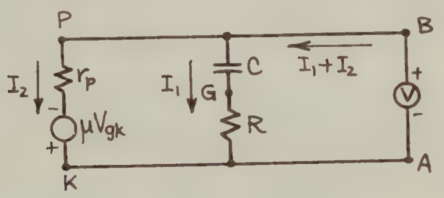
b) V2 acts as a dynamic resistance $r_p = R_{K1}$ in the cathode of V1. Hence, the impedance R_{AN} seen looking into the plate of V1 is

$$R_{AN} = r_p + (\mu+1)R_{K1} = (\mu+2)r_p = (17+2)(7.7K) = 147K \quad \longleftarrow$$

c) Looking into the cathode of V1, we see $\frac{r_p}{\mu+1}$. This is in parallel with the plate resistance of V2. Hence, the impedance R_{BN} is

$$R_{BN} = \frac{r_p\left(\frac{r_p}{\mu+1}\right)}{r_p + \frac{r_p}{\mu+1}} = \frac{r_p}{\mu+2} = \frac{7.7K}{17+2} = 404\ \Omega \quad \longleftarrow$$

(8-31) a)



b) $I_1 = VY_{CR}$, $V_{gk} = I_1 R = VRY_{CR}$, $I_2 = (V + \mu V_{gk})Y_p = (V + \mu VRY_{CR})Y_p$

$I_1 + I_2 = V(Y_{CR} + Y_p + g_m R Y_{CR})$ since $\mu = g_m r_p$ or $g_m = \frac{\mu}{r_p} = \mu Y_p$

$$\therefore Y_i = \frac{I_1 + I_2}{V} = Y_p + (1 + g_m R)Y_{CR} \qquad \text{QED}$$

c) If $g_m R \gg 1$, $Y_i \approx Y_p + g_m R Y_{CR} = \frac{1}{r_p} + \frac{g_m R}{R - j\frac{1}{\omega C}} = \frac{1}{r_p} + \frac{g_m \omega CR}{\omega CR - j} = \frac{\omega CR - j + g_m \omega CR r_p}{r_p(\omega CR - j)}$

let $\alpha \equiv \omega CR$, then, $Y_i = \frac{(\mu+1)\alpha - j}{r_p(\alpha - j)}$

(8-31) (Cont'd) $Y_i = \dfrac{[(\mu+1)\alpha - j](\alpha+j)}{r_p(\alpha^2+1)} = \dfrac{(\mu+1)\alpha^2+1}{r_p(\alpha^2+1)} + j\dfrac{\mu\alpha}{r_p(\alpha^2+1)} \equiv \dfrac{1}{R_i} + j\omega C_i$

$\therefore R_i = \dfrac{r_p(\alpha^2+1)}{1+(\mu+1)\alpha^2}$ and $C_i = \dfrac{g_m\alpha}{\omega(\alpha^2+1)}$

d) For maximum C_i, set $\dfrac{dC_i}{d\alpha} = 0$ or $\dfrac{1}{\alpha^2+1} - \dfrac{2\alpha^2}{(\alpha^2+1)^2} = 0 = \dfrac{1-\alpha^2}{(\alpha^2+1)^2}$ or $\alpha = 1$

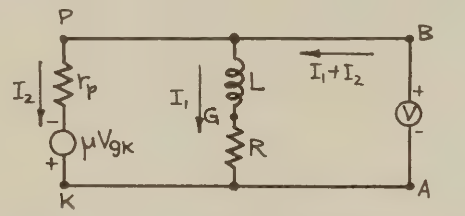
$\therefore (C_i)_{max} = \dfrac{g_m}{2\omega}$ and $(R_i)_{max} = \dfrac{2r_p}{\mu+2}$ QED

e) From Fig. 7-8, over the range 2 to 20 mA, g_m varies from 1.1 to 3.5 m$\mho$, r_p varies from 15K to 6K and μ varies from 17.3 to 21.

$\therefore (C_i)_{max}$ varies from $\dfrac{1.1\times10^{-3}}{2\pi(5000)} = 0.0176\ \mu F$ to $\dfrac{3.5\times10^{-3}}{2\pi(5000)} = 0.0557\ \mu F$ $\longleftarrow$

$(R_i)_{max}$ varies from $\dfrac{2\times15K}{2+17.3} = 1.56K$ to $\dfrac{2\times6K}{2+21} = 0.52K$ $\longleftarrow$

(8-32) a)



b) $I_1 = VY_{LR}$, $V_{gK} = I_1R = VY_{LR}R$; $I_2 = (V+\mu V_{gK})Y_p = (Y_p + g_mRY_{LR})$ since $g_m = \mu Y_p$

$Y_i = \dfrac{I_1+I_2}{V} = Y_{LR} + Y_p + g_mRY_{LR} = Y_p + (1+g_mR)Y_{LR}$ $\longleftarrow$

c) $Y_i \approx Y_p + g_mRY_{LR} = \dfrac{1}{r_p} + \dfrac{g_mR}{R+j\omega L} = \dfrac{R+j\omega L+g_mRr_p}{r_p(R+j\omega L)}$. Let $\alpha \equiv \dfrac{\omega L}{R}$

Then, $Y_i = \dfrac{\mu+1+j\alpha}{r_p(1+j\alpha)} = \dfrac{\mu+1+\alpha^2}{r_p(1+\alpha^2)} - j\dfrac{g_m\alpha}{1+\alpha^2} \equiv \dfrac{1}{R_i} - j\dfrac{1}{\omega L_i}$ (i.e. R_i in parallel with L_i)

$\therefore L_i = \dfrac{1+\alpha^2}{g_m\alpha\omega}$ and $R_i = r_p\left(\dfrac{1+\alpha^2}{1+\alpha^2+\mu}\right)$ $\longleftarrow$

d) For maximum L_i, set $\dfrac{dL_i}{d\alpha} = 0$ or $-\dfrac{1}{\alpha^2} + 1 = 0$ or $\alpha = 1$

$\therefore (L_i)_{max} = \dfrac{2}{g_m\omega}$ and $(R_i)_{max} = \dfrac{2r_p}{\mu+2}$

e) From Fig. 7-8, over the range* 2 to 20 mA (on the $V_p = 250V^*$ curves), g_m varies from 1.1 to 3.5 m$\mho$, r_p varies from 15K to 6K and μ from 17.3 to 21.

$\therefore (L_i)_{max}$ varies from $\dfrac{2}{1.1\times10^{-3}\times2\pi(5000)} = 0.0578\,H$ to $\dfrac{2}{3.5\times10^{-3}\times2\pi(5000)} = 0.0182\,H$ $\longleftarrow$

$(R_i)_{max}$ varies from $\dfrac{2\times15K}{17.3+2} = 1.56K$ to $\dfrac{2\times6K}{21+2} = 0.52K$ $\longleftarrow$

(8-33) From Eqs. (8-39) and (8-42), $A = \dfrac{-g_m+Y_{gp}}{Y_p+Y_L+Y_{pk}+Y_{gp}}$, and $Y_i = Y_{gK} + (1-A)Y_{gp}$ respectively.

For $f = 100\,kHz$

a) $Y_{gK} = j\omega C_{gK} = j(2\pi\times10^5)(4\times10^{-12}) = j2.51\times10^{-6}\ \mho$; $Y_{pk} = j\omega C_{pk} = j(2\pi\times10^5)(3.6\times10^{-12}) = j2.26\times10^{-6}\ \mho$

$Y_{gp} = j\omega C_{gp} = j(2\pi\times10^5)(2.4\times10^{-12}) = j1.51\times10^{-6}\ \mho$; $Y_L = 10^{-5}\ \mho$; $Y_p = \dfrac{1}{66{,}000} = 1.51\times10^{-5}\ \mho$

(8-33) (Cont'd) $A = \frac{-1.5\times10^{-3}+j1.51\times10^{-6}}{(1.51+1)10^{-5}+j(2.26+1.51)10^{-6}} = \frac{-1.5\times10^{-3}}{(25.1+j3.77)10^{-6}} = -58.5+j8.78 \quad \longleftarrow$

$Y_i = j2.51\times10^{-6} + (1+58.5-j8.78)(j1.51\times10^{-6}) = (1.32+j9.23)\times10^{-5}\ \mho$

b) $Y_i = \frac{1}{R_i} + j\omega C_i \quad \therefore \frac{1}{R_i} = 1.32\times10^{-5}$ or $R_i = 75.7\text{K} \quad \longleftarrow$

$C_i = \frac{9.23\times10^{-5}}{2\pi\times10^{5}} = 1.47\times10^{-10}\,\text{F} = 147\ \text{pF} \quad \longleftarrow$

For $f = 100$ Hz

a) $A = \frac{-1.5\times10^{-3}+j1.51\times10^{-9}}{(1.51+1)10^{-5}+j3.77\times10^{-9}} \approx \frac{-1.5\times10^{-3}}{2.51\times10^{-5}} = -59.8 \quad \longleftarrow$

This is the value obtained when all the interelectrode capacitances are neglected.

$Y_i = j2.51\times10^{-9} + (1+59.8)(j1.51\times10^{-9}) = j9.45\times10^{-8}\ \mho$

b) $Y_i = \frac{1}{R_i} + j\omega C_i \quad \therefore R_i = \infty$ and $C_i = \frac{9.45\times10^{-8}}{2\pi\times100} = 151\ \text{pF} \quad \longleftarrow$

(8-34) From Eq. (8-39) $A = \frac{-g_m + Y_{gp}}{Y_p + Y_L + Y_{pk} + Y_{gp}}$ and from Eq. (8-42) $Y_i = Y_{gk} + (1-A)Y_{gp}$

a) $R_L = 50\text{K}$

For $f = 10^3$ Hz

$Y_{gk} = j\omega C_{gk} = j(2\pi\times10^3)(3\times10^{-12}) = j1.88\times10^{-8}$; $Y_{pk} = j\omega C_{pk} = j(2\pi\times10^3)(11\times10^{-12}) = j6.91\times10^{-8}$

$Y_{gp} = j\omega C_{gp} = j(2\pi\times10^3)(2\times10^{-12}) = j1.26\times10^{-8}$; $Y_L = 2\times10^{-5}$, $Y_p = 10^{-4}$

$A = \frac{-2\times10^{-3}+j1.26\times10^{-8}}{1.2\times10^{-4}+j(6.91+1.26)\times10^{-8}} \approx \frac{-2\times10^{-3}}{1.2\times10^{-4}} = -16.7$

$Y_i = j1.88\times10^{-8} + (1+16.7)(j1.26\times10^{-8}) = j2.42\times10^{-7} = \frac{1}{R_i} + j\omega C_i \quad \therefore R_i = \infty \quad \longleftarrow$

and $C_i = \frac{2.42\times10^{-7}}{2\pi\times10^{3}} = 38.5\ \text{pF} \quad \longleftarrow$

For $f = 10^6$ Hz

$Y_{gk} = j1.88\times10^{-5}$, $Y_{pk} = j6.91\times10^{-5}$, $Y_{gp} = j1.26\times10^{-5}$

$A = \frac{-2\times10^{-3}+j1.26\times10^{-5}}{1.2\times10^{-4}+j(6.91+1.26)\times10^{-5}} = \frac{-200+j1.26}{12+j8.17} = -11.4+j7.74$

$Y_i = j1.88\times10^{-5} + (1+11.4-j7.74)(j1.26\times10^{-5}) = (9.75+j17.6)\times10^{-5} = \frac{1}{R_i} + j\omega C_i$

$\therefore R_i = \frac{10^5}{9.75} = 10.3\ \text{K}$ and $C_i = \frac{17.6\times10^{-5}}{2\pi\times10^{6}} = 28\ \text{pF} \quad \longleftarrow$

b) $Z_L = j5\times10^4$

For $f = 10^3$ Hz

$Y_L = -j2\times10^{-5}$, $A \approx \frac{-g_m}{Y_p + Y_L} = \frac{-2\times10^{-3}}{10^{-4}-j2\times10^{-5}} = -19.2-j3.84$

$Y_i = j1.88\times10^{-8} + (1+19.2+j3.84)(j1.26\times10^{-8}) = (-4.82+j27.3)\times10^{-8} = \frac{1}{R_i} + j\omega C_i$

$\therefore R_i = \frac{-10^8}{4.82} = -2.08\times10^{7} = -20.8\ \text{M}$ and $C_i = \frac{27.3\times10^{-8}}{2\pi\times10^{3}} = 43.5\ \text{pF} \quad \longleftarrow$

For $f = 10^6$ Hz $\quad A = \frac{-2\times10^{-3}+j1.26\times10^{-5}}{10^{-4}+j6.17\times10^{-5}} = -14.5+j8.94$

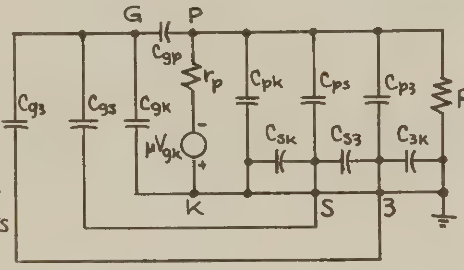
(8-34) (Cont'd) $Y_i = j1.88\times10^{-5} + (1+14.5-j8.94)(j1.26\times10^{-5}) = (1.13+j2.14)\times10^{-4} = \frac{1}{R_i} + j\omega C_i$

$\therefore R_i = \frac{10^4}{1.13} = 8.83\text{ K}$ and $C_i = \frac{2.14\times10^{-4}}{2\pi\times10^6} = 34.1\text{ pF}$ ⟵

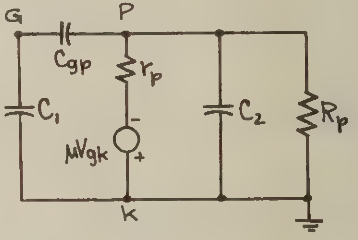
(8-35) From Eq. (8-5), $A = \frac{-\mu R_L}{R_L + r_p}$ and from Eq. (8-44), $C_i = C_{gk} + (1-A)C_{gp}$.

At $R_L = 0$, $A = 0$ and $C_i = C_{gk} + C_{gp} = 2.4 + 3.9 = 6.3\text{ pF}$. If this is multiplied by 6, then $C_i = 37.8 = C_{gk} + (1-A)C_{gp}$ or $1-A = \frac{37.8-2.4}{3.9} = 9.08$ or $A = -8.08 = -\frac{\mu R_L}{R_L + r_p}$ or $8.08R_L + 8.08\times7.7\text{K} = 20R_L$ or $R_L = 5.22\text{K}$ ⟵

(8-36) The screen S and the suppressor 3 are at ground potential as far as signal is concerned. Hence, the equivalent circuit is as shown in (a). Since some of the capacitors are short-circuited, the equivalent circuit is reduced to that in (b), where $C_1 = C_{g3} + C_{gs} + C_{gk} \approx C_{gs} + C_{gk}$ and $C_2 = C_{pk} + C_{ps} + C_{p3}$.



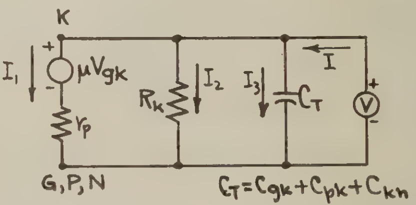
(a)



(b)

The purpose of the screen is to isolate the grid from the plate. Hence, C_{gp} and C_{g3} are very small.

(8-37) The impedance Z between K and N of Fig. 8-22 is obtained by shorting V_i, applying an external voltage V and evaluating $Z = V/I$. Since $V_{gk} = -V$, we have $I_1 = V(1+\mu)Y_p = V(Y_p + g_m)$ because $g_m = \mu Y_p$. $I_2 = VY_K$, $I_3 = VY_T$. Thus, $Y = \frac{1}{Z} = \frac{I_1+I_2+I_3}{V} = Y_p + g_m + Y_K + Y_T$



The short-circuit current across the output terminals in Fig. 8-22b (with V_i present but not V) is, $I_{sc} = V_iY_{gk} + \frac{\mu V_{gk}}{r_p} = V_i(Y_{gk} + g_m)$ since $V_{gk} = V_i$

Hence, $A = \frac{V_o}{V_i} = \left(\frac{I_{sc}}{Y}\right)\frac{1}{V_i} = \frac{g_m + Y_{gk}}{Y_K + Y_p + g_m + Y_T}$

(8-38) The output impedance of a cathode follower is given by the impedance Z in Prob. 8-37 with R_K considered external to the circuit. Thus, taking R_K to be infinite or $Y_K = 0$, we have

$Y_o = \frac{1}{Z_o} = g_m + Y_p + Y_T$ where $Y_p = \frac{1}{r_p}$ and $Y_T = j\omega C_T$ QED.

CHAPTER 9

(9-1) From Eq. (9-11), $I_{np}(0) = \dfrac{AeD_n n_{po}}{L_n}(e^{V_E/V_T} - 1) = I_{nE}$, since $n_{Eo} = n_{po}$ and $L_E = L_n$.
If in Eq. (9-17) $|V_C| \gg V_T$, then $I_{pn}(0) = \dfrac{AeD_p p_{no}}{W} e^{V_E/V_T} = I_{pE}$

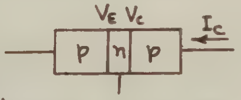
Hence,
$$\frac{I_{pE}}{I_{nE}} = \frac{AeD_p p_{no} e^{V_E/V_T}}{W} \frac{L_n}{AeD_n n_{po} e^{V_E/V_T}} \quad \text{assuming that } e^{V_E/V_T} \gg 1$$
$$= \frac{L_n D_p p_{no}}{W D_n n_{po}} \qquad (1)$$

Now, $\sigma_p = \mu_p e p_{po} = \mu_p e \dfrac{n_i^2}{n_{po}}$ and $\sigma_n = \mu_n e n_{no} = \mu_n e \dfrac{n_i^2}{p_{no}}$

or $n_{po} = \mu_p e \dfrac{n_i^2}{\sigma_p}$ and $p_{no} = \mu_n e \dfrac{n_i^2}{\sigma_n}$ (2)

Substituting Eqs. (2) into (1) we have:
$$\frac{I_{pE}}{I_{nE}} = \frac{L_n D_p}{W D_n} \frac{\mu_n e n_i^2 \sigma_p}{\sigma_n \mu_p e n_i^2} = \frac{L_n}{W} \frac{D_p/\mu_p}{D_n/\mu_n} \frac{\sigma_p}{\sigma_n} = \left(\frac{L_n}{W}\right) \frac{\sigma_p}{\sigma_n} \quad \text{since from}$$
Eq. (5-33) $\dfrac{D_p}{\mu_p} = \dfrac{D_n}{\mu_n}$. QED

(9-2) Since the conditions at the collector are identical with those at the emitter, then Eqs. (9-11) and (9-17) remain valid, if the subscripts E and C are interchanged. These equations now become:



$$I_{np} = \frac{AeD_n n_{co}}{L_C}(e^{V_C/V_T} - 1) \quad \text{and} \quad I_{pn} = \frac{-AeD_p p_{no}}{W}\left[(e^{V_E/V_T} - 1) - (e^{V_C/V_T} - 1)\right]$$

$$I_C = I_{np} + I_{pn} = a_{21}(e^{V_E/V_T} - 1) + a_{22}(e^{V_C/V_T} - 1)$$

where $a_{21} = -\dfrac{AeD_p p_{no}}{W}$ and $a_{22} = \dfrac{AeD_p p_{no}}{W} + \dfrac{AeD_n n_{co}}{L_C}$

(9-3) a) From Eq. (9-13), $p_n - p_{no} = K_1 e^{-x/L_B} + K_2 e^{+x/L_B}$. From the boundary conditions given by Eqs. (9-14), $p_n = p_{no} e^{V_E/V_T}$ @ $x = 0$ and $p_n = p_{no} e^{V_C/V_T}$ @ $x = W$ we obtain:
$$p_{no} e^{V_E/V_T} - p_{no} = K_1 + K_2 \quad \text{and} \quad p_{no} e^{V_C/V_T} - p_{no} = K_1 e^{-W/L_B} + K_2 e^{+W/L_B}$$

Let $\psi_E \equiv p_{no}(e^{V_E/V_T} - 1)$, $\psi_C \equiv p_{no}(e^{V_C/V_T} - 1)$ and $y \equiv \dfrac{W}{L_B}$, Then $\psi_E = K_1 + K_2$ and
$\psi_C = K_1 e^{-y} + K_2 e^{+y} = K_1 e^{-y} + (\psi_E - K_1) e^{y} = -K_1(e^{y} - e^{-y}) + \psi_E e^{y}$

Thus, $K_1 = \dfrac{\psi_E e^{y} - \psi_C}{2\sinh y}$, $K_2 = \psi_E - K_1 = \dfrac{2\psi_E \sinh y - \psi_E e^{y} + \psi_C}{2\sinh y} = \dfrac{-\psi_E e^{-y} + \psi_C}{2\sinh y}$

From Eq. (9-12), $I_{pn}(0) = -AeD_p \dfrac{dp_n}{dx}\bigg|_{x=0} = -AeD_p\left(-\dfrac{K_1}{L_B} + \dfrac{K_2}{L_B}\right)$
$$= -\frac{AeD_p}{2L_B \sinh y}(-\psi_E e^{y} + \psi_C - \psi_E e^{-y} + \psi_C) = -\frac{AeD_p}{2L_B \sinh y}(-2\psi_E \cosh y + 2\psi_C)$$
$$= -\frac{AeD_p}{L_B}(-\psi_E \coth y + \psi_C \operatorname{csch} y)$$
$$= -\frac{AeD_p}{L_B}\left[-\left(\coth\frac{W}{L_B}\right)(e^{V_E/V_T} - 1) + \left(\operatorname{csch}\frac{W}{L_B}\right)(e^{V_C/V_T} - 1)\right]$$

(9-3) (Cont'd) From Eq. (9-11), $I_{np}(0) = \frac{AeD_n n_{E0}}{L_E}(e^{V_E/V_T}-1)$

$$I_E = I_{pn}(0) + I_{np}(0) = a_{11}(e^{V_E/V_T}-1) + a_{12}(e^{V_C/V_T}-1)$$

where $a_{11} = Ae\left(\frac{D_p p_{no}}{L_B}\coth\frac{W}{L_B} + \frac{D_n n_{E0}}{L_E}\right)$ and $a_{12} = -AeD_p\frac{p_{no}}{L_B}\operatorname{csch}\frac{W}{L_B}$ QED

Since conditions at the collector are the same as those at the emitter, then the expression for I_C is obtained from that of I_E by interchanging the subscripts E and C. Thus, if we write $I_C = a_{21}(e^{V_E/V_T}-1) + a_{22}(e^{V_C/V_T}-1)$, then a_{21} is the same as a_{12} and a_{22} is the same as a_{11} with E and C interchanged. Hence,

$$a_{21} = -AeD_p\frac{p_{no}}{L_B}\operatorname{csch}\frac{W}{L_B} \quad \text{and} \quad a_{22} = Ae\left(D_p\frac{p_{no}}{L_B}\coth\frac{W}{L_B} + \frac{D_n n_{C0}}{L_C}\right) \quad \text{QED}$$

b) If $\frac{W}{L_B} \equiv y \ll 1$, then $\coth y = \frac{\cosh y}{\sinh y} \cong \frac{1 + \cancel{\frac{y^2}{2}} + \cdots}{y + \cdots} \approx \frac{1}{y} = \frac{L_B}{W}$

and $\operatorname{csch} y = \frac{1}{\sinh y} \approx \frac{1}{y} = \frac{L_B}{W}$

$$\therefore a_{12} = a_{21} = -\frac{AeD_p p_{no}}{W}, \quad a_{11} = Ae\left(\frac{D_p p_{no}}{W} + \frac{D_n n_{E0}}{L_E}\right), \quad a_{22} = Ae\left(\frac{D_p p_{no}}{W} + \frac{D_n n_{C0}}{L_C}\right)$$

(9-4) Eq. (9-27) is identical with Eq. (9-26) provided that $\frac{D_n n_{E0}}{D_p p_{no}} = \frac{\sigma_B}{\sigma_E}$ which we shall now prove:

From Eq. (5-19) $p_{no} = \frac{n_i^2}{n_{no}}$ and $n_{E0} = \frac{n_i^2}{p_{E0}}$. Also from Eq. (5-33) $\frac{D_n}{D_p} = \frac{\mu_n}{\mu_p}$

Hence, $\frac{D_n n_{E0}}{D_p p_{no}} = \frac{D_n}{D_p}\frac{n_{no}}{p_{E0}} = \frac{\mu_n n_{no} e}{\mu_p p_{E0} e} = \frac{\sigma_n}{\sigma_p} = \frac{\sigma_B}{\sigma_E}$

(9-5) Using Eq. (9-24) and the results of Prob. 9-3a we have:

$$\alpha = -\frac{a_{21}}{a_{11}} = \frac{D_p\frac{p_{no}}{L_B}\frac{1}{\sinh W/L_B}}{D_p\frac{p_{no}}{L_B}\frac{\cosh W/L_B}{\sinh W/L_B} + \frac{D_n n_{E0}}{L_E}} = \frac{1}{\cosh\frac{W}{L_B} + \frac{D_n n_{E0} L_B}{D_p p_{no} L_E}\sinh\frac{W}{L_B}}$$

$$= \frac{\operatorname{sech} W/L_B}{1 + \frac{D_n L_B n_{E0}}{D_p L_E p_{no}}\tanh\frac{W}{L_B}} = \beta^*\gamma \quad \text{from Eqs. (9-28) and (9-29)}$$

(9-6) From Eq. (9-29), $\beta^* = \operatorname{sech}\frac{W}{L_B} = \frac{1}{\cosh\frac{W}{L_B}} \cong \frac{1}{1 + \frac{1}{2}\frac{W^2}{L_B^2}} \approx 1 - \frac{1}{2}\left(\frac{W}{L_B}\right)^2$, if $\frac{W}{L_B} \ll 1$

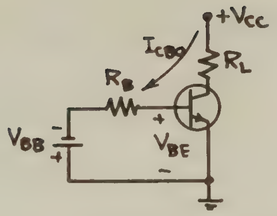
From Eq. (9-28), $\gamma = \frac{1}{1 + \frac{D_n L_B n_{E0}}{D_p L_E p_{no}}\tanh\frac{W}{L_B}} \approx \frac{1}{1 + \frac{D_n L_B n_{E0} W}{D_p L_E p_{no} L_B}} = \frac{1}{1 + \frac{W\sigma_B}{L_E\sigma_E}}$

The last equality is obtained by comparing Eqs. (9-26) and (9-27) and is verified in Prob. 9-4.

If $\frac{W\sigma_B}{L_E\sigma_E} \ll 1$, then $\gamma \approx 1 - \frac{W\sigma_B}{L_E\sigma_E}$ QED

(9-7) a) $I_{CBO}(@75°C) = I_{CBO}(@25°C)\cdot 2^{\frac{\Delta T}{10}}$ where $\Delta T = 50°C$

$= (2\mu A)\cdot(2^5) = 64\ \mu A$



Applying KVL in the base circuit, we have:

$-V_{BB} + (R_B)_{max} I_{CBO}(@75°C) = V_{BE}(\text{cutoff}) = -0.1\ V$

or $-5 + (R_B)_{max} \times 64\times10^{-6} = -0.1$ $\quad \therefore (R_B)_{max} = 76.5K$ ⟵

b) $V_{BB} = 1V$, $R_B = 50K$, Now we have:

$-1 + 50\times10^3\, I_{CBO} = -0.1$ or $I_{CBO} = 18\ \mu A = 2\times 2^{\frac{\Delta T}{10}}$; $2^{\frac{\Delta T}{10}} = 9$

or $\frac{\Delta T}{10} = 3.2$ and $\Delta T = 32°C$. Hence, $T = 25+32 = 57°C$ ⟵

(9-8) From Eq. (9-47), $\beta' = \beta + (I_{CBO} + I_B)\dfrac{\partial\beta}{\partial I_B}$ (1)

Consider, $\dfrac{\partial\beta}{\partial I_B} = \dfrac{\partial\beta}{\partial I_C}\dfrac{\partial I_C}{\partial I_B} = \beta'\dfrac{\partial\beta}{\partial I_C}$ (2) Substituting (2) in (1)

we have: $\beta' = \beta + (I_{CBO}+I_B)\beta'\dfrac{\partial\beta}{\partial I_C}$ or $\beta'\left[1-(I_{CBO}+I_B)\dfrac{\partial\beta}{\partial I_C}\right] = \beta$

Hence, $\beta' = h_{fe} = \dfrac{h_{FE}}{1-(I_{CBO}+I_B)\dfrac{\partial h_{FE}}{\partial I_C}}$ QED

(9-9) a) If $I_B \gg I_{CBO}$, Eq. (9-48) becomes: $h_{fe}\left(1-I_B\dfrac{\partial h_{FE}}{\partial I_C}\right) \approx h_{FE}$

or $\dfrac{h_{fe}-h_{FE}}{h_{fe}} \approx I_B\dfrac{\partial h_{FE}}{\partial I_C} = \dfrac{I_C}{h_{FE}}\dfrac{\partial h_{FE}}{\partial I_C}$

b) At $I_C = -80\ mA$, we find from Fig. 9-12 (Medium beta curve) that $h_{FE} = 82$. From 30 to 130 mA (centered around 80 mA), we find that h_{FE} varies from 92 to 70.

Thus, $\dfrac{\Delta h_{FE}}{\Delta(-I_C)} = \dfrac{70-92}{130-30} = -0.22$ and $\dfrac{h_{fe}-h_{FE}}{h_{fe}} = \left(\dfrac{-I_C}{h_{FE}}\right)\left(\dfrac{\Delta h_{FE}}{\Delta(-I_C)}\right) = \dfrac{80}{82}(-.22)$

$= -0.21$

or h_{fe} is about 20% less than h_{FE}.

(9-10) To the first approximation, we neglect V_{BE} and V_{CE}. Then $I_B \approx \frac{-9}{30K} = -0.3mA$ and $I_C \approx -9\ mA$. From Fig. 9-11 we find that $V_{CE} \approx -0.07V$. Thus, to the second approximation, $I_C = \frac{-9+0.07}{1K} = -8.93\ mA$

From Fig. 9-9, where the input characteristics of a 2N404 transistor are plotted, we find at $I_B = -0.3\ mA$ and $V_{CE} = -0.07V$ that $V_{BE} \approx -0.21\ V$. Thus, to the second approximation, $I_B = \frac{-9+0.21}{30K} = -0.293$

Checking back Figs. 9-9 and 9-11, we find that

$V_{CE} = -0.07V$, $V_{BE} = -0.21V$. Hence, $V_{BC} = V_{BE} - V_{CE} = -0.14V$ ⟵

(9-11) a) $I_C = -\alpha_N I_E + I_{CO}(1-e^{V_C/V_T})$ .....(9-49), $I_E = -\alpha_I I_C + I_{EO}(1-e^{V_E/V_T})$ ....(9-50)

Substituting Eq. (9-50) into Eq. (9-49) we have:

$$I_C = -\alpha_N\left[-\alpha_I I_C + I_{EO}(1-e^{V_E/V_T})\right] + I_{CO}(1-e^{V_C/V_T})$$

$$\text{or}\quad I_C = \frac{\alpha_N I_{EO}}{1-\alpha_N\alpha_I}(e^{V_E/V_T}-1) - \frac{I_{CO}}{1-\alpha_N\alpha_I}(e^{V_C/V_T}-1) \quad \ldots\ (9\text{-}51)$$

Substituting Eq. (9-49) into Eq. (9-50) we have:

$$I_E = -\alpha_I\left[-\alpha_N I_E + I_{CO}(1-e^{V_C/V_T}\right) + I_{EO}(1-e^{V_E/V_T})$$

$$\text{or}\quad I_E = \frac{\alpha_I I_{CO}}{1-\alpha_N\alpha_I}(e^{V_C/V_T}-1) - \frac{I_{EO}}{1-\alpha_N\alpha_I}(e^{V_E/V_T}-1) \quad \ldots\ (9\text{-}52)$$

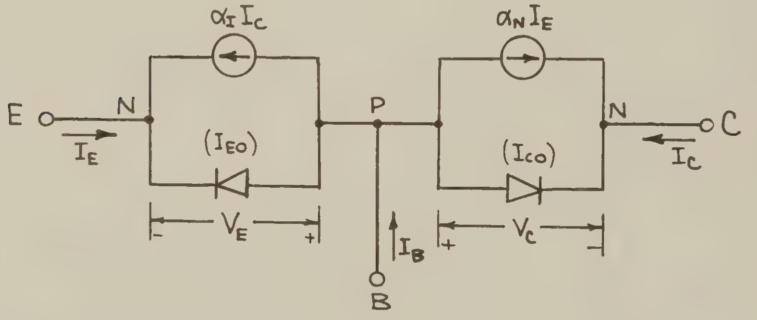
b) From Eq. (9-49), $\frac{I_C + \alpha_N I_E}{I_{CO}} = 1 - e^{V_C/V_T}$ or $e^{V_C/V_T} = \left(1 - \frac{I_C + \alpha_N I_E}{I_{CO}}\right)$

$$\text{Hence,}\quad V_C = V_T \ln\left(1 - \frac{I_C + \alpha_N I_E}{I_{CO}}\right) \quad \ldots\ (9\text{-}56)$$

From Eq. (9-50), $\frac{I_E + \alpha_I I_C}{I_{EO}} = 1 - e^{V_E/V_T}$ or $e^{V_E/V_T} = \left(1 - \frac{I_E + \alpha_I I_C}{I_{EO}}\right)$

$$\text{Hence,}\quad V_E = V_T \ln\left(1 - \frac{I_E + \alpha_I I_C}{I_{EO}}\right) \quad \ldots\ (9\text{-}55)$$

(9-12)



Note: For an NPN transistor I_{EO} and I_{CO} are +, I_C is + and I_E is −.

(9-13) a) For a p-n-p transistor $V_{CE} = V_C - V_E$ where V_E and V_C are given by Eqs. (9-55) and (9-56) respectively. Substituting, we have:

$$V_{CE} = V_T \ln\left(\frac{I_{CO} - I_C - \alpha_N I_E}{I_{EO} - I_E - \alpha_I I_C}\right)\cdot\left(\frac{I_{EO}}{I_{CO}}\right)$$

From Eq. (9-54), $I_E = -(I_B + I_C)$ and from Eq. (9-53) $\frac{I_{EO}}{I_{CO}} = \frac{\alpha_I}{\alpha_N}$. Substituting,

$$V_{CE} = V_T \ln\left(\frac{\alpha_I}{\alpha_N}\right)\cdot\left(\frac{I_{CO} + \alpha_N I_B - I_C(1-\alpha_N)}{I_{EO} + I_B + I_C(1-\alpha_I)}\right) = V_T \ln\frac{\alpha_I}{\alpha_N} + V_T \ln\frac{I_{CO} + \alpha_N I_B - I_C(1-\alpha_N)}{I_{EO} + I_B + I_C(1-\alpha_I)}$$

b) If $I_B \gg I_{EO}$ and $I_B \gg \frac{I_{CO}}{\alpha_N}$, we have:

$$V_{CE} \approx V_T \ln\left(\frac{\alpha_I}{\alpha_N}\cdot\frac{\alpha_N I_B - I_C(1-\alpha_N)}{I_B + I_C(1-\alpha_I)}\right) = V_T \ln\frac{I_B - \left(\frac{1-\alpha_N}{\alpha_N}\right)I_C}{I_B/\alpha_I + \left(\frac{1-\alpha_I}{\alpha_I}\right)I_C} = V_T \ln\frac{1 - \frac{1}{\beta}\frac{I_C}{I_B}}{\frac{1}{\alpha_I} + \frac{1}{\beta_I}\frac{I_C}{I_B}}$$

$$\text{or}\quad V_{CE} = -V_T \ln\left(\frac{\frac{1}{\alpha_I} + \frac{1}{\beta_I}\frac{I_C}{I_B}}{1 - \frac{1}{\beta_N}\frac{I_C}{I_B}}\right) \qquad \text{QED}$$

(9-14) a) If $|V_C| \gg V_T$ and $|V_E| \gg V_T$, then $e^{V_C/V_T} \approx 0$ and $e^{V_E/V_T} \approx 0$. Then Eq. (9-51) becomes

$$I_E = \frac{I_{EO}}{1-\alpha_N\alpha_I} - \frac{\alpha_I I_{CO}}{1-\alpha_N\alpha_I} = \frac{I_{EO} - \alpha_N I_{EO}}{1-\alpha_N\alpha_I} = \frac{I_{EO}(1-\alpha_N)}{1-\alpha_N\alpha_I}$$

From Eq. (9-52), $I_C = \frac{I_{CO}}{1-\alpha_N\alpha_I} - \frac{\alpha_N I_{EO}}{1-\alpha_N\alpha_I} = \frac{I_{CO} - \alpha_I I_{CO}}{1-\alpha_N\alpha_I} = \frac{I_{CO}(1-\alpha_I)}{1-\alpha_N\alpha_I}$ QED

b) Under the conditions that $I_E = 0$ and $e^{V_C/V_T} \approx 0$, Eqs. (9-49) and (9-50) become: $I_C = I_{CO}$, and $0 = -\alpha_I I_C + I_{EO}(1 - e^{V_E/V_T})$

or $(1 - e^{V_E/V_T}) = \alpha_I \frac{I_{CO}}{I_{EO}} = \alpha_N$ from Eq. (9-53)

Then, $e^{V_E/V_T} = 1 - \alpha_N$ and $V_E = V_T \ln(1-\alpha_N)$ QED

(9-15) a) From Prob. 9-14$\underline{a}$, $I_C = \frac{I_{CO}(1-\alpha_I)}{1-\alpha_N\alpha_I}$ where $\alpha_I = \frac{\alpha_N I_{EO}}{I_{CO}} = \frac{0.98 \times 3.57}{5} = 0.7$

Hence, $I_C = \frac{5(1-0.7)}{1-0.98\times0.7} = 4.78\ \mu A$ ⟵

b) Also from Prob. 9-14$\underline{a}$, $I_E = \frac{I_{EO}(1-\alpha_N)}{1-\alpha_N\alpha_I} = \frac{3.57(1-0.98)}{1-0.98\times0.7} = 0.227\ \mu A$ ⟵

(9-16) In the active region the collector junction is reverse biased and assuming that $|V_C| \gg V_T$, then $e^{V_C/V_T} \approx 0$. From Eq. (9-51) we have:

$$I_E \approx \frac{-\alpha_I I_{CO}}{1-\alpha_N\alpha_I} - \frac{I_{EO}}{1-\alpha_N\alpha_I}(e^{V_E/V_T}-1) = \frac{-\alpha_N I_{EO} + I_{EO} - I_{EO}e^{V_E/V_T}}{1-\alpha_N\alpha_I} \quad \text{since } \alpha_I I_{CO} = \alpha_N I_{EO}$$

$$\text{or } I_E \approx \frac{I_{EO}}{1-\alpha_N\alpha_I}\left[(1-\alpha_N) - e^{V_E/V_T}\right] \approx -\frac{I_{EO}}{1-\alpha_N\alpha_I} e^{V_E/V_T} = I_o e^{V_E/V_T} \quad \text{since } \alpha_N \approx 1.$$

(9-17) a) Neglecting $V_{CE}(sat)$, we have that $I_C(sat) \approx \frac{12}{4K} = 3\,mA$. Then, $(I_B)_{min} = \frac{I_C(sat)}{h_{FE}}$ where $h_{FE} = \frac{0.98}{1-0.98} = 49$. Thus, $(I_B)_{min} = \frac{3}{49} = 61.3\ \mu A$

b) From Eq. (9-55), $V_E = V_T \ln\left(1 - \frac{I_E + \alpha_I I_C}{I_{EO}}\right)$ where $\alpha_I = \frac{0.98\times1.6}{2} = 0.784$, $I_E = -3.06\,mA$

Thus, $V_E = 26 \ln\left(1 - \frac{-3.06 + 0.784\times3}{1.6\times10^{-3}}\right) = 159\,mV$

From Eq. (9-56) $V_C = V_T \ln\left(1 - \frac{I_C + \alpha_N I_E}{I_{CO}}\right) = 26 \ln\left(1 - \frac{3 - 0.98\times3.06}{2\times10^{-3}}\right) \approx 0$

Hence, $V_{CB} = -V_C \approx 0$, $V_{EB} = -V_E = -159\,mV$ and $V_{CE} = -V_C + V_E = 159\,mV$ ⟵

c) $I_B = 0.2\,mA$, $I_C = 3\,mA$, $I_E = -3.2\,mA$

From Eq. (9-55), $V_E = 26 \ln\left(1 - \frac{-3.2 + 0.784\times3}{1.6\times10^{-3}}\right) = 167\,mV$

(9-56), $V_C = 26 \ln\left(1 - \frac{3 + 0.98(-3.2)}{2\times10^{-3}}\right) = 110\,mV$

Thus, $V_{CB} = -V_C = -110\,mV$, $V_{EB} = -V_E = -167\,mV$, $V_{CE} = V_E - V_C = 167 - 110 = 57\,mV$

d) V_C, V_E and V_{CE} are unchanged.

$V_{CB} = -V_C - I_B r_{bb'}$: for $\underline{b}$, $V_{CB} = -(0.061)(250) = -15.3\,mV$; for $\underline{c}$, $V_{CB} = -110 - 0.2\times250 = 160\,mV$

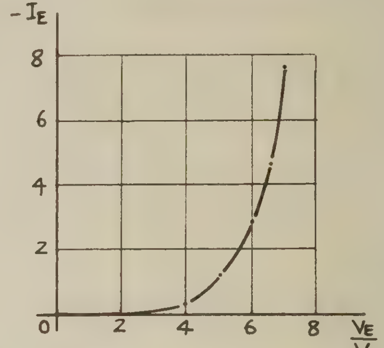
$V_{EB} = -V_E - I_B r_{bb'}$: for $\underline{b}$, $V_{EB} = -159 - 0.061\times250 = -174\,mV$; for $\underline{c}$, $V_{EB} = -167 - 50 = -217\,mV$ ⟵

(9-18) a) From Eq. (9-51), $I_E = \frac{\alpha_I I_{CO}}{1-\alpha_N\alpha_I}(e^{V_C/V_T}-1) - \frac{I_{EO}}{1-\alpha_N\alpha_I}(e^{V_E/V_T}-1)$; If $V_C = 0$, the

$$I_E = -\frac{I_{EO}}{1-\alpha_N\alpha_I}(e^{V_E/V_T}-1) = -\frac{1.6\times10^{-3}}{1-0.98\times0.784}(e^{V_E/V_T}-1) = -6.89\times10^{-3}(e^{V_E/V_T}-1) \text{ mA}$$

or $-I_E = 6.89\times10^{-3}(e^{V_E/V_T}-1)$ mA

V_E/V_T	0	2	4	5	6	6.5	7
$-I_E$	0	0.044	0.37	1.2	2.75	4.6	7.6



b) If the collector junction is reverse-biased by more than a few tenths of a volt, $e^{V_C/V_T} \approx 0$ and $I_E = \frac{-\alpha_I I_{CO}}{1-\alpha_N\alpha_I} - \frac{I_{EO}}{1-\alpha_N\alpha_I}(e^{V_E/V_T}-1)$

where $\frac{\alpha_I I_{CO}}{1-\alpha_N\alpha_I} = \frac{0.784\times2\times10^{-3}}{1-0.98\times0.784} = 6.82\times10^{-3}$ mA.

Thus, $-I_E = 6.89\times10^{-3}(e^{V_E/V_T}-1) + 6.82\times10^{-3}$ mA. We note that at each value of V_E, the current differs from that found in part (a) by the constant amount of 6.82×10^{-3} mA $= 6.82$ μA. This is too small to be indicated on the above plot. In other words, the emitter current is almost independent of collector voltage in the active region. (This calculation neglects the Early effect.)

(9-19) From Eq. (9-57) we have: $V_{CE} = V_T\left[\ln\left(\frac{1}{\alpha_I} + \frac{1}{\beta_I}\frac{I_C}{I_B}\right) - \ln\left(1 - \frac{1}{\beta}\frac{I_C}{I_B}\right)\right]$

where $\frac{1}{\alpha_I} = \frac{1}{0.9} = 1.11$, $\beta = \frac{0.9}{1-0.9} = 9 = \beta_I$

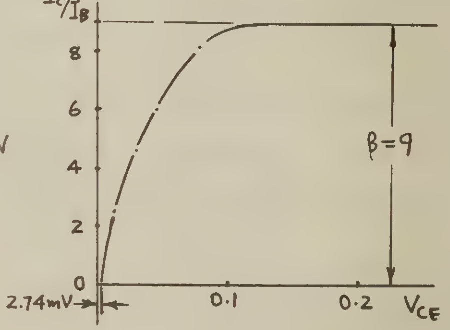
Thus, $V_{CE} = 26\left[\ln\left(1.11 + \frac{1}{9}\frac{I_C}{I_B}\right) - \ln\left(1 - \frac{1}{9}\frac{I_C}{I_B}\right)\right]$ mV

If $I_C/I_B = 0$, $V_{CE} = 26\ln 1.11 = 2.74$ mV

$= 4.5$, $V_{CE} = 26(\ln 1.61 - \ln 0.5) = 30.5$ mV

$= 6$, $V_{CE} = 26\times1.68 = 43.7$ mV

$= 8.1$, $V_{CE} = 26\times3 = 78$ mV



As $V_{CE}\to\infty$, $\frac{I_C}{I_B}\to\beta = 9$. Note that these calculations assume that α_N and α_I are constant for all values of V_{CE}.

(9-20) a) From Eq. (9-62), $I_{CEO} = \frac{I_{CO}}{1-\alpha_N}$ from which $\alpha_N = \frac{I_{CEO} - I_{CO}}{I_{CEO}}$ QED

b) From Eq. (9-61), $I_{CES} = \frac{I_{CO}}{1-\alpha_N\alpha_I}$ from which $\alpha_I = \frac{I_{CES} - I_{CO}}{\alpha_N I_{CES}}$.

Substituting the value of α_N from part (a), we have:

$$\alpha_I = \frac{(I_{CES} - I_{CO})}{(I_{CEO} - I_{CO})I_{CES}}\cdot I_{CEO} = \frac{1 - I_{CO}/I_{CES}}{1 - I_{CO}/I_{CEO}} \quad \text{QED}$$

(9-21) Since $e^{V_C/V_T} \approx 0$, Eq. (9-49) becomes: $I_C = -\alpha_N I_E + I_{CO}$ (1)

For small V_E we can write, $1 - e^{V_E/V_T} \approx 1 - \left(1 + \frac{V_E}{V_T} + \cdots\right) = -\frac{V_E}{V_T}$. Then Eq. (9-50) can be written: $I_E = -\alpha_I I_C - I_{EO}\frac{V_E}{V_T}$ (2) Also, $I_B = -(I_C + I_E)$ (3)

From the circuit, $V_E = -I_B R = (I_C + I_E)R$ (4) Substituting (4) into (2) we get:

$$I_E = -\alpha_I I_C - \frac{I_{EO}R}{V_T}(I_C + I_E) \quad \text{or} \quad I_E = \frac{-\left(\alpha_I + \frac{I_{EO}R}{V_T}\right)}{1 + \frac{I_{EO}R}{V_T}} I_C \qquad (5)$$

Substituting (5) into (1) we obtain:

$$I_C = \frac{I_{CO}}{1 - \frac{\alpha_N\left(\alpha_I + \frac{I_{EO}R}{V_T}\right)}{1 + \frac{I_{EO}R}{V_T}}} = \frac{I_{CO}\left(1 + \frac{I_{EO}R}{V_T}\right)}{1 - \alpha_N\alpha_I + \frac{I_{EO}R}{V_T}(1 - \alpha_N)} = I_{CER} \qquad \text{QED}$$

(9-22) The condition $V_o = V_{CC}$ implies that $V_{CE} = 0$. From Eq. (9-57) we have:

$$V_{CE} = V_T \ln\left(\frac{\frac{1}{\alpha_I} + \frac{1}{\beta_I}\frac{I_C}{I_B}}{1 - \frac{1}{\beta_N}\frac{I_C}{I_B}}\right) = 0 \qquad \therefore \quad \frac{1}{\alpha_I} + \frac{1}{\beta_I}\frac{I_C}{I_B} = 1 - \frac{1}{\beta_N}\frac{I_C}{I_B}$$

or $\frac{I_C}{I_B}\left(\frac{1}{\beta_I} + \frac{1}{\beta_N}\right) = 1 - \frac{1}{\alpha_I} = \frac{\alpha_I - 1}{\alpha_I} = -\frac{1}{\beta_I}$ and $I_B = -\left(1 + \frac{\beta_I}{\beta_N}\right) I_C$. (1)

But $I_C = -I_B - I_E = -I_B + \frac{V_o}{R_e} = -I_B + \frac{V_{CC}}{R_e}$ (2)

Substituting (2) into (1) we get: $I_B = \left(1 + \frac{\beta_I}{\beta_N}\right)\left(I_B - \frac{V_{CC}}{R_e}\right)$

$$\text{or} \quad I_B = \frac{V_{CC}}{R_e}\left(1 + \frac{\beta_N}{\beta_I}\right) = \frac{V_{CC}}{R_e}\left(1 + \frac{\frac{\alpha_N}{1-\alpha_N}}{\frac{\alpha_I}{1-\alpha_I}}\right) = \frac{V_{CC}}{R_e}\left(1 + \frac{\alpha_N}{\alpha_I}\,\frac{1-\alpha_I}{1-\alpha_N}\right) \qquad \text{QED}$$

(9-23) a) From Eq. (9-49) with $e^{V_C/V_T} \approx 0$, we have: $I_C \approx -\alpha_N I_E + I_{CO} \approx -\alpha_N I_E$

Thus, $I_{C2} = -\alpha_2 I_E = -(0.96)(100) = 96\ mA$ $\longleftarrow$

From $I_C \approx \beta I_B = \frac{\alpha}{1-\alpha} I_B$ we have: $I_{B2} \approx -(1-\alpha_2) I_E = -(1-0.96)(100) = 4\ mA$

$I_{E1} = -I_{B2} = -4\ mA$; $I_{C1} = -\alpha_1 I_{E1} = -(0.98)(-4) = 3.92\ mA$ $\longleftarrow$

$I_{B1} = I_B = -(1-\alpha_1) I_{E1} = -(1-0.98)(-4) = 80\ \mu A$; Then $I_C = I_{C1} + I_{C2} = 96 + 3.92 = 99.92\ mA$

b) $V_{CE} = V_{CC} - I_C R_c = 24 - 99.92 \times 0.12 \approx 12\,V$ $\longleftarrow$

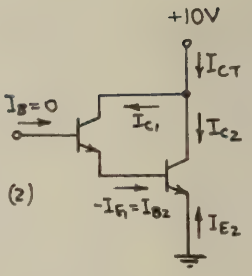
c) $\frac{I_C}{I_B} = \frac{99.92}{80 \times 10^{-3}} = 1249$; $\frac{I_C}{I_E} = \frac{99.92}{100} = 0.9992$ $\longleftarrow$

(9-24) With $I_E = 0$ and $e^{V_C/V_T} \approx 0$, Eq. (9-49) becomes: $I_C = I_{CO}$. Then from Eq (9-50)

$I_E = 0 = -\alpha_I I_{CO} + I_{EO}\left(1 - e^{V_E/V_T}\right)$ or $1 - e^{V_E/V_T} = \frac{\alpha_I I_{CO}}{I_{EO}} = \alpha_N$ (from Eq. (9-53)

or $e^{V_E/V_T} = 1 - \alpha_N$ and $V_E = V_{EBF} = V_T \ln(1 - \alpha_N)$ QED

(9-25) If $I_B = 0$, then $I_{C_1} = -I_{E_1}$ and with $e^{V_{C1}/V_T} \approx 0$, Eq. (9-49) becomes: $I_{C_1} = -\alpha_N(-I_{C_1}) + I_{CO}$ or $I_{C_1} = \dfrac{I_{CO}}{1-\alpha_N}$ (1)



For the other transistor, $e^{V_{C2}/V_T} \approx 0$ and

$$I_{C2} = -\alpha_N I_{E2} + I_{CO} = -\alpha_N(-I_{C_1} - I_{C2}) + I_{CO} \text{ or } I_{C2} = \frac{\alpha_N I_{C_1} + I_{CO}}{1-\alpha_N} \quad (2)$$

Substituting (1) into (2): $I_{C2} = \dfrac{I_{CO}}{(1-\alpha_N)^2}$ (3)

From the circuit we have: $I_{CT} = I_{C_1} + I_{C2} = \dfrac{I_{CO}}{1-\alpha_N}\left(1 + \dfrac{1}{1-\alpha_N}\right) = \dfrac{2-\alpha_N}{(1-\alpha_N)^2} I_{CO}$

(9-26) a) $V_{BE} = I_B r_{bb'} - I_E R_E + V_E$ (1) where V_E is given by Eq. (9-55), namely,

$V_E = V_T \ln\left(1 - \dfrac{I_E + \alpha_I I_C}{I_{EO}}\right)$. (2) If $|V_{CB}| >> V_T$, then $e^{V_C/V_T} \approx 0$ and Eq. (9-49) becomes: $I_C = -\alpha_N I_E + I_{CO} = -I_B - I_E$ or $I_E = \dfrac{I_B + I_{CO}}{\alpha_N - 1}$

Consider $(I_E + \alpha_I I_C) = I_E + \alpha_I(-I_B - I_E) = I_E(1-\alpha_I) - \alpha_I I_B = \dfrac{-(1-\alpha_I)-(1-\alpha_N)\alpha_I}{1-\alpha_N} I_B - \dfrac{1-\alpha_I}{1-\alpha_N} I_{CO}$

or $(I_E + \alpha_I I_C) = -\left[\dfrac{(1-\alpha_N\alpha_I) I_B + (1-\alpha_I) I_{CO}}{1-\alpha_N}\right]$ (3)

Substituting (3) into (2) we get: $V_E = V_T \ln\left[1 + \dfrac{(1-\alpha_N\alpha_I) I_B}{(1-\alpha_N) I_{EO}} + \dfrac{I_{CO}}{I_{EO}} \dfrac{1-\alpha_I}{1-\alpha_N}\right]$ (4)

Consider $(I_B r_{bb'} - I_E R_E) = I_B r_{bb'} + \left(\dfrac{I_B + I_{CO}}{1-\alpha_N}\right) R_E = I_B\left(r_{bb'} + \dfrac{R_E}{1-\alpha_N}\right) + \dfrac{I_{CO} R_E}{1-\alpha_N}$ (5)

Substituting (4) & (5) into (1) we obtain, since $\dfrac{I_{CO}}{I_{EO}} = \dfrac{\alpha_N}{\alpha_I}$:

$$V_{BE} = I_B\left(r_{bb'} + \frac{R_E}{1-\alpha_N}\right) + \frac{I_{CO} R_E}{1-\alpha_N} + V_T \ln\left[1 + \frac{I_B(1-\alpha_N\alpha_I)}{I_{EO}(1-\alpha_N)} + \frac{\alpha_N(1-\alpha_I)}{\alpha_I(1-\alpha_N)}\right] \quad \text{QED}$$

b) If $I_C = 0$, then $I_E = -I_B$ and $V_E = V_T \ln\left(1 + \dfrac{I_B}{I_{EO}}\right)$

Thus, $V_{BE} = I_B(r_{bb'} + R_E) + V_T \ln\left(1 + \dfrac{I_B}{I_{EO}}\right)^*$ QED

(9-27) If $I_E = -2$ mA, then $I_B = -(1-\alpha)I_E = -(1-0.98)(-2) = 40\ \mu A$ and $I_C = 1.96$ mA

$V_{BN} = V_{BE} - I_E R_e = 0.6 - (-2 \times 0.1) = 0.8$ V. $I_{R_2} = \dfrac{V_{BN}}{R_2} = \dfrac{0.8}{20K} = 40\ \mu A$

$I_{R_1} = I_B + I_{R2} = 40 + 40 = 80\ \mu A$. $I_{3.3K} = I_{R_1} + I_C = 80 \times 10^{-3} + 1.96 = 2.04$ mA

$V_{R_1} = V_{CN} - V_{BN} = (12 - 2.04 \times 3.3) - 0.8 = 4.47$ V

Hence, $R_1 = \dfrac{V_{R_1}}{I_{R_1}} = \dfrac{4.47}{0.08} = 56$ K ←

(9-28) a) With $I_C = 0$ and $I_E = 2$ mA, Eq. (9-55) becomes: $V_E = V_T \ln\left(1 - \dfrac{I_E}{I_{EO}}\right)$ where for a p-n-p transistor I_{EO} is negative i.e. $I_{EO} = -1.6\ \mu A$.

Thus, $V_E = 26 \ln\left(1 + \dfrac{2}{1.6 \times 10^{-3}}\right) = 26 \ln 1250 = 26 \times 7.12 = 185.3$ mV forward bias.

From Eq. (9-56), $V_C = V_T \ln\left(1 - \dfrac{\alpha_N I_E}{I_{CO}}\right) = 26 \ln\left(1 - \dfrac{0.98 \times 2}{2 \times 10^{-3}}\right) = 26 \ln 981 = 26 \times 6.9$ or $V_C = 179$ mV forward bias. Hence, the transistor is in saturation. ←

b) $V_{CE} = V_C - V_E = 179 - 185.3 = -6.3$ mV. ←

Check: From Eq. (9-57) with $I_C = 0$, $V_{CE} = -V_T \ln \dfrac{1}{\alpha_I} = -26 \ln \dfrac{1}{0.784} = -6.3$ mV

(9-29) a) From Eq. (6-47), $W^2 = \frac{2\epsilon V_B}{eN_A}$ where $\epsilon = \frac{12}{36\pi \times 10^{11}}$ F/cm and

$eN_A = \frac{\sigma_B}{\mu_p} = \frac{1}{\rho_B \mu_p}$

Thus, $V_B = \frac{eN_A W^2}{2\epsilon} = \frac{W^2}{2\epsilon \mu_p \rho_B} = \frac{W^2 (\text{mils})^2 \times (2.54 \frac{\text{cm}}{\text{in}})^2 \times 10^{-6} (\frac{\text{in}}{\text{mil}})^2}{\frac{2\times 12}{36\pi\times 10^{11}} \text{F/cm} \times 500 \frac{\text{cm}^2}{\text{V-sec}} \rho_B \, \Omega\text{-cm}}$

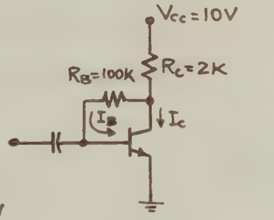
or $V_B = 6.46\times 10^{2} \times 3\pi \frac{W^2}{\rho_B} = 6.1\times 10^3 \frac{W^2}{\rho_B}$ * where W is in mils and ρ_B in Ω-cm.

b) Punch-through occurs when the depletion region of the collector junction reaches the emitter junction.

With $W = 1\,\mu = 10^{-6}\,\text{m} = 10^{-4}\,\text{cm} = \frac{10^{-4}\,\text{cm}}{2.54\,\text{cm/in}} \frac{10^3\,\text{mils}}{\text{in}} = 3.93\times 10^{-2}$ mil

From part a then, we have: $V = 6.1\times 10^3 \times \frac{(3.93)^2 \times 10^{-4}}{0.5} = 18.9\ \text{V}$ $\longleftarrow$

CHAPTER 10



(10-1) a) KVL in the collector circuit gives: $V_{cc} - (I_c + I_B)R_c - I_B R_B = 0$

Neglecting I_{co}, $I_c = \beta I_B$. Then $(\beta+1) I_B R_c + I_B R_B = V_{cc}$

or $(51\times 2\times 10^3 + 100\times 10^3)\, I_B = 10$ $\therefore$ $I_B = \frac{10}{202\times 10^{3}} = 49.5\ \mu\text{A}$

Thus, $I_c = 50 \times 49.5 \times 10^{-6} = 2.475$ mA and $V_{CE} = I_B R_B = 4.95$ V

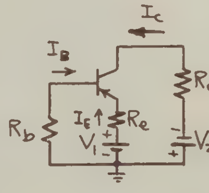
b) From Eq. (10-11), $S = \frac{\beta+1}{1+\beta R_c/(R_c+R_B)} = \frac{51}{1 + \frac{50\times 2}{102}} = 25.6$

(10-2) a) KVL in the collector circuit gives: $V_{cc} - (\beta+1) I_B R_c - V_{CE} = 0$

or $10 - 101\times 10^3 I_B - 4 = 0$ $\therefore$ $I_B = \frac{6}{101\times 10^3} = 59.4\ \mu\text{A}$. But $V_{CE} = I_B R_b = 4$ V

$\therefore$ $R_b = \frac{4}{59.4\times 10^{-6}} = 67.3$ K

b) From Eq. (10-11), $S = \frac{101}{1 + 100\times 1/68.3} = 41$



(10-3) Neglecting V_{BE} we obtain from the circuit: $V_1 = I_E R_e - I_B R_b$;

but $I_E = -(I_B + I_c)$, thus $V_1 = -(I_B + I_c) R_e - I_B R_b$ or $I_B = -\frac{I_c R_e + V_1}{R_e + R_b}$

From Eq. (9-43), $I_c = (1+\beta) I_{co} + \beta I_B = (1+\beta) I_{co} - \frac{\beta}{R_e + R_b}(I_c R_e + V_1)$

or $I_c \left(1 + \frac{\beta R_e}{R_e + R_b}\right) = (1+\beta) I_{co} - \frac{\beta V_1}{R_e + R_b}$; $S = \frac{\partial I_c}{\partial I_{co}} = \frac{1+\beta}{1 + \beta R_e/(R_e + R_b)}$ QED

(10-4) a) Solving for I_B in Eq. (10-15) we get: $I_B = \frac{V - V_{BE} - I_c R_e}{R_e + R_b}$

From Eq. (9-43), $I_c = (1+\beta) I_{co} + \beta I_B = (1+\beta) I_{co} + \frac{\beta}{R_e + R_b}(V - V_{BE} - I_c R_e)$

or $I_c \left(1 + \frac{\beta R_e}{R_e + R_b}\right) = (1+\beta) I_{co} + \frac{\beta}{R_e + R_b}(V - V_{BE})$

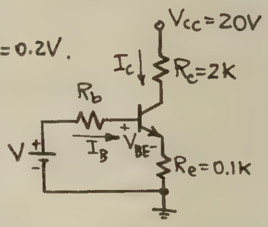
(10-4) (Cont'd) Then $S = \frac{\partial I_c}{\partial I_{co}} = \frac{1+\beta}{1+\beta R_e/(R_e+R_b)} = (1+\beta)\frac{1+R_b/R_e}{1+\beta+R_b/R_e}$

b) $S = \frac{(1+\beta)(R_e+R_b)}{R_e+R_b+\beta R_e} = \frac{(1+\beta)(R_e+R_b)}{(1+\beta)R_e+R_b} = \frac{R_e+R_b}{R_e+\frac{R_b}{(1+\beta)}}$ Dividing by R_eR_b,

$$S = \frac{\frac{1}{R_b}+\frac{1}{R_e}}{\frac{1}{R_b}+\frac{1}{R_e(1+\beta)}} = \frac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_e}}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{1}{R_e(1+\beta)}} = \frac{G_1+G_2+G_e}{G_1+G_2+G_e/(1+\beta)} \qquad \text{QED}$$

(10-5) a) From Eqs. (10-14), $V = \frac{R_2V_{cc}}{R_1+R_2} = \frac{5\times 20}{100+5} = 0.952\,V$

and $R_b = \frac{R_1R_2}{R_1+R_2} = \frac{5\times 100}{105}K = 4.76K$. For a Ge* transistor $V_{BE} = 0.2V$.



Using KVL in the base-to-emitter loop, we have:

$V = I_BR_b + V_{BE} + (I_B+I_c)R_e = I_BR_b + V_{BE} + (1+\beta)I_BR_e$ or

$0.952 = I_B[4.76 + 51\times 0.1]\,I_B + 0.2 = 9.86\,I_B + 0.2$

$\therefore\ I_B = \frac{0.952-0.2}{9.86} = 76.2\ \mu A$ and $I_c = \beta I_B = 50\times 76.2\times 10^{-3} = 3.81\ mA$ ◁—

Then $I_E = -(I_B+I_c) \doteq -3.89\ mA$. ◁—

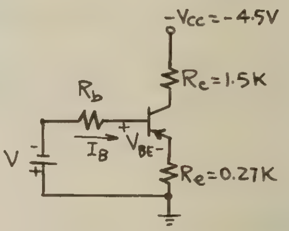
$V_{CE} = V_{cc} - I_cR_c - I_ER_e = 20 - 3.81\times 2 - 3.89\times 0.1 = 20 - 7.62 - 0.39 = 12\,V$ ◁—

b) From Eq. (10-17), $S = (1+\beta)\frac{1+R_b/R_e}{1+\beta+R_b/R_e} = 51\,\frac{1+4.76/0.1}{51+4.76/0.1} \doteq 25$ ◁—

(10-6) a) From Eqs. (10-14), $V = \frac{R_2V_{cc}}{R_1+R_2} = \frac{2.7\times 4.5}{27+2.7} = 0.409V$

and $R_b = \frac{R_1R_2}{R_1+R_2} = \frac{27\times 2.7}{27+2.7}K = 2.46K$; For Ge, $|V_{BE}| = 0.2V$

From Eq. (10-17), $S = (45)\,\frac{1+2.46/0.27}{45+2.46/0.27} = 8.4$ ◁—



b) KVL in the base-emitter circuit gives:

$-V = I_BR_b + V_{BE} + (1+\beta)I_BR_e$ or $-0.409 = I_B(2.46 + 45\times 0.27) - 0.2$

or $I_B = \frac{-0.409+0.2}{14.6} = -14.3\ \mu A$, $I_c = \beta I_B = 44(-14.3\times 10^{-3}) = -0.63\ mA$ ◁—

$V_{CE} = -V_{cc} - I_cR_c - (I_B+I_c)R_e = -4.5 + 0.63\times 1.5 + 0.644\times 0.27 = -3.38\ V$ ◁—

c) R_b must now be increased by $690\,\Omega$ to $2.46 + 0.69 = 3.15K$

$S = 45\,\frac{1+3.15/0.27}{45+3.15/0.27} = 10.1$ ◁—

$-4.09 = I_B(15.29) - 0.2$ or $I_B = \frac{-0.209}{15.29} \doteq -13.7\ \mu A$, $I_c = -0.60\ mA$ ◁—

$V_{CE} = -4.5 + 0.6\times 1.5 + 0.602\times 0.27 = -3.43\ V$ ◁—

(10-7) a) $I_B = \frac{I_c}{\beta} = \frac{4}{50} = 80\ \mu A$, $R_e = \frac{V_{cc}-V_{CE}-I_cR_c}{I_B+I_c} = \frac{16-8-4\times 1.5}{4.08} = 0.49K$ ◁—

From Eq. (10-17) $S = 12 = 51\,\frac{1+R_b/R_e}{51+R_b/R_e}$ or $R_b/R_e = 14.4$ and $R_b = 7.05K$

The base-to-ground voltage, $V_{BN} = V_{BE} - I_ER_e = 0.2 + 4.08\times 0.49 = 2.2V$

The Thévenin voltage in Fig. 10-6b is $V = V_{BN} + I_BR_b = 2.2 + 0.08\times 7.05 = 2.76V$

From Eqs. (10-14), $\frac{V}{R_b} = \frac{V_{cc}}{R_1} = \frac{16}{R_1}$ $\therefore\ R_1 = \frac{16R_b}{V} = \frac{16\times 7.05}{2.76} = 41\ K$ ◁—

(10-7) (Cont'd) Then $I_{R_1} = \dfrac{V_{CC} - V_{BN}}{R_1} = \dfrac{16 - 2.2}{41K} = 0.337\,mA$ and $I_{R_2} = I_{R_1} - I_B = 0.257\,mA$

Hence, $R_2 = \dfrac{V_{BN}}{I_{R_2}} = \dfrac{2.2}{0.257}K = 8.56\,K$ $\longleftarrow$

b) Re is the same as in part a i.e. $R_e = 0.49K$ $\longleftarrow$

$S = 3 = \dfrac{51\,(1 + R_b/R_e)}{51 + R_b/R_e}$ or $\dfrac{R_b}{R_e} = 2.13$ and $R_b = 1.04K$. Then $V = 2.2 + 0.08 \times 1.04$

or $V = 2.28\,V$. Thus $\dfrac{V}{R_b} = \dfrac{V_{CC}}{R_1}$ or $R_1 = \dfrac{V_{CC}}{V} R_b = \dfrac{16}{2.28}(1.04K) = 7.3K$ $\longleftarrow$

$I_{R_1} = \dfrac{16 - 2.2}{7.3K} = 1.89\,mA$; $I_{R_2} = 1.89 - 0.08 = 1.81\,mA$ $\therefore R_2 = \dfrac{2.2}{1.81} = 1.21K$ $\longleftarrow$

c) The power drain from the battery taken by the biasing resistors R_1 and R_2 is increased as the stability factor is improved.

(10-8) a) From Eqs. (10-14), $V = \dfrac{R_2 V_{CC}}{R_1 + R_2} = \dfrac{1}{2}(3) = 1.5\,V$ and $R_b = \dfrac{R_1 R_2}{R_1 + R_2} = 2.5K$

then from Eq. (10-17), $S = 45 \dfrac{1 + 2.5}{45 + 2.5} = 3.31$ $\longleftarrow$

b) KVL in the collector-emitter circuit of Fig. 10-6b gives:

$3 + V_{CE} + (I_B + I_C) \times 1 = 0$; KVL in the base-emitter circuit gives:

$1.5 - 0.2 + 2.5 I_B + (I_B + I_C) = 0$, since $V_{BE} = 0.2$ for Ge.

subtracting, we get: $V_{CE} = -1.7 + 2.5 I_B$. Also from the second eq.n:

$1.3 = -(2.5 + 1 + 44) I_B = -47.5 I_B$ or $I_B = -27.4\,\mu A$ and $I_C = -1.2\,mA$ $\longleftarrow$

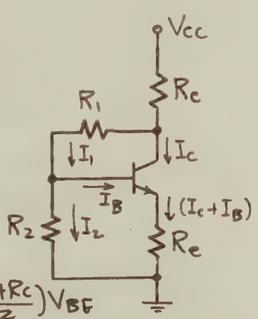
Then $V_{CE} = -1.78\,V$ $\longleftarrow$

c) R_b is increased to $(2.5 + 0.69)K = 3.19K$ $\therefore S = 45 \dfrac{1 + 3.19}{45 + 3.19} = 3.92$

Base-emitter circuit: $1.3 + (3.19 + 45) I_B = 0$ or $I_B = -27\,\mu A$ $\longleftarrow$

$I_C = 44 \times (-27 \times 10^{-3}) = -1.19\,mA$. Then $V_{CE} \doteq -1.78\,V$ $\longleftarrow$

(10-9) KVL around the collector-base-emitter loop gives:



$V_{CC} = (I_C + I_1) R_c + (I_C + I_B) R_e + I_1 R_1 + V_{BE}$

but $I_2 = \dfrac{V_{BE} + (I_B + I_C) R_e}{R_2}$ and $I_1 = I_B + I_2$

or $I_1 = \dfrac{I_B R_2 + V_{BE} + (I_B + I_C) R_e}{R_2}$. Substituting for I_1 & I_2,

we get after some simplification:

$$V_{CC} = I_C\left[R_c + R_e + \frac{R_e(R_1 + R_c)}{R_2}\right] + I_B\left[R_e + \frac{(R_e + R_2)(R_1 + R_c)}{R_2}\right] + \left(1 + \frac{R_1 + R_c}{2}\right) V_{BE}$$

Since $I_C = \beta I_B + (1 + \beta) I_{CO}$, we have $I_B = \dfrac{I_C - (1+\beta) I_{CO}}{\beta}$. Substituting,

$$V_{CC} = I_C\left[R_c + R_e + \frac{R_e(R_1 + R_c)}{R_2}\right] + \frac{I_C - (1+\beta) I_{CO}}{\beta}\left[R_e + \frac{(R_e + R_2)(R_1 + R_c)}{R_2}\right] + V_{BE}\left(1 + \frac{R_1 + R_c}{2}\right)$$

$S = \dfrac{dI_C}{dI_{CO}}$. Differentiating the above eq'n and assuming V_{BE} constant, we obtain after simplification:

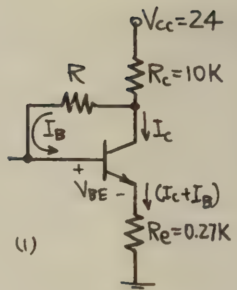
$$S = \frac{(\beta + 1)\left[R_2(R_c + R_1) + R_e(R_1 + R_2 + R_c)\right]}{R_1 R_2 + (\beta + 1)\left[R_2 R_c + R_e(R_1 + R_2 + R_c)\right]} \quad \longleftarrow$$

(10-10) a) KVL in the collector-emitter circuit gives:

$V_{CC} = (I_B+I_C)(R_C+R_e)+V_{CE}$ or $24-5=(I_B+I_C)(10.27K)$

thus, $(I_B+I_C)=(1+45)I_B=\frac{19}{10.27}=1.85$ mA or $I_B \doteq 40\mu A$

Then, $I_B R = V_{CE}-V_{BE}=5-0.6=4.4V$ ∴ $R=\frac{4.4}{40\times10^{-3}}=110K$ ←



b) From the collector-base circuit:

$V_{CC}=(I_B+I_C)(R_C+R_e)+I_BR+V_{BE}$ or $I_B=\frac{V_{CC}-V_{BE}-I_C(R_C+R_e)}{R_C+R_e+R}$ (1)

From Eq. (10-4) $I_C=\beta I_B+(\beta+1)I_{CO}$ (2) Substituting (1) into (2) and simplifying, we get $I_C\left[1+\frac{\beta(R_C+R_e)}{R_C+R_e+R}\right]=(\beta+1)I_{CO}+\frac{\beta(V_{CC}-V_{BE})}{R_C+R_e+R}$

Hence, $S=\frac{dI_C}{dI_{CO}}=\frac{\beta+1}{1+\beta\left(\frac{R_C+R_e}{R_C+R_e+R}\right)}=\frac{46}{1+45\,\frac{10.27}{120.27}}=9.5$ ←

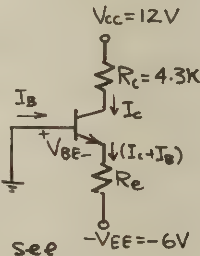
(10-11) a) Collector-emitter circuit: $V_{CC}+V_{EE}=R_CI_C+R_e(I_C+I_B)+V_{CE}$

or $18=4.3I_C+R_e(1+\frac{1}{50})I_C+4$ (1)

Base-emitter circuit: $V_{EE}=(I_C+I_B)R_e+V_{BE}$

or $6=(1+\frac{1}{50})I_CR_e+0.6^*$ (2)

Solving (1) & (2) simultaneously, we get $I_C=2mA$, $R_e=2.7K$ ←



b) Comparing the circuit at hand with that of Fig. 10-6b, we see they are similar except here $R_b=0$. Eq. (10-17), therefore, is valid with $R_b=0$. Under this condition, $S=1$ ←

(10-12) a) Collector-emitter circuit of Q1: $V_{CC}=R_{C1}(I_{C1}+\cancel{I_{B2}})+R_{e1}(I_{C1}+\cancel{I_{B1}})+V_{CE1}$

or $24-4=-(17.8+2.2)I_{C1}$ ∴ $I_{C1}=-1$ mA

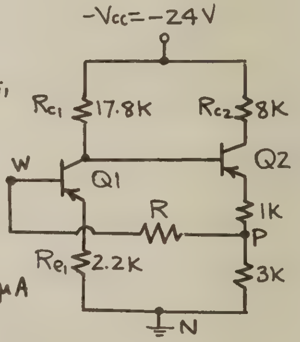
Similarly for Q2: $24-6=-(1+3+8)I_{C2}=-12I_{C2}$ ∴ $I_{C2}=-1.5mA$

Voltage at pt. P is $-3\times1.5=-4.5V$; $V_{BE}=0.2V^*$

Voltage at pt W is $-(2.2\times1+0.2)=-2.4V$

Voltage across R is $-2.4-(-4.5)=2.1V$; $I_{B1}=\frac{I_{C1}}{\beta}=-10\mu A$

∴ $R=\frac{2.1}{10\times10^{-6}}=210K$ ←



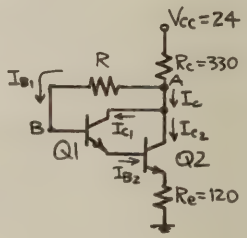
b) Bias stabilization is obtained in this circuit by the mechanism of negative dc feedback through R as well as the emitter resistors. For example, if I_{C1} increases, V_{CN1} decreases (in magnitude) thus reducing I_{B2}, I_{E2} and consequently the voltage at pt. P. This, in turn, reduces I_{B1} and, hence, offsets the initial increase in I_{C1}.

(10-13) a) Collector-emitter circuit of Q2: $V_{CC}=(I_{B1}+I_{C1}+I_{C2})(R_C+R_e)+V_{CE2}$

or $(I_{B1}+I_{C1}+I_{C2})=-I_{E2}=\frac{24-6}{330+120}=40$ mA ; $I_{B2}=-I_{E1}=\frac{-I_{E2}}{1+\beta_2}=1mA$

and $I_{B1}=\frac{-I_{E1}}{1+\beta_1}=\frac{1mA}{1+24}=40\mu A$; $V_R=V_A-V_B$

where $V_A=24-330\times40\times10^{-3}=10.8V$, $V_B=120\times40\times10^{-3}+2\times0.6=6V$



(10-13) (Cont'd) Thus $V_R = 10.8 - 6 = 4.8V$ ∴ $R = \frac{V_R}{I_{B1}} = \frac{4.8V}{40\times10^{-6}} = 120K$ ⟵

b) Treating the two transistors as one unit, we see that this circuit is similar to that of Prob. 10-10, where we obtained S to be:

$S = \frac{\beta+1}{1+\beta\frac{(R_c+R_e)}{(R_c+R_e+R)}}$. Note that $\beta = \frac{I_c}{I_{B1}} = \frac{I_{c1}+I_{c2}}{I_{B1}} \doteq \frac{40\times10^{-3}}{40\times10^{-6}} = 1000$

Thus, $S = \frac{1001}{1+1000\frac{(.33+.12)}{(.33+.12+120)}} = \frac{1001}{1+\frac{450}{120.45}} = \frac{1001}{1+3.73} = 211$ ⟵

(10-14) From Eq. (10-28), $I_c = \frac{\beta(V+V'-V_{BE})}{R_b+R_e(1+\beta)}$. Since V' is independent of β, then

$$S'' = \frac{\partial I_c}{\partial \beta} = \frac{[R_b+R_e(1+\beta)](V+V'-V_{BE})-\beta(V+V'-V_{BE})R_e}{[R_b+R_e(1+\beta)]^2} = \frac{(V+V'-V_{BE})(R_b+R_e)}{[R_b+R_e(1+\beta)]^2}$$

$= \frac{I_c}{\beta}\cdot\frac{R_b+R_e}{R_b+R_e(1+\beta)} = \frac{I_c}{\beta}\left(\frac{1+R_b/R_e}{1+\beta+R_b/R_e}\right)$. From Eq. (10-17) we note that the term in the parenthesis is identically $\frac{S}{1+\beta}$

∴ $S'' = \frac{I_c S}{\beta(1+\beta)}$ QED

(10-15) From Eq. (10-14), $R_b = R_1 \| R_2 = \frac{90\times10}{100}K = 9K$; $\frac{R_b}{R_e} = \frac{9K}{1K} = 9$. Then from Eq. (10-17)

$S(25^\circ C) = \frac{(1+\beta)(1+R_b/R_e)}{(1+\beta+R_b/R_e)} = \frac{56(1+9)}{56+9} = \frac{560}{65} = 8.61$

From Eq. (10-26), $S'(25^\circ C) = \frac{-S}{(R_b+R_e)}\frac{\beta}{(1+\beta)} = \frac{-8.61}{(9+1)}\frac{55}{56} = -0.845$ mA/V

$S_2(+175^\circ C) = \frac{(1+\beta_2)(1+R_b/R_e)}{(1+\beta_2+R_b/R_e)} = \frac{101\times10}{101+9} = 9.18$

From Eq. (10-34), $S''(175^\circ C) = \frac{I_{c1}S}{\beta_1(1+\beta_2)} = \frac{1.5\times9.18}{55\times101} = 2.48\times10^{-3}$ mA

also $S_2(-65^\circ C) = \frac{26\times10}{26+9} = 7.43$; $S''(-65^\circ C) = \frac{1.5\times7.43}{55\times26} = 7.8\times10^{-3}$ mA

Hence, from Eq. (10-36), $\Delta I_c(+175^\circ C) = S\Delta I_{co} + S'\Delta V_{BE} + S''\Delta\beta$

$= (8.61\times33\times10^{-3}) + (-0.845)\times(-0.375) + (2.48\times10^{-3})(45)$

$= 0.284 + 0.317 + 0.111 = 0.712$ mA

∴ $I_c = 1.5 + 0.712 = 2.21$ mA ⟵

$\Delta I_c(-65^\circ C) = (8.61)\times(-10^{-6}) + (-0.845)(0.18) + (7.8\times10^{-3})(-30) = 0.000 - 0.152 - 0.234$

$= -0.386$ mA

∴ $I_c = 1.5 - 0.386 = 1.11$ mA ⟵

(10-16) From Prob. 10-15 $S = 8.61$, $S' = -0.845$

$S_2(75^\circ C) = \frac{91\times10}{91+9} = 9.1$; $S_2(-65^\circ C) = \frac{21\times10}{21+9} = 7$; $S''(75^\circ C) = \frac{1.5\times9.1}{55\times91} = 2.72$ μA

$S''(-65^\circ C) = \frac{1.5\times7}{55\times21} = 9.12\times10^{-3}$ mA

(10-16) (Cont'd) $\Delta I_c(75°C) = 8.61 \times 31 \times 10^{-3} + (-0.85) \times (-0.1) + (2.72 \times 10^{-3})(35)$

$= 0.267 + 0.085 + 0.095 = 0.447$ mA

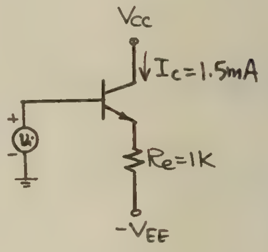
$\therefore I_c(75°C) = 1.5 + 0.447 \doteq 1.95$ mA $\longleftarrow$

$\Delta I_c(-65°C) = 8.61 \times (-10^{-3}) + (-0.85) \times (0.18) + 9.12 \times 10^{-3} \times (-35)$

$= -0.0086 - 0.153 - 0.319 = -0.481$

$\therefore I_c(-65°C) = 1.5 - 0.481 = 1.02$ mA $\longleftarrow$

(10-17) Note that $R_b = 0$. Hence from Eq. (10-17), $S(25°C) = 1$



From Eq. (10-26), $S'(25°C) = \frac{-S}{(R_b + R_e)} \frac{\beta}{1+\beta} = \frac{-1}{1K} \frac{55}{56} = -0.982$ mA/V

$S_2(175°C) = 1$. Then, from Eq. (10-34), $S''(175°C) = \frac{1.5 \times 1}{55(101)} = 0.27\ \mu A$

$S_2(-65°C) = 1$, $S''(-65°C) = \frac{1.5 \times 1}{55 \times 26} = 1.05 \times 10^{-3}$ mA

Hence, from Eq. (10-36), $\Delta I_c(175°C) = S \Delta I_{co} + S' \Delta V_{BE} + S'' \Delta\beta$

$= 1 \times 33 \times 10^{-3} + (-0.982)(-0.375) + (0.27 \times 10^{-3})(45)$

$= 0.033 + 0.368 + 0.012 = 0.413$ mA

$\therefore I_c(175°C) = 1.5 + 0.413 = 1.913$ mA $\longleftarrow$

$\Delta I_c(-65°C) = 1(-10^{-6}) + (-0.982)(0.18) + (1.05 \times 10^{-3})(-30)$

$= 0.000 - 0.177 - 0.031 = -0.208$

$\therefore I_c(-65°C) = 1.5 - 0.208 = 1.292$ mA $\longleftarrow$

Comparing with the results of Prob. 10-15 we see that the change in I_c is smaller in this circuit, because R_b is zero, resulting is smaller stability factors.

(10-18) From Prob. 10-17, $S(25°C) = 1$, $S'(25°C) = -0.985$ mA/V

$S''(75°C) = \frac{1.5 \times 1}{55 \times 91} = 3 \times 10^{-4}$ mA ; $S''(-65°C) = \frac{1.5}{55 \times 21} = 1.3 \times 10^{-3}$ mA

$\Delta I_c(75°C) = 1 \times 31 \times 10^{-3} + (-0.985)(-0.1) + 3 \times 10^{-4} \times 35 = 0.031 + 0.098 + 0.01$

$= 0.14$ mA

$\therefore I_c(75°C) = 1.5 + 0.14 = 1.64$ mA $\longleftarrow$

$\Delta I_c(-65°C) = 1(-0.95 \times 10^{-4}) + (-0.985)(.18) + 1.3 \times 10^{-3}(-35) = 0.000 - 0.175 - 0.045$

$= -0.22$ mA

$\therefore I_c(-65°C) = 1.5 - 0.22 = 1.28$ mA $\longleftarrow$

(10-19) With $R_b/R_e = \frac{7.75K}{1K} = 7.75$, Eq. (10-17) becomes: $S(25°C) = \frac{56(1+7.75)}{56+7.75} = 7.68$

From Eq. (10-26), $S'(25°C) = \frac{-S}{R_b + R_e} \cdot \frac{\beta}{1+\beta} = \frac{-7.68}{8.75} \frac{55}{56} = -0.862$ mA/V

$S_2(+175°C) = \frac{101 \times 8.75}{101 + 7.75} = 8.14$. From Eq. (10-34), $S''(175°C) = \frac{1.5 \times 8.14}{55 \times 101} = 2.2 \times 10^{-3}$ mA

(10-19) (Cont'd) $S_2(-65°C) = \dfrac{26\times 8.75}{26+7.75} = 6.75$; $S''(-65°C) = \dfrac{1.5\times 6.75}{55\times 26} = 7.08\times 10^{-3}$ mA

Hence, from Eq. (10-36), $\Delta I_c(175°C) = 7.68\times 33\times 10^{-3} + (-0.862)(-0.375) + 2.2\times 10^{-3}\times 45$

$= 0.253 + 0.323 + 0.099 = 0.675$ mA

$\therefore\ I_c(+175°C) = 1.5 + 0.675 = 2.175$ mA

$\Delta I_c(-65°C) = 7.68\times(-10^{-6}) + (-0.862)(0.18) + 7.08\times 10^{-3}(-30)$

$= 0.000 - 0.155 - 0.212 = -0.367$ mA

$\therefore\ I_c(-65°C) = 1.5 - 0.367 = 1.133$ mA

(10-20) From Prob. 10-19, $S(25°C) = 7.68$; $S'(25°C) = -0.862$ mA/V .

$S(75°C) = \dfrac{91\times 8.75}{91+7.75} = 8.05$; $S(-65°C) = \dfrac{21\times 8.75}{21+7.75} = 6.39$; $S''(75°C) = \dfrac{1.5\times 8.05}{55\times 91}$

or $S''(75°C) = 2.42\times 10^{-3}$; $S''(-65°C) = \dfrac{1.5\times 6.39}{55\times 21} = 8.27\times 10^{-3}$

From Eq. (10-36), $\Delta I_c(75°C) = 7.68\times 31\times 10^{-3} + (0.862)(-0.1) + 2.42\times 10^{-3}\times 35$

$= 0.237 + 0.086 + 0.085 = 0.408$ mA

$\therefore\ I_c(+75°C) = 1.5 + 0.408 = 1.908$ mA

$\Delta I_c(-65°C) = 7.68\times(-10^{-3}) + (-0.862)(0.18) + 8.27\times 10^{-3}(-35)$

$= -0.008 - 0.155 - 0.290 = -0.453$ mA

$\therefore\ I_c(-65°C) = 1.5 - 0.453 = 1.047$ mA

(10-21) From Eq. (10-28), $I_c = \dfrac{\beta(V+V'-V_{BE})}{R_b+R_e(1+\beta)}$ where $V' = (R_b+R_e)\left(\dfrac{1+\beta}{\beta}\right)I_{co}$

$S'' = \dfrac{\partial I_c}{\partial\beta} = \dfrac{[R_b+R_e(1+\beta)][(V+V'-V_{BE})+\beta\frac{\partial V'}{\partial\beta}] - \beta(V+V'-V_{BE})R_e}{[R_b+R_e(1+\beta)]^2}$ where $\dfrac{\partial V'}{\partial\beta} = -\dfrac{(R_b+R_e)I_{co}}{\beta^2}$

Substituting and regrouping terms we have:

$S'' = \dfrac{(V+V'-V_{BE})(R_b+R_e)}{[R_b+R_e(1+\beta)]^2} - \dfrac{(R_b+R_e)I_{co}}{\beta[R_b+R_e(1+\beta)]} = \dfrac{I_c S}{\beta(1+\beta)} - \dfrac{I_{co}S}{\beta(1+\beta)}$ using Eqs. (10-17) and (10-28). $\therefore\ S'' = \dfrac{(I_c - I_{co})}{\beta(1+\beta)}S$ QED

(10-22) Substituting Eq. (10-24) into (10-28) we obtain: $I_c = \dfrac{\beta[V-V_{BE}+(R_b+R_e)\frac{1+\beta}{\beta}I_{co}]}{R_b+R_e(1+\beta)}$

Now form the expression $I_c - I_{co} = \dfrac{\beta[V-V_{BE}+(R_b+R_e)\frac{1+\beta}{\beta}I_{co}] - [R_b+R_e(1+\beta)]I_{co}}{R_b+R_e(1+\beta)}$

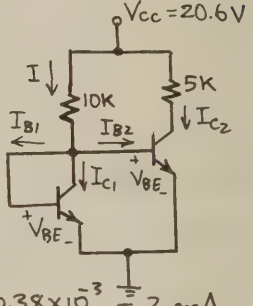
or $I_c - I_{co} = \dfrac{\beta(V-V_{BE})}{R_b+R_e(1+\beta)}$

Using the hint, we have: $\dfrac{I_{c2}-I_{co2}}{I_{c1}-I_{co1}} = \dfrac{\beta_2}{\beta_1}\dfrac{R_b+R_e(1+\beta_1)}{R_b+R_e(1+\beta_2)}$. Subtracting unity from both sides of the eq'n:

$$\frac{I_{c2}-I_{co2}}{I_{c1}-I_{co1}} - 1 = \frac{\beta_2}{\beta_1}\frac{R_b+R_e(1+\beta_1)}{R_b+R_e(1+\beta_2)} - 1 = \frac{(\beta_2-\beta_1)(R_b+R_e)}{\beta_1[R_b+R_e(1+\beta_2)]} = \frac{\beta_2-\beta_1}{\beta_1(1+\beta_2)}S_2$$

(10-22) (Cont'd) $\dfrac{(I_{C2}-I_{CO2})-(I_{C1}-I_{CO1})}{I_{C1}-I_{CO1}} = \dfrac{\beta_2-\beta_1}{\beta_1(1+\beta_2)} S_2$

or $\dfrac{(I_{C2}-I_{CO2})-(I_{C1}-I_{CO1})}{\beta_2-\beta_1} = \dfrac{\Delta(I_C-I_{CO})}{\Delta\beta} \equiv S'' = \dfrac{(I_{C1}-I_{CO1})S_2}{\beta_1(1+\beta_2)}$ QED



(10-23) a) From the circuit we have: $I = \dfrac{20.6-0.6}{10K} = 2\,mA$

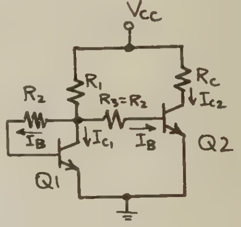
Since $I_{B1} = I_{B2} = I_B$, then $I = 2I_B + I_{C1} = 2I_B + \beta I_B$

or $2mA = (2+48)I_B \quad \therefore I_B = \dfrac{2}{50}mA = 40\,\mu A$

Then, $I_{C1} = I_{C2} = \beta I_B = 48\times40\times10^{-3} = 1.92\,mA$

b) $I = \dfrac{V_{CC}-V_{BE}}{10K} = \dfrac{20.6-0.22}{10K} = 2.038\,mA = (2+\beta)I_B$

or $I_B = \dfrac{2.038}{2+98}mA = 20.38\,\mu A \quad \therefore I_{C1} = I_{C2} = \beta I_B = 98\times20.38\times10^{-3} \cong 2\,mA$



(10-24) Applying KVL in the collector-base circuit of Q1 we obtain: $V_{CC} = (I_{C1}+2I_B)R_1 + R_2I_B + V_{BE}$

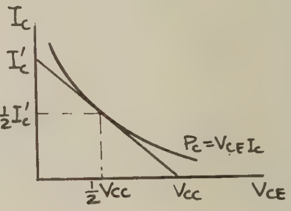
Solving for I_{C1} we have: $I_{C1} = \dfrac{V_{CC}-V_{BE}}{R_1} - (2+\dfrac{R_2}{R_1})I_B$

QED

(10-25) a) From Eq. (10-42), $T_j - T_A = \Theta P_D$ or $150-25 = \Theta\times(125)$

$\therefore \Theta = 1\,°C/mW$

b) $T_j - 25 = 1\times75 \qquad \therefore T_j = 100°C$



(10-26) For the hyberbola, $P_C = V_{CE}I_C = \text{constant}$ or $V_{CE} = \dfrac{P_C}{I_C}$

slope $= \dfrac{dV_{CE}}{dI_C} = -\dfrac{P_C}{I_C^2} = -\dfrac{V_{CE}I_C}{I_C^2} = -\dfrac{V_{CE}}{I_C}$ (1)

At the point of tangency, $\dfrac{dV_{CE}}{dI_C} = -R_L = -\dfrac{V_{CC}}{I'_C}$ (2)

From (1) & (2) $\dfrac{V_{CC}}{I'_C} = \dfrac{V_{CE}}{I_C}$ (3). Also $V_{CE} = V_{CC} - I_CR_L = V_{CC} - I_C\dfrac{V_{CC}}{I'_C}$

or $V_{CE} = V_{CC}(1-\dfrac{I_C}{I'_C})$.(4). From (3) $\dfrac{V_{CE}}{V_{CC}} = \dfrac{I_C}{I'_C}$ and from (4) $\dfrac{V_{CE}}{V_{CC}} = (1-\dfrac{I_C}{I'_C})$

Hence, $\dfrac{I_C}{I'_C} = \dfrac{1}{2} = \dfrac{V_{CE}}{V_{CC}}$ and $V_{CE} = \frac{1}{2}V_{CC}$ at the pt. of tangency. QED.

(10-27) a) Since the transistor is cutoff, $P_C = I_{CO}V_{CB} \approx I_{CO}(V_{CC} - I_{CO}R_C)$ (1)

To avoid thermal runaway, $\dfrac{\partial P_C}{\partial T_j} = \dfrac{\partial P_C}{\partial I_{CO}}\dfrac{\partial I_{CO}}{\partial T_j} < \dfrac{1}{\Theta}$ Eq. (10-47)

From (1) and Eq. (10-51) $\dfrac{\partial P_C}{\partial I_{CO}}\cdot\dfrac{\partial I_{CO}}{\partial T_j} = (V_{CC}-2I_{CO}R_C)(0.07I_{CO}) < \dfrac{1}{\Theta}$ (2)

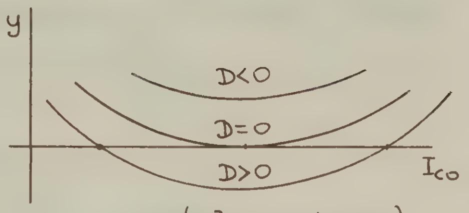
(10-27) (Cont'd) At runaway, $\frac{\partial P_c}{\partial I_{co}} \cdot \frac{\partial I_{co}}{\partial T_j} = \frac{1}{\theta} = 0.07 I_{co} V_{cc} - 2(0.07) I_{co}^2 R_c$

or $2\theta R_c (0.07) I_{co}^2 - (0.07)\theta V_{cc} I_{co} + 1 = y = 0$

Then, $I_{co} = \frac{V_{cc} \pm \sqrt{V_{cc}^2 - 8R_c/(0.07\theta)}}{4R_c}$

i.e. runaway will occur for values of I_{co} in the range:

$$\frac{V_{cc} - \sqrt{V_{cc}^2 - 8R_c/0.07\theta}}{4R_c} \leq I_{co} \leq \frac{V_{cc} + \sqrt{V_{cc}^2 - 8R_c/0.07\theta}}{4R_c}$$



$D = (V_{cc}^2 - 8R_c/0.07\theta)$

b) From (2) we see that if $V_{cc} \leq 2 I_{co} R_c$, the left side is always negative and runaway is impossible. Under this condition, $I_{co} = \frac{V_{cc}}{2R_c}$ QED

(10-28) a) From Prob. 10-27a, at the point of runaway, $I_{co} = \frac{V_{cc} - \sqrt{V_{cc}^2 - 8R_c/0.07\theta}}{4R_c}$

or $I_{co} = \frac{30 - \sqrt{900 - 8000/0.07 \times 250}}{4000} = \frac{30 - \sqrt{900 - 456}}{4000} = \frac{30 - 21.1}{4000} = 2.23 \text{ mA} \longleftarrow$

b) $I_{co}(@T_j) = I_{co}(@25°C) \cdot 2^{\frac{\Delta T}{10}}$ or $2.23 = 0.01 \times 2^{\frac{\Delta T}{10}}$; $2^{\frac{\Delta T}{10}} = 223$

$\ln 223 = \frac{\Delta T}{10} \ln 2$ or $5.41 = 0.0693 \Delta T$ $\therefore \Delta T = 78°$

Hence, $T_j = 25 + 78 = 103°C$

The ambient temperature at which runaway will occur can be found from Eq. (10-42) : $T_j - T_a = \theta P_D$

where $P_D = (V_{cc} - I_{co} R_c) I_{co} = (30 - 2.23 \times 1)(2.23 \times 10^{-3}) = 0.062 \text{ W}$

Thus, $T_a = T_j - \theta P_D = 103 - 0.062 \times 250 = 103 - 15.5 = 87.5°C \longleftarrow$

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CHAPTER 11

(11-1) a) From Fig. 9-14a we obtain:

$$h_{fe} \approx \left.\frac{\Delta I_c}{\Delta I_B}\right|_{V_{CE}} \div \frac{39 - 22}{200 \times 10^{-3}} = 85 \longleftarrow$$

$$h_{oe} \approx \left.\frac{\Delta I_c}{\Delta V_c}\right|_{I_B} \div \frac{(32.5 - 31) \times 10^{-3}}{4} = 375 \ \mu\mho \longleftarrow$$

b) $h_{fb} = -\frac{h_{fe}}{1 + h_{fe}} = -\frac{85}{86} \div -\left(1 - \frac{1}{86}\right) = -1 + 0.0116 = -0.9884 \longleftarrow$

$h_{ob} = \frac{h_{oe}}{1 + h_{fe}} = \frac{375}{86} = 4.36 \ \mu\mho \longleftarrow$

(11-2) a) By definition $h_{fe} = \left.\frac{I_c}{I_b}\right|_{V_{ce}=0}$ where from Fig. 11-9b we get:

$I_c = h_{fb}I_e - I = h_{fb}I_e - V_{bc}h_{ob}$. Note that with $V_{ce}=0$, $V_{be} = V_{bc} = -V_{cb}$

Also, $I_e = -\frac{V_{be} + h_{rb}V_{cb}}{h_{ib}} = -\frac{(h_{rb}-1)V_{cb}}{h_{ib}}$. Substituting, we have:

$$I_c = \left[\frac{h_{fb}(1-h_{rb})}{h_{ib}} + h_{ob}\right]V_{cb} \; ; \; I_b = -I_c - I_e$$

$$\text{or } I_b = -\left[\frac{h_{fb}(1-h_{rb})}{h_{ib}} + h_{ob}\right]V_{cb} - \frac{(1-h_{rb})}{h_{ib}}V_{cb} = -\left[\frac{(1+h_{fb})}{h_{ib}}(1-h_{rb}) + h_{ob}\right]V_{cb}$$

$$\therefore h_{fe} = \frac{I_c}{I_b} = -\frac{\frac{h_{fb}}{h_{ib}}(1-h_{rb}) + h_{ob}}{\frac{1+h_{fb}}{h_{ib}}(1-h_{rb}) + h_{ob}} = -\frac{h_{fb}(1-h_{rb}) + h_{ib}h_{ob}}{(1+h_{fb})(1-h_{rb}) + h_{ib}h_{ob}} \qquad \text{QED}$$

b) For the typical values of Table 11-2, $h_{rb} \ll 1$ and $h_{ib}h_{ob} \ll (1+h_{fb})$

hence, $h_{fe} \approx -\frac{h_{fb}}{1+h_{fb}}$

(11-3) a) By definition $h_{fb} = \left.\frac{I_c}{I_e}\right|_{V_{cb}=0}$ where from Table 11-1a we get:

$$I_b = -(I_c+I_e) \; ; \; I_c = h_{fe}I_b + V_{ce}h_{oe} = -h_{fe}(I_c+I_e) + V_{ce}h_{oe} \qquad (1)$$

$$V_{be} = h_{ie}I_b + h_{re}V_{ce} = -h_{ie}(I_c+I_e) + h_{re}V_{ce} \qquad (2)$$

For $V_{cb}=0$, then $V_{ce} = \cancel{V_{cb}} + V_{be} = V_{be}$. Thus (2) becomes:

$$(1-h_{re})V_{ce} = -h_{ie}(I_c+I_e) \quad \text{or} \quad V_{ce} = -\frac{h_{ie}}{1-h_{re}}(I_c+I_e) \qquad (3)$$

Substituting (3) into (1), we have:

$$I_c = -h_{fe}(I_c+I_e) - \frac{h_{ie}h_{oe}}{1-h_{re}}(I_c+I_e)$$

$$\text{or} \quad I_c\left(1 + h_{fe} + \frac{h_{ie}h_{oe}}{1-h_{re}}\right) = -\left(h_{fe} + \frac{h_{ie}h_{oe}}{1-h_{re}}\right)I_e$$

$$\text{Hence, } h_{fb} = \frac{I_c}{I_e} = -\frac{\left(h_{fe} + \frac{h_{ie}h_{oe}}{1-h_{re}}\right)}{1 + h_{fe} + \frac{h_{ie}h_{oe}}{1-h_{re}}} = -\frac{h_{fe}(1-h_{re}) + h_{ie}h_{oe}}{(1+h_{fe})(1-h_{re}) + h_{ie}h_{oe}}$$

QED

b) From Table 11-2 we see that $h_{re} = 2.5\times10^{-4} \ll 1$

$h_{ie}h_{oe} = 1100\times25\times10^{-6} \approx 3\times10^{-2} \ll h_{fe} = 50$

Hence, $h_{fb} \approx -\frac{h_{fe}}{1+h_{fe}}$

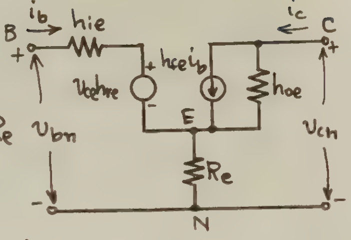
(11-4) Refer to the CC hybrid model of Table 11-1.

$h_{ie} = \left.\frac{v_{be}}{i_b}\right|_{v_{ce}=0}$, where $v_{be} = i_bh_{ic} - h_{rc}\cancel{v_{ce}} + \cancel{v_{ce}} = i_bh_{ic}$ $\quad\therefore h_{ie} = h_{ic}$

$h_{fe} = \left.\frac{i_c}{i_b}\right|_{v_{ce}=0}$, where $i_c = -i_b - i_e = -i_b - h_{fc}i_b$ $\quad\therefore h_{fe} = \frac{-i_b - h_{fc}i_b}{i_b} = -(1+h_{fc})$

$h_{re} = \left.\frac{v_{be}}{v_{ce}}\right|_{i_b=0}$, where $v_{be} = \cancel{i_b}h_{ic} - h_{rc}v_{ce} + v_{ce}$ $\quad\therefore h_{re} = \frac{-h_{rc}v_{ce} + v_{ce}}{v_{ce}} = 1 - h_{rc}$

(11-4) (Cont'd) $h_{oe} = \left.\frac{i_c}{v_{ce}}\right|_{i_b=0}$ where $i_c = -i_e = -h_{fc}\cancel{i_b} + v_{ce}h_{oc}$ $\therefore h_{oe} = h_{oc}$



(11-5) a) $h'_{ie} = \left.\frac{v_{bn}}{i_b}\right|_{v_{cn}=0}$. With $v_{cn}=0$, $v_{ce} = -(i_b+i_c)R_e$

$= -i_bR_e - h_{fe}i_bR_e - v_{ce}h_{oe}R_e$

Thus, $v_{ce} = -\frac{i_bR_e(1+h_{fe})}{1+h_{oe}R_e} = -v_{en}$

and $v_{bn} = i_bh_{ie} + h_{re}v_{ce} + v_{en} = i_bh_{ie} + \frac{i_bR_e(1+h_{fe})}{1+h_{oe}R_e}(1-h_{re})$

Hence, $h'_{ie} = h_{ie} + \frac{R_e(1+h_{fe})}{1+h_{oe}R_e}(1-h_{re}) \approx h_{ie} + \frac{R_e(1+h_{fe})}{1+h_{oe}R_e}$ since $h_{re} \ll 1$

b) $h'_{re} = \left.\frac{v_{bn}}{v_{cn}}\right|_{i_b=0}$ where $v_{bn} = h_{ie}\cancel{i_b} + h_{re}v_{ce} + v_{en}$.

Since $i_b = 0$, $i_c = v_{ce}h_{oe} = \frac{v_{en}}{R_e}$. Also, $v_{cn} = v_{ce} + v_{en}$; $v_{ce} = \frac{v_{en}}{h_{oe}R_e}$;

$v_{cn} = v_{en}\left(1 + \frac{1}{h_{oe}R_e}\right)$ or $v_{en} = v_{cn}\frac{h_{oe}R_e}{1+h_{oe}R_e}$

Thus, $v_{bn} = \frac{h_{re}v_{cn}}{1+h_{oe}R_e} + \frac{h_{oe}R_ev_{cn}}{1+h_{oe}R_e}$ $\therefore h'_{re} = \frac{h_{re} + h_{oe}R_e}{1+h_{oe}R_e}$

c) $h'_{fe} = \left.\frac{i_c}{i_b}\right|_{v_{cn}=0}$ where $i_c = h_{fe}i_b + h_{oe}v_{ce}$; $v_{ce} = -\frac{i_bR_e(1+h_{fe})}{1+h_{oe}R_e}$ (part $\underline{a}$)

or $i_c = i_b\left[h_{fe} - \frac{h_{oe}R_e(1+h_{fe})}{1+h_{oe}R_e}\right]$ $\therefore h'_{fe} = \frac{h_{fe} - h_{oe}R_e}{1+h_{oe}R_e}$

d) $h'_{oe} = \left.\frac{i_c}{v_{cn}}\right|_{i_b=0}$ where $i_c = \frac{v_{en}}{R_e} = \frac{v_{cn}}{\cancel{R_e}}\,\frac{h_{oe}\cancel{R_e}}{1+h_{oe}R_e} = \frac{h_{oe}}{1+h_{oe}R_e}v_{cn}$ (part $\underline{b}$)

Hence, $h'_{oe} = \frac{h_{oe}}{1+h_{oe}R_e}$

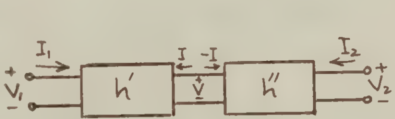
e) If $h_{oe}R_e \ll 1$, then $h'_{ie} \approx h_{ie} + (1+h_{fe})R_e$, $h'_{re} \approx h_{re} + h_{oe}R_e$

$h'_{fe} \approx h_{fe} - h_{oe}R_e \approx h_{fe}$, $h'_{oe} \approx h_{oe}$

(11-6)

$V_1 = h'_{11}I_1 + h'_{12}V$ (1) $V = -h''_{11}I + h''_{12}V_2$ (3)

$I = h'_{21}I_1 + h'_{22}V$ (2) $I_2 = -h''_{21}I + h''_{22}V_2$ (4)



Eliminate V and I and reduce the above equations to the form: $V_1 = h_{11}I_1 + h_{12}V_2$ and $I_2 = h_{21}I_1 + h_{22}V_2$

Substituting (3) into (1) and (2), we have:

$V_1 = h'_{11}I_1 + h'_{12}(-h''_{11}I + h''_{12}V_2) = h'_{11}I_1 + h'_{12}h''_{12}V_2 - h'_{12}h''_{11}I$ (5)

$I = h'_{21}I_1 + h'_{22}(-h''_{11}I + h''_{12}V_2) = h'_{21}I_1 + h''_{12}h'_{22}V_2 - h''_{11}h'_{22}I$ (6)

From (6) $I = (h'_{21}I_1 + h''_{12}h'_{22}V_2)/(1+h''_{11}h'_{22})$ (7)

Substituting (7) into (5) and simplifying, we obtain

$$V_1 = \left(h'_{11} - \frac{h'_{12}h''_{11}h'_{21}}{1+h''_{11}h'_{22}}\right)I_1 + \left(\frac{h'_{12}h''_{12}}{1+h''_{11}h'_{22}}\right)V_2 \equiv h_{11}I_1 + h_{12}V_2$$

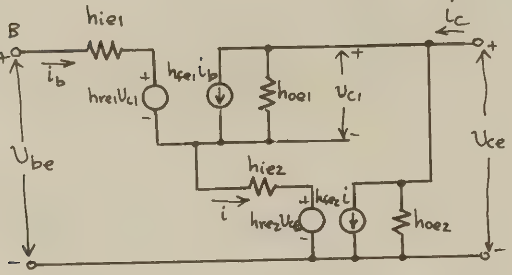
(11-6) (Cont'd) Substituting (2) into (3) and (4) we have:

$$V = -h''_{11}(h'_{21}I_1 + h'_{22}V) + h''_{12}V_2 \qquad \text{or} \qquad V = (-h''_{11}h'_{21}I_1 + h''_{12}V_2)/(1+h''_{11}h'_{22})$$

$$I_2 = -h''_{21}(h'_{21}I_1 + h'_{22}V) + h''_{22}V_2 = -h'_{21}h''_{21}I_1 - h''_{21}h'_{22}\left(\frac{-h''_{11}h'_{21}I_1 + h''_{12}V_2}{1+h''_{11}h'_{22}}\right) + h''_{22}V_2$$

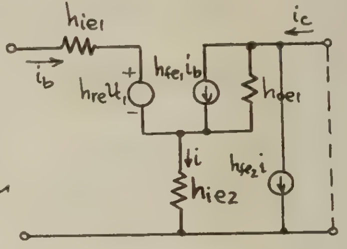
or
$$I_2 = \left(\frac{-h'_{21}h''_{21}}{1+h''_{11}h'_{22}}\right)I_1 + \left(h''_{22} - \frac{h''_{12}h''_{21}h'_{22}}{1+h''_{11}h'_{22}}\right)V_2 \equiv h_{21}I_1 + h_{22}V_2$$

(11-7)



a) $h_{ie} = \left.\frac{v_{be}}{i_b}\right|_{v_{ce}=0}$

With $v_{ce}=0$, the circuit on the left reduces to that on the right. From Prob. 11-5a we have: $h_{ie} = h_{ie1} + \frac{h_{ie2}(1+h_{fe1})(1-h_{re1})}{1+h_{oe1}h_{ie2}}$



b) $h_{fe} = \left.\frac{i_c}{i_b}\right|_{v_{ce}=0}$ where $i_c = h_{fe2}i + h_{fe1}i_b + v_{c1}h_{oe1}$; $v_{c1} = -ih_{ie2}$

and $i = i_b + h_{fe1}i_b + v_{c1}h_{oe1} = i_b(1+h_{fe1}) - ih_{ie2}h_{oe1}$

or $i = i_b(1+h_{fe1})/(1+h_{oe1}h_{ie2})$. Thus, $i_c = h_{fe1}i_b + \frac{h_{fe2}(1+h_{fe1})i_b}{1+h_{oe1}h_{ie2}} - \frac{h_{oe1}h_{ie2}(1+h_{fe1})i_b}{1+h_{oe1}h_{ie2}}$

Hence, $h_{fe} = h_{fe1} + \frac{(h_{fe2} - h_{oe1}h_{ie2})(1+h_{fe1})}{1+h_{oe1}h_{ie2}}$

c) $h_{oe} = \left.\frac{i_c}{v_{ce}}\right|_{i_b=0}$ where $i_c = h_{oe1}v_{c1} + h_{fe2}i + h_{oe2}v_{ce}$; $i = h_{oe1}v_{c1}$

and $v_{ce} = v_{c1} + ih_{ie2} + h_{re2}v_{ce} = v_{c1}(1+h_{ie2}h_{oe1}) + v_{ce}h_{re2}$

or $v_{ce} = v_{c1}(1+h_{ie2}h_{oe1})/(1-h_{re2})$. Thus, $i_c = \left[h_{oe2} + \frac{(1+h_{fe2})(1-h_{re2})}{1+h_{ie2}h_{oe1}}h_{oe1}\right]v_{ce}$

Hence, $h_{oe} = h_{oe2} + \frac{(1+h_{fe2})(1-h_{re2})h_{oe1}}{1+h_{ie2}h_{oe1}}$

d) $h_{re} = \left.\frac{v_{be}}{v_{ce}}\right|_{i_b=0}$ where $v_{be} = h_{re1}v_{c1} + h_{ie2}i + h_{re2}v_{ce}$; $i = v_{c1}h_{oe1}$

and $v_{c1} = \frac{(1-h_{re2})}{1+h_{ie2}h_{oe1}}v_{ce}$ as in part c above.

Thus, $v_{be} = \left[\frac{h_{re1}(1-h_{re2})}{1+h_{ie2}h_{oe1}} + h_{re2} + \frac{h_{oe1}h_{ie2}(1-h_{re2})}{1+h_{ie2}h_{oe1}}\right]v_{ce}$

Hence, $h_{re} = h_{re2} + \frac{(h_{re1} + h_{oe1}h_{ie2})(1-h_{ie2})}{1+h_{ie2}h_{oe1}}$

e) $h_{ie} = 1.1K + \frac{1.1K(1+50)(1-2.5\times10^{-4})}{1+25\times10^{-6}\times1.1\times10^{3}} = 1.1K + \frac{56.1K}{1.0275} = 55.7\,K \longleftarrow$

$h_{fe} = 50 + \frac{(50-0.0275)(51)}{1.0275} = 2530 \longleftarrow$

$h_{oe} = 25\times10^{-6} + \frac{51(1-2.5\times10^{-4})(25\times10^{-6})}{1.0275} = 25\times10^{-6} + 1240\times10^{-6} = 1.265\times10^{-3} \longleftarrow$

$h_{re} = 2.5\times10^{-4} + \frac{(2.5\times10^{-4}+0.0275)(1-2.5\times10^{-4})}{1.0275} = 2.5\times10^{-4} + 270\times10^{-4} = 27.25\times10^{-3} \longleftarrow$

(11-8) From Table 11-4 we have: $A_I = -\dfrac{h_{fc}}{1+h_{oc}R_L} = +\dfrac{51}{1+25\times10^{-6}\times10^4} = 40.8$

$R_i = h_{ic} + h_{rc}A_I R_L = 1.1K + 1\times 40.8\times10^4 = 409.1\,K$; $A_V = \dfrac{A_I R_L}{R_i} = \dfrac{40.8\times10^4}{409.1} = 0.998$

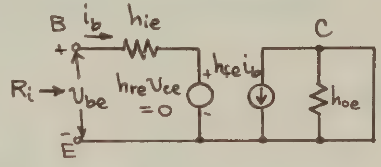
$Y_o = h_{oc} - \dfrac{h_{fc}h_{rc}}{h_{ic}+R_s} = 25\times10^{-6} + \dfrac{51\times1}{(1.1+10)10^3} = 25\times10^{-6} + 4.6\times10^{-3} = 4.625\times10^{-3}$

Thus, $R_o = 0.217\times10^3 = 217\,\Omega$; $A_{V_s} = \dfrac{A_V R_i}{R_i+R_s} = \dfrac{0.998\times409.1}{419.1} = 0.974$

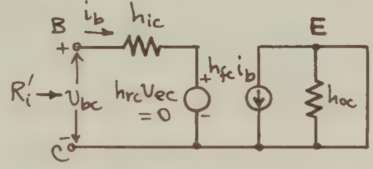
These values agree with the results of Figs. 11-14 to 11-17.

(11-9)
a) $R_i = \left.\dfrac{v_{be}}{i_b}\right|_{v_{ce}=0}$, but by definition

$h_{ie} = \left.\dfrac{v_{be}}{i_b}\right|_{v_{ce}=0}$ $\therefore R_i = h_{ie}$



Similarly, $R_i' = \left.\dfrac{v_{bc}}{i_b}\right|_{v_{ec}=0} = h_{ic}$

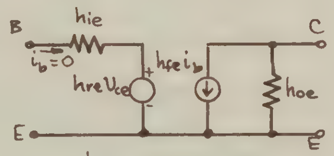


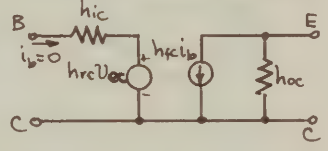
From Table 11-3, we have that $h_{ie} = h_{ic}$

$\therefore R_i = R_i'$ QED

b) From Table 11-3, we have that $h_{oe} = h_{oc}$

The output impedance of either circuit with the input open-circuited $(i_b = 0)$ is by definition $h_{oe} = h_{oc}$





(11-10) From Table 11-4, $R_i = h_i + h_r A_I R_L$ and $R_L = \dfrac{A_V R_i}{A_I}$

Substituting, $R_i = h_i + \dfrac{h_r A_I A_V R_i}{A_I} = h_i + h_r A_V R_i$ $\therefore R_i = \dfrac{h_i}{1-h_r A_V}$

(11-11) From Fig. 11-12, the short-circuit current is I_L when $Z_L = 0$

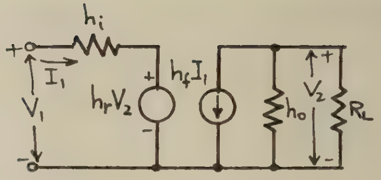
Thus, $I_{sc} = h_f I_1$ where $I_1 = V_s/(h_i+R_s)$ since $V_2 = 0$

so, $I_{sc} = h_f V_s/(h_i+R_s)$. The open-circuit voltage, V_{oc}, is V_2 when $Z_L = \infty$. Then, $V_{oc} = -\dfrac{h_f I_1}{h_o}$ where $I_1 = (V_s - h_r V_{oc})/(h_i+R_s)$

so, $V_{oc} = -\dfrac{h_f}{h_o}\left(\dfrac{V_s - h_r V_{oc}}{h_i+R_s}\right)$ or $V_{oc} = \dfrac{-h_f V_s}{h_o(h_i+R_s) - h_f h_r}$

$$\therefore Y_o = \frac{I_{sc}}{V_{oc}} = \frac{\dfrac{h_f}{h_i+R_s}V_s}{\dfrac{h_f V_s}{h_o(h_i+R_s)-h_f h_r}} = h_o - \frac{h_f h_r}{h_i+R_s} \qquad \text{QED}$$

(11-12) If $R_L = 0$, $R_i = R_{io} = h_i$ by definition.



If $R_L = \infty$, $V_2 = -\frac{h_f I_1}{h_o}$ where $I_1 = \frac{V_1 - h_r V_2}{h_i}$

or $V_1 = h_i I_1 + h_r V_2 = h_i I_1 - \frac{h_r h_f}{h_o} I_1 = (h_i - \frac{h_r h_f}{h_o}) I_1$

Thus, $R_{i\infty} = \frac{V_1}{I_1} = h_i - \frac{h_r h_f}{h_o}$. Substituting, we obtain:

$$Y_o = h_o\left(\frac{R_s + R_{i\infty}}{R_s + R_{io}}\right) = h_o\left(\frac{R_s + h_i - \frac{h_r h_f}{h_o}}{R_s + h_i}\right) = h_o\left(1 - \frac{h_f h_r}{h_o(R_s + h_i)}\right) = h_o - \frac{h_f h_r}{R_s + h_i}$$

Comparing with Table 11-4, we see that the equality is true.

(11-13) For Fig. 11-14

a) $A_I = -\frac{h_{fe}}{1 + h_{oe}R_L} = -\frac{50}{1 + 25\times10^{-6}\times10} \approx -50$; From figure: $R_L = 10\Omega$, $A_I = -50$

$R_L = 10^7\Omega$, $A_I = -0.2$; $A_I = -\frac{50}{1 + 25\times10^{-6}\times10^7} = -\frac{50}{251} \approx -0.2$

b) $R_L = 10\Omega$, $A_I = 0.98$; $A_I = -\frac{h_{fb}}{1 + h_{ob}R_L} = -\frac{-0.98}{1 + 0.49\times10^{-6}\times10} \approx 0.98$

$R_L = 10^7\Omega$, $A_I \approx 0.17$; $A_I = \frac{0.98}{1 + 0.49\times10^{-6}\times10^7} = \frac{0.98}{5.9} = 0.166$

c) $R_L = 10\Omega$, $A_I = 51$; $A_I = \frac{51}{1 + 25\times10^{-6}\times10} \approx 51$

$R_L = 10^7\Omega$, $A_I = 0.2$; $A_I = \frac{51}{1 + 25\times10^{-6}\times10^7} = \frac{51}{251} = 0.203$

For Fig. 11-16

a) $R_L = 10\Omega$, $R_i = 1100\Omega$; $R_i = h_{ie} + h_{re}A_I R_L = 1100 + 2.5\times10^{-4}(-50)10 \approx 1100\Omega$

$R_L = 10^7\Omega$, $R_i = 600\Omega$; $R_i = 1100 + 2.5\times10^{-4}(-0.2)10^7 = 1100 - 500 = 600\Omega$

b) $R_L = 10\Omega$, $R_i = 21.6\Omega$; $R_i = 21.6 + 2.9\times10^{-4}(0.98)10 \approx 21.6\Omega$

$R_L = 10^7\Omega$, $R_i = 503\Omega$; $R_i = 21.6 + 2.9\times10^{-4}(0.166)10^7 = 21.6 + 482 = 503.6\Omega$

c) $R_L = 10\Omega$, $R_i = 1600\Omega$; $R_i = 1100 + 1\times51\times10 = 1610\Omega$

$R_L = 10^7\Omega$, $R_i = 2.03M$; $R_i = 1100 + 1\times0.203\times10^7 \approx 2.03M$

For Fig. 11-15

a) $R_L = 10\Omega$, $A_V = -0.45$; $A_V = \frac{A_I R_L}{R_i} = \frac{-50\times10}{1100} = -0.455$

$R_L = 10^7\Omega$, $A_V = -3300$; $A_V = -\frac{0.2\times10^7}{600} \simeq -3300$

b) $R_L = 10\Omega$, $A_V = +0.45$; $A_V = \frac{0.98\times10}{21.6} = 0.454$

$R_L = 10^7\Omega$, $A_V = +3300$; $A_V = \frac{0.166\times10^7}{503.6} = \frac{1660\times10^3}{503.6} = 3300$

c) $R_L = 10\Omega$, $A_V = 0.31$; $A_V = \frac{51\times10}{1610} = 0.317$

$R_L = 10^7\Omega$, $A_V = 1$; $A_V = \frac{0.203\times10^7}{2.03\times10^6} = 1$

For Fig. 11-17

a) $R_s = 10\Omega$, $R_o = 75K$; $Y_o = h_o - \frac{h_f h_r}{h_i + R_s} = 25\times10^{-6} - \frac{50\times2.5\times10^{-4}}{1110} = 0.137\times10^{-4}$; $R_o = 72.6K$

$R_s = 10^7\Omega$, $R_o = 40k$; $Y_o = 25\times10^{-6} - \frac{50\times2.5\times10^{-4}}{1100 + 10^7} \approx 0.25\times10^{-4}$; $R_o = 40K$

b) $R_s = 10\Omega$, $R_o = 105k$; $Y_o = 0.49\times10^{-6} + \frac{0.98\times2.9\times10^{-4}}{31.6} = 0.095$; $R_o = 105k$

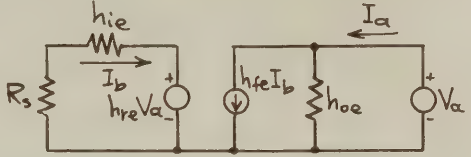
$R_s = 10^7\Omega$, $R_o = 2.04M$; $Y_o = 0.49\times10^{-6} + \frac{0.98\times2.9\times10^{-4}}{10^7} \approx 0.49\times10^{-6}$; $R_o = 2.04\times10^6$

(11-13) (Cont'd) c) CC. $R_s=10\Omega$, $R_o=21.7\Omega$; $Y_o=25\times10^{-6}+\frac{51\times1}{1110}=0.046$; $R_o=21.7\Omega$

$R_s=10^7\Omega$, $R_o=33.2K$; $Y_o=25\times10^{-6}+\frac{51\times1}{10^7}=0.301\times10^{-4}$; $R_o=33.2K$

(11-14) $I_b=-\frac{h_{re}V_a}{h_{ie}+R_s}$; $I_a=h_{oe}V_a+h_{fe}I_b$

$$=\left(h_{oe}-\frac{h_{fe}h_{re}}{h_{ie}+R_s}\right)V_a$$



Hence, $Y_o=\frac{I_a}{V_a}=\frac{1}{Z_o}=\left(h_{oe}-\frac{h_{fe}h_{re}}{h_{ie}+R_s}\right)$. For the example of Sec. 11-6:

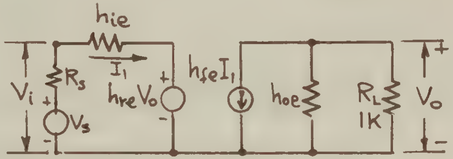
$Y_o=25\times10^{-6}-\frac{50\times2.5\times10^{-4}}{(1.1+1)10^3}=19.1\times10^{-6}$; $\therefore Z_o=\frac{1}{19.1\times10^{-6}}=52.3K$

(11-15) From the circuit we have: $V_o=-h_{fe}I_1R_L'$

where $R_L'=R_L\|\frac{1}{h_{oe}}=1K\|40K=\frac{40}{41}K$

Also, $V_i=I_1h_{ie}+h_{re}V_o=I_1(h_{ie}-h_{re}h_{fe}R_L')$

Thus, $\frac{V_o}{V_i}=\frac{-h_{fe}R_L'}{h_{ie}-h_{re}h_{fe}R_L'}=\frac{(-50\times40/41)}{1.1-2.5\times10^{-4}\times50\times40}=-48.8$ which checks.



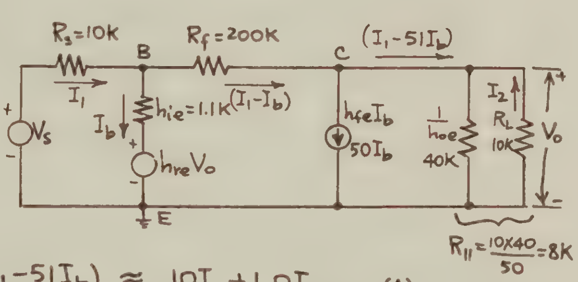
(11-16) From the left-hand mesh:

$V_s=10I_1+1.1I_b+2.5\times10^{-4}V_o$ (1)

Outside mesh:

$V_s=10I_1+200(I_1-I_b)+8(I_1-51I_b)$ (2)

Output: $V_o=8(I_1-51I_b)$ (3)



From (1) & (3) : $V_s=10I_1+1.1I_b+2\times10^{-3}(I_1-51I_b)\approx10I_1+1.0I_b$ (4)

From (2) : $V_s=218I_1-608I_b$ (5)

(5) − (4) gives: $0=208I_1-609I_b$ or $I_1=\frac{609}{208}I_b=2.93I_b$

$\therefore V_s=29.2I_b+I_b=30.2I_b$ or $I_b=\frac{V_s}{30.2}=0.0331V_s$

Then, $I_1=2.93\times0.0331V_s=0.097V_s$; $V_o=8(0.097-51\times0.0331)V_s=-12.7V_s$

Also, $V_{BE}=1.1I_b+2.5\times10^{-4}V_o=(1.1\times0.0331-2.5\times10^{-4}\times12.7)V_s=0.0332V_s$

Hence, $A_{V_s}=\frac{V_o}{V_s}=-12.7$; $R_i=\frac{V_{BE}}{I_1}=\frac{0.0332V_s}{0.097V_s}=0.342K=342\Omega \longleftarrow$

$A_I=-\frac{I_2}{I_1}=\frac{-V_o/10}{I_1}=\frac{1.27V_s}{0.097V_s}=13.1 \longleftarrow$

$A_V=\frac{V_o}{V_{BE}}=\frac{-12.7V_s}{0.0332V_s}=-382 \longleftarrow$

(11-17) a) From Eq. (11-27), $R_i=h_{ie}-\frac{h_{fe}h_{re}}{h_{oe}+1/R_L}$; If $R_L=0$, $R_i=h_{ie}$. The value of R_L for which $R_i=0.9h_{ie}$ is found from $0.9h_{ie}=h_{ie}-\frac{h_{fe}h_{re}}{h_{oe}+1/R_L}$

(11-17) (Cont'd) or $R_L = \dfrac{0.1\,h_{ie}}{h_{fe}h_{re} - 0.1 h_{ie}h_{oe}} = \dfrac{0.1 \times 1100}{50 \times 2.5 \times 10^{-4} - \frac{110}{40 \times 10^3}} = 11.3\,K$ ◁—

b) For a CE configuration, if R_s increases, then R_o decreases slightly. See Fig. 11-17. If R_o decreases by 10%, $Y_o = \frac{1}{R_o}$ increases by 11%. From Eq. (11-34), $Y_o\big|_{R_s=0} = h_{oe} - \dfrac{h_{fe}h_{re}}{h_{ie}} = 25\times10^{-6} - \dfrac{50 \times 2.5 \times 10^{-4}}{1100}$

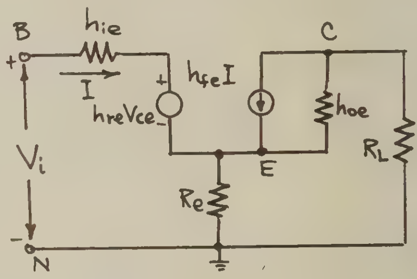
or $Y_o\big|_{R_s=0} = 13.63 \times 10^{-6}$

Thus, $(1.11) \times (13.63 \times 10^{-6}) = 25 \times 10^{-6} - \dfrac{125 \times 10^{-4}}{(1.1 + R_s)10^3} = 15.15 \times 10^{-6}$

or $(25 - 15.15) \times 10^{-6} = \dfrac{12.5 \times 10^{-6}}{1.1 + R_s} = 9.85 \times 10^{-6}$

or $1.1 + R_s = \dfrac{12.5}{9.85} = 1.27 \qquad \therefore\ R_s = 0.17K = 170\Omega$ ◁—

(11-18) a) $R_i = \dfrac{V_i}{I}$ where $V_i = I h_{ie} + h_{re}V_{ce} + V_{en}$



$V_{en} = (I + h_{fe}I + V_{ce}h_{oe})R_e$

$V_{ce} = -V_{en} - R_L(h_{fe}I + V_{ce}h_{oe})$

$\quad = -(I + h_{fe}I + V_{ce}h_{oe})R_e - R_L(h_{fe}I + V_{ce}h_{oe})$

or $V_{ce}(1 + h_{oe}R_e + h_{oe}R_L) = -I[(1+h_{fe})R_e + h_{fe}R_L]$

Thus, $V_i = I[h_{ie} + (1+h_{fe})R_e] + V_{ce}(h_{re} + h_{oe}R_e)$

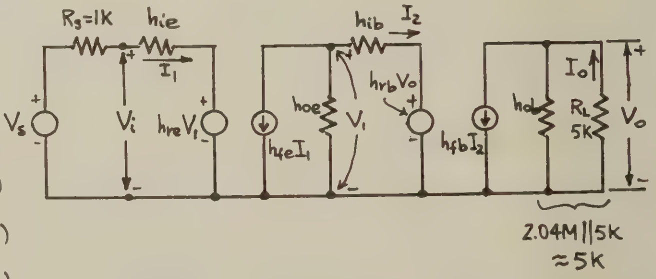
or $V_i = I\left[h_{ie} + (1+h_{fe})R_e - (h_{re} + h_{oe}R_e)\dfrac{(R_e + h_{fe}R_e + h_{fe}R_L)}{1 + h_{oe}R_e + h_{oe}R_L}\right]$

Hence, $R_i = h_{ie} + (1+h_{fe})R_e - (h_{re} + h_{oe}R_e)\dfrac{[(1+h_{fe})R_e + h_{fe}R_L]}{1 + h_{oe}(R_e + R_L)}$ ◁—

b) If $R_L = R_e = 1K$ and using the h-parameters in Table 11-2, we have:

$$R_i = 1.1K + 51K - \frac{(2.5\times10^{-4} + 0.025)(51+50)K}{1 + 25\times10^{-6}(2\times10^3)} = \left(52.1 - \frac{0.02525 \times 101}{1.05}\right)K$$
$$= (52.1 - 2.4)K = 49.7K$$

(11-19) a) From the equivalent circuit and the values of h-parameters in Table 11-2, we have:



$V_i = h_{ie}I_1 + h_{re}V_1 = 1.1I_1 + 2.5\times10^{-4}V_1$ (1)

$V_1 = h_{ib}I_2 + h_{rb}V_o = 2.16\times10^{-2}I_2 + 2.9\times10^{-4}V_o$ (2)

$V_o = -I_oR_L \approx -h_{fb}I_2R_L = 0.98 \times 5\,I_2$ (3)

$I_2 = -h_{fe}I_1 - V_1h_{oe} = -50I_1 - 25\times10^{-6}V_1$ (4) Substituting (2) & (3) into (4) we have:

$I_2 = -50I_1 - 25\times10^{-6}(2.16\times10^{-2} + 2.9\times10^{-4}\times0.98\times5)I_2 = -50I_1 - (5.4\times10^{-7} + 0.355\times10^{-7})I_2 \approx -50I_1$

Thus, $I_o \approx -h_{fb}I_2 = -0.98(-50I_1) = 49\,I_1 \qquad \therefore\ A_I = -\dfrac{I_o}{I_1} \doteq -49$ ◁—

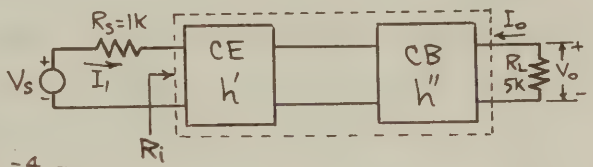
(11-19) (Cont'd) Substituting (2) and (3) into (1) we obtain:

$$V_i = 1.1\,I_1 + 2.5\times10^{-4}(2.16\times10^{-2}\times 49 I_1 + 2.9\times10^{-4}\times 4.9\times 49\, I_1) \approx 1.1\,I_1$$

Hence, $R_i = \frac{V_i}{I_1} = 1.1K$; Then $A_V = \frac{A_I R_L}{R_i} = -\frac{49\times5}{1.1} = -223$ $\longleftarrow$

and $A_{Vs} = A_V \frac{R_i}{R_i + R_s} = (-223)\frac{1.1}{1.1+1} = -117$ $\longleftarrow$

b) The network in dotted lines may be replaced by a single two-port with h-parameters given in Prob. 11-6.



Thus, $h_{11} = h'_{11} - \frac{h'_{12}h'_{21}h''_{11}}{1 + h''_{11}h'_{22}} = 1.1K - \frac{2.5\times10^{-4}\times50\times21.6\Omega}{1 + 21.6\times25\times10^{-6}} \approx 1.1K$

$h_{12} = \frac{h'_{12}h''_{12}}{1 + h''_{11}h'_{22}} = \frac{2.5\times10^{-4}\times2.9\times10^{-4}}{1 + 21.6\times25\times10^{-6}} \approx 7.25\times10^{-8}$; $h_{21} = -\frac{h'_{21}h''_{21}}{1+h''_{11}h'_{22}} \approx \frac{-50\times(-0.98)}{1+\dots}$

or $h_{21} \approx 49$ and $h_{22} = h''_{22} - \frac{h''_{12}h''_{21}h'_{22}}{1+h''_{11}h'_{22}} = 0.49\times10^{-6} + \frac{2.9\times10^{-4}\times0.98\times25\times10^{-6}}{1+\dots} \approx 0.49\times10^{-6}$

From Table 11-4 we have: $A_I = -\frac{h_{21}}{1+h_{22}R_L} = -\frac{49}{1+0.49\times10^{-6}\times5\times10^{3}} \approx -49$ $\longleftarrow$

$R_i = h_{11} + h_{12}A_I R_L = 1.1K + 7.25\times10^{-8}(-49)(5\times10^3) \approx 1.1\,K$ $\longleftarrow$

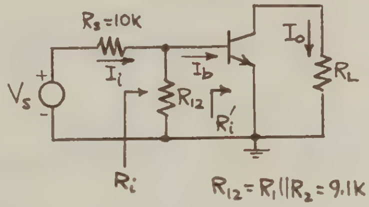
$A_V = \frac{A_I R_L}{R_i} = \frac{-49\times5}{1.1} = -223$ and $A_{Vs} = \frac{A_I R_L}{R_i + R_s} = -\frac{49\times5}{1.1+1} = -117$ $\longleftarrow$

(11-20) Let $A'_I \equiv \frac{I_o}{I_b} = \frac{-h_{fe}}{1+h_{oe}R_L} = \frac{-50}{1+5/40} = -44.5$

$R'_i = h_{ie} + h_{re}(A'_I)R_L = 1.1K + 2.5\times10^{-4}(-44.5)(5K)$
$\cong 1.05K$

Then, $A_I = \frac{I_o}{I_i} = \frac{I_o}{I_b}\cdot\frac{I_b}{I_i} = A'_I\frac{R_{12}}{R_{12}+R'_i} = -44.5\frac{9.1}{9.1+1.05}$

or $A_I = -39.8$ $\longleftarrow$



$R_{12} = R_1 \| R_2 = 9.1K$

$R_i = R_{12}\|R'_i = 9.1K\|1.05K = 943\ \Omega$ $\longleftarrow$

$A_V = \frac{A_I R_L}{R_i} = -\frac{39.8\times5K}{0.943} = -211$; $A_{Vs} = \frac{A_V R_i}{R_i+R_s} = \frac{-211\times0.943}{0.943+10} = -18.2$ $\longleftarrow$

$Y_o = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie}+R'_s} = 25\times10^{-6} - \frac{50\times2.5\times10^{-4}}{1.1K+(10\|9.1)K} = (25-2.12)10^{-6} = 22.9\times10^{-6}\ \mho$

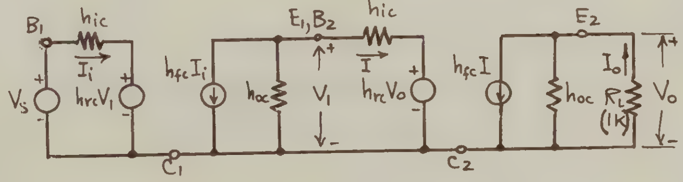
or $R_o = \frac{1}{Y_o} = \frac{10^6}{22.9} = 43.7\,K$ $\longleftarrow$

(11-21) a) From the equivalent circuit,

$I_o = h_{fc}I + h_{oc}V_o = -51I + 25\times10^{-6}V_o$ (1)

$V_o = -I_oR_L = -10^3 I_o$ (2)

$I = -h_{fc}I_i - h_{oc}V_1 = +51I_i - 25\times10^{-6}V_1$ (3)



(11-21) (Cont'd) Substituting (2) & (3) into (1) we obtain:

$I_o = -51I - 25\times10^{-6}\times10^3 I_o = -51I - 0.025 I_o$ or $I_o = -49.8\,I$ (4)

$I = 51I_i - 25\times10^{-6}(1.1\times10^3 I + V_o) = 51I_i - 0.0275\,I - 25\times10^{-6}\times49.8\times10^3\,I$

or $I = \frac{51}{2.27} I_i = 22.5\,I_i$. Thus, $I_o = -49.8\,(22.5)I_i = -1120\,I_i$ (5)

Hence, $A_I = -\frac{I_o}{I_i} = +1120 \quad \longleftarrow$

From the circuit we also have that: $V_s = h_{ic}I_i + h_{rc}V_1 = 1.1\times10^3 I_i + V_1$

and $V_1 = h_{ic}I + h_{rc}V_o = 1.1\times10^3 I + V_o = 1.1\times10^3 I + 10^3(49.8)I = 50.9\times10^3 I$

or $V_1 = 50.9\times10^3(22.5\,I_i)$. Thus, $V_s = (1.1 + 50.9\times22.5)10^3\,I_i = 1.146\times10^6\,I_i$

Hence, $R_i = \frac{V_s}{I_i} = 1.146\,M$ and $A_V = \frac{A_I R_L}{R_i} = \frac{1120\times10^3}{1.146\times10^6} \doteq 0.98 \quad \longleftarrow$

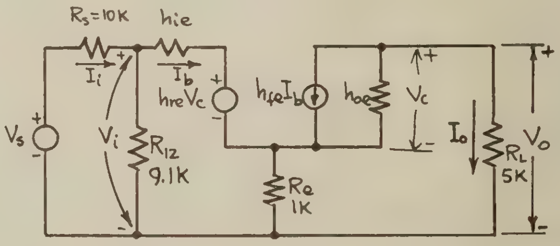
b) Replacing the compound connection by a single transistor whose h-parameters are given in Prob. 11-7e, we have:

$$A_I = \frac{-h_{fc}}{1 + h_{oc}R_L} = \frac{+2531}{1 + 1.265\times10^{-3}\times10^3} = +1120 \quad \longleftarrow$$

$$R_i = h_{ic} + h_{rc}A_I R_L = 55.7K + 0.973\times1120\times1K = (55.7 + 1090)K = 1.146M \longleftarrow$$

$$A_V = \frac{A_I R_L}{R_i} = \frac{1120\times10^3}{1.146\times10^6} \doteq 0.98 \quad \longleftarrow$$

(11-22) From the circuit we have:



$-I_o = h_{fe}I_b + h_{oe}V_c = 50I_b + 25\times10^{-6}V_c$ (1)

$V_c = V_o - (I_b - I_o)R_e = V_o - 10^3(I_b - I_o)$ (2)

$V_o = +I_oR_L = +5\times10^3 I_o$ (3)

Substituting (2) & (3) into (1) we have:

$-I_o = 50I_b + 25\times10^{-6}(+5\times10^3 I_o - 10^3 I_b + 10^3 I_o) = 49.75\,I_b + 0.15\,I_o$ or $I_o = -43.3\,I_b$

Also, $V_i = h_{ie}I_b + h_{re}V_c + (I_b - I_o)R_e$; $V_c = -5\times10^3(43.3I_b) - 10^3(44.3I_b) = -26.1\times10^4 I_b$

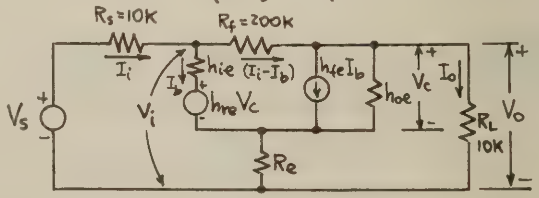
so $V_i = 1.1\times10^3 I_b + 2.5\times10^{-4}(-26.1\times10^4 I_b) + 44.3\times10^3 I_b \doteq 45.3\times10^3\,I_b$

Thus, $A_I = \frac{I_o}{I_i} = \frac{I_o}{I_b}\cdot\frac{I_b}{I_i} = (-43.3)\frac{9.1}{9.1 + 45.3} = -7.25 \quad \longleftarrow$

$R_i = R_{12}\|(\frac{V_i}{I_b}) = 9.1K\|45.3 = \frac{9.1\times45.3}{9.1 + 45.3} = 7.6K \quad \longleftarrow$

$A_V = \frac{A_I R_L}{R_i} = -\frac{7.25\times5}{7.6} = -4.77$; $A_{Vs} = A_V\frac{R_i}{R_i + R_s} = \frac{-4.77\times7.6}{7.6 + 10} = -2.05$

(11-23) From the circuit we have:



$I_o = (I_i - I_b) - h_{fe}I_b - h_{oe}V_c = -51I_b + I_i - 25\times10^{-6}V_c$ (1)

$V_c = V_o + (I_o - I_i)R_e = V_o + 100I_o - 100I_i$ (2)

$V_o = I_oR_L = 10^4 I_o$ (3)

(11-23) (Cont'd) Substituting (2) & (3) into (1) we have:

$$I_o = -51 I_b + I_i - 25\times10^{-6}(10^4 I_o + 100 I_o - 100 I_i) = -51 I_b + 0.9975 I_i - 0.2525 I_o$$

Also from the circuit: $V_i = h_{ie} I_b + h_{re} V_c + (I_i - I_o) R_e$ (4)

from the outside loop: $V_i = R_f (I_i - I_b) + V_o$ (5)

(4) − (5) : $0 = h_{ie} I_b + h_{re} V_c + (I_i - I_o) R_e - R_f (I_i - I_b) - V_o$

or $1.1\times10^3 I_b + 2.5\times10^{-4}(10.1\times10^3 I_o - 100 I_i) + 100 I_i - 100 I_o - 200\times10^3 (I_i - I_b) - 10^4 I_o = 0$

or $I_b = 0.0503 I_o + 0.994 I_i$; Then, $I_o = -51(0.0503 I_o + 0.994 I_i) + 0.9975 I_i - 0.2525 I_o$

or $I_o \doteq -13 I_i$ and $I_b = 0.339 I_i$ (6)

From (6), $A_I = \dfrac{I_o}{I_i} = -13$ ⟵

Substituting (6) into (5): $V_i = 200\times10^3 (1 - 0.339) I_i + 10^4(-13 I_i) = 2.2\times10^3 I_i$

Hence, $R_i = \dfrac{V_i}{I_i} = 2.2\,K$ and $A_V = \dfrac{A_I R_L}{R_i} = \dfrac{-13\times10}{2.2} = -59$; $A_{Vs} = -\dfrac{59\times2.2}{2.2+10} = -10.65$

⟵

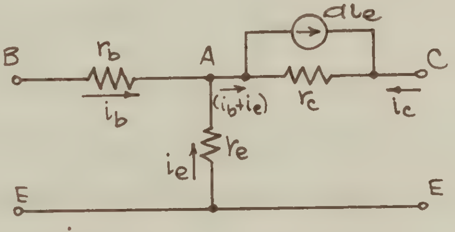
(11-24) From Table 11-4, $R_i = h_{ib} - \dfrac{h_{fb} h_{rb}}{h_{ob} + \frac{1}{R_L}} = 50\,\Omega$. Using the h-parameters in Table 11-2, we have:

$$50 - 21.6 = \frac{0.98\times2.9\times10^{-4}}{0.49\times10^{-6} + \frac{1}{R_L}} = 28.4$$

or $\dfrac{1}{R_L} = 0.994\times10^{-5} - 0.049\times10^{-5} = 0.945\times10^{-5}$ ∴ $R_L = 105.8\,K$ ⟵

(11-25) Fig. 11-20 redrawn as CE is shown in (a). Conversion of current sources of circuit (a) and of the given circuit is shown in (b) and (c), respectively.

(a)

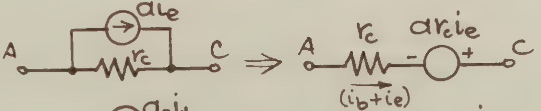


For equivalence, $V_{CA} \equiv V_{C'A'}$

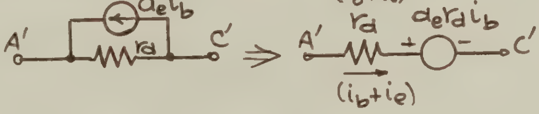
or $\alpha i_e r_c - r_c(i_b+i_e) \equiv -\alpha_e r_d i_b - r_d(i_b+i_e)$

or $i_b r_c + i_e(1-\alpha) r_c \equiv i_b(1+\alpha_e) r_d + i_e r_d$

(b)



(c)

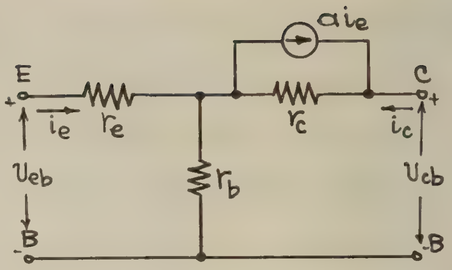


Hence, $r_d = (1-\alpha) r_c$

and $r_c = (1+\alpha_e) r_d = (1+\alpha_e)(1-\alpha) r_c = r_c$ or $(1+\alpha_e)(1-\alpha) = 1$

or $1 + \alpha_e = \dfrac{1}{1-\alpha}$ and $\alpha_e = \dfrac{1}{1-\alpha} - 1 = \dfrac{\alpha}{1-\alpha}$ QED

(11-26) $h_{ib} = \left.\frac{v_{eb}}{i_e}\right|_{v_{cb}=0}$ where $v_{eb} = i_e r_e + i_e(1-a)(r_b \| r_c)$



From Table 11-3 we see that typically $r_b = 590\,\Omega \ll r_c = 2.04\,M$. Thus, $r_b \| r_c \approx r_b$

Then, $h_{ib} = \frac{[r_e + (1-a) r_b] i_e}{i_e} = r_e + (1-a) r_b$

$h_{rb} = \left.\frac{v_{eb}}{v_{cb}}\right|_{i_e=0}$ where $v_{eb} = \frac{r_b}{r_b + r_c} v_{cb} \approx \frac{r_b}{r_c}$ ∴ $h_{rb} = \frac{r_b}{r_c}$

$h_{fb} = \left.\frac{i_c}{i_e}\right|_{v_{cb}=0}$ where $i_c = -a i_e$, since $r_b \ll r_c$ ∴ $h_{fb} = -a$

$h_{ob} = \left.\frac{i_c}{v_{cb}}\right|_{i_e=0}$ where $i_c = \frac{v_{cb}}{r_b + r_c} \approx \frac{v_{cb}}{r_c}$ ∴ $h_{ob} = \frac{1}{r_c}$

(11-27) From Fig. 11-19 we have: $v_{eb'} = i_e r_e + \mu v_{cb'}$ (1)

and $i_c = -\alpha i_e + \frac{v_{cb'}}{r_c'}$ (2)

From (2) $v_{cb'} = \alpha i_e r_c' + i_c r_c'$. Substituting in (1), we obtain:

$v_{eb'} = i_e(r_e' + \mu\alpha r_c') + i_c r_c' \mu$ (3)

From Fig. 11-20, we have: $v_{eb'} = i_e(r_e + r_b') + i_c r_b'$ (4)

Equating (3) & (4), we get: $r_b' = \mu r_c'$

and $r_e + r_b' = r_e' + \mu\alpha r_c' = r_e' + \alpha r_b'$ ∴ $r_e = r_e' - (1-\alpha) r_b' = r_e' - (1-\alpha)\mu r_c'$

From (2) above we found that: $v_{cb'} = \alpha i_e r_c' + i_c r_c'$ (5)

From Fig. 11-20 we have: $v_{cb'} = (i_c + a i_e) r_c + (i_c + i_e) r_b'$

or $v_{cb'} = i_e(a r_c + r_b') + i_c(r_c + r_b')$ (6)

Equating (5) & (6), we get:

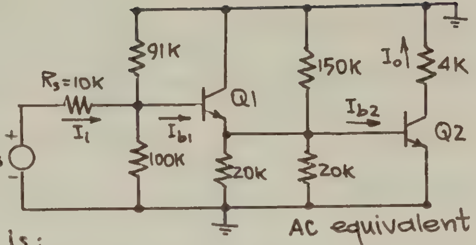
$r_c' = r_c + r_b' = r_c + \mu r_c'$ ∴ $r_c = r_c'(1-\mu)$

and $\alpha r_c' = a r_c + r_b'$ or $a r_c = \alpha r_c' - r_b'$

$$\text{or} \quad \alpha = \frac{\alpha r_c' - r_b'}{r_c} = \frac{1}{r_c}\left(\frac{\alpha r_c}{1-\mu} - \frac{\mu r_c}{1-\mu}\right)$$

$$\therefore \alpha = \frac{\alpha - \mu}{1 - \mu} \qquad \text{QED}$$

CHAPTER 12



(12-1) a) Starting with the second stage, we have: $A_{I_2} = -\frac{I_o}{I_{b2}} = \frac{-h_{fe}}{1+h_{oe}R_{L2}}$

or $A_{I_2} = \frac{-50}{1+4K/40K} = -45.5$

The input impedance of the transistor itself is:

$R_{i2} = h_{ie} + h_{re}A_{I_2}R_{L_2} = 1.1K + 2.5\times10^{-4}(-45.5)(4K) = 1100 - 45 = 1055\,\Omega$

Then, $A_{V_2} = \frac{A_{I_2}R_{L2}}{R_{i2}} = -\frac{45.5\times4}{1.055} = -172$

Q1 is in CC configuration. The effective load is $R_{L_1} = 20K\|20K\|150K\|R_{i2}$

or $R_{L_1} = 0.960K$. Thus, $A_{I_1} = \frac{-h_{fc}}{1+h_{oc}R_{L1}} = \frac{I_{e1}}{I_{b1}} = \frac{51}{1+0.96/40} = 49.8$

$R_{i_1} = h_{ic} + h_{rc}A_{I_1}R_{L_1} = 1.1K + 1\times49.8\times0.96K = 1.1K + 48K = 49.1K$

$A_{V_1} = \frac{A_{I_1}R_{L1}}{R_{i1}} = \frac{49.8\times0.96}{49.1} = 0.977$

$Y_{o1} = h_{oc} - \frac{h_{fc}h_{rc}}{h_{ic}+R_{s1}}$ where $R_{s_1} = 10K\|91K\|100K = 8.26K$; $Y_{o1} = 2.5\times10^{-5} - \frac{(-51)\times1}{1100+8260}$

or $Y_{o1} = (2.5 + 544)\times10^{-5} = 5.46\times10^{-3}$ $\therefore R_{o1} = 183\,\Omega$

$Y_{o2} = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie}+R_{s2}}$ where $R_{s2} = 183\Omega\|20K\|20K\|150K = 180\Omega$

or $Y_{o2} = 2.5\times10^{-5} - \frac{50\times2.5\times10^{-4}}{1100+180} = (2.5-0.98)\times10^{-5} = 1.52\times10^{-5}$ $\therefore R_{o2} = 65.7K$

Now we can calculate the parameters of the entire amplifier.

Thus, $R_i = R_{i_1}\|91K\|100K = 24K$; $R_o' = R_{o2}\|4K = 65.7K\|4K = 3.77K$ ◁—

$A_V = A_{V_1}A_{V_2} = (0.977)\times(-172) = -168$; $A_{Vs} = A_V\frac{R_i}{R_i+R_s} = -168\times\frac{24}{34} = -119$ ◁—

$A_I = -\frac{I_o}{I_{b1}} = \frac{A_V R_{i1}}{R_{L2}} = \frac{-168\times49.1}{4} = -2070$; $A_{Is} = \frac{A_{Vs}(R_s+R_i)}{R_{L2}} = -1010$ ◁—

b) Using Table 12-2 we have: Stage II. ($h_{oe}R_{L_2} = 0.1$). $A_{I_2} = -h_{fe} = -50$;

$R_{i2} = h_{ie} = 1.1K$; $A_{V2} = -\frac{h_{fe}R_L}{h_{ie}} = \frac{-50\times4}{1.1} = -182$; $R_{L_1} = 20K\|20K\|150K\|1.1K = 985\Omega$

Stage I. ($h_{oc}R_{L_1} = 2.5\times10^{-5}\times985 = 0.0246$) . $A_{I_1} = 1+h_{fe} = 51$; $R_{i_1} = h_{ie} + (1+h_{fe})R_{L_1}$

or $R_{i_1} = 1.1K + 51\,(0.985K) = 51.4K$; $A_{V_1} = 1 - \frac{h_{ie}}{R_{i_1}} = 1 - \frac{1.1}{51.4} = 0.978$

$R_{o2} = \infty$, $R_{o1} = \frac{R_{s1}+h_{ie}}{1+h_{fe}} = \frac{(8.26+1.1)}{51}K = 184\,\Omega$

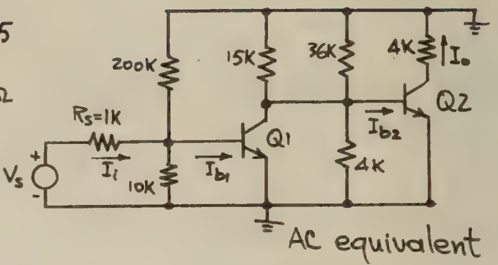
For the entire amplifier:

$R_i = 51.4K\|91K\|100K = 24.7K$; $R_o' = 4K$ ◁—

$A_V = A_{V_1}A_{V_2} = 0.978(-182) = -178$; $A_{Vs} = \frac{-178\times24.7}{34.7} = -127.5$ ◁—

$A_I = \frac{-178\times51.4}{4} = -2280$; $A_{I_s} = \frac{(-127.5\times34.7)}{4} = -1100$ ◁—

(12-2) a) Stage II : $A_{I_2} = -\frac{I_o}{I_{b2}} = \frac{-h_{fe}}{1+h_{oe}R_{L2}} = \frac{-50}{1+4/40} = -45.5$



AC equivalent

$R_{i2} = h_{ie} + h_{re}A_{I_2}R_{L2} = 1.1K + (2.5\times10^{-4})(-45.5)(4K) = 1055\,\Omega$

$A_{V2} = A_{I_2}\frac{R_{L2}}{R_{i2}} = (-45.5)\frac{4K}{1.055K} = -172$

Stage I : $R_{L1} = 15K \| 36K \| 4K \| 1.055K = 0.776K$

$A_{I_1} = -\frac{I_{c1}}{I_{b1}} = \frac{-50}{1+\frac{0.776}{40}} = -49.1$

$R_{i1} = 1100 + (2.5\times10^{-4})(-49.1)(0.776K) = 1100 - 9.5 = 1090.5\,\Omega$

$A_{V1} = -49.1\,\frac{776}{1090.5} = -34.9$; $Y_{o1} = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie}+R_{s1}}$ where $R_{s1} = (1K \| 10K \| 200K) = 905\,\Omega$

$Y_{o1} = 2.5\times10^{-5} - \frac{50\times2.5\times10^{-4}}{1100+905} = (2.5-0.625)\times10^{-5} = 1.875\times10^{-5}$ $\therefore R_{o1} = 53.4K$

For the second stage, $R_{s2} = (53.4K \| 15K \| 36K \| 4K) = 2.75K$

$Y_{o2} = 2.5\times10^{-5} - \frac{125\times10^{-4}}{1100+2750} = (2.5-0.32)\times10^{-5} = 2.18\times10^{-5}$ $\therefore R_{o2} = 46K$

For the entire amplifier : $A_V = A_{V1}A_{V2} = (-34.9)(-172) = 6000$; $R_o' = 46K \| 4K = 3.68K$

$R_i = (R_{i1} \| 10K \| 200K) = 980\,\Omega$; $A_{Vs} = A_V\frac{R_i}{R_i+R_s} = 6000\,\frac{0.98K}{1.98K} = 2970$ $\longleftarrow$

$A_I = -\frac{I_o}{I_{b1}} = \frac{A_V R_{i1}}{R_{L2}} = \frac{6000\times1.09K}{4K} = 1635$; $A_{Is} = \frac{A_{Vs}(R_s+R_i)}{R_{L2}} = \frac{2970\times1.98K}{4K} = 1470$ $\longleftarrow$

b) Stage II : $A_{I_2} = -h_{fe} = -50$; $R_{i2} = h_{ie} = 1.1K$; $A_{V2} = -\frac{h_{fe}R_{L2}}{h_{ie}} = \frac{-50\times4}{1.1} = -182$

$R_{o2} = \infty$; $R_{L1} = (15K \| 36K \| 4K \| 1.1K) = 0.8K$

Stage I : $A_{I_1} = -50$; $R_{i1} = 1.1K$; $A_{V1} = -\frac{50\times0.8K}{1.1K} = -36.3$; $R_{o1} = \infty$

Overall : $A_V = A_{V1}A_{V2} = (-36.3)(-182) = 6600$; $R_i = (R_{i1} \| 10K \| 200K) = 985\,\Omega$; $R_o' = 4K$

$A_{Vs} = A_V\frac{R_i}{R_i+R_s} = 6600\times\frac{985}{1985} = 3280$; $A_I = -\frac{I_o}{I_{b1}} = \frac{6600\times1.1}{4} = 1815$ $\longleftarrow$

$A_{Is} = -\frac{I_o}{I_i} = A_{Vs}\frac{(R_s+R_i)}{R_{L2}} = \frac{3280\,(1985)}{4000} = 1625$ $\longleftarrow$

(12-3) a) 0 dBm → 10^{-3}W in a 600 Ω resistor. $P = \frac{V^2}{R}$ or $V = (10^{-3}\times600)^{1/2}$

or $V = (0.6)^{1/2} = 0.775\,V$ $\longleftarrow$

b) 20 dBm corresponds to a power ratio of $100 = \frac{P_2}{P_1}$ where $P_1 = 10^{-3}W$

Thus, $P_2 = 100\times10^{-3}W = 0.1W = \frac{V^2}{60}$ or $V = (6)^{1/2} = 2.45V$ $\longleftarrow$

If R = 600K, $V^2 = 0.1\times600\times10^3 = 6\times10^4$ $\therefore V = (6\times10^4)^{1/2} = 245V$ $\longleftarrow$

(12-4) Stage II : $(R_{c_2}+R_{e_2})h_{oe} = 3.1\times10^3\times\frac{1}{40\times10^3} = 0.0775 < 0.1$. Thus, we can use the approximate analysis, i.e. Table 12-2.

$A_{I_2} = -h_{fe} = -50$; $R_{i2} = h_{ie} + (1+h_{fe})R_e = 1.1K + 51\times0.1K = 6.2K$;

$A_{V_2} = -\frac{h_{fe}R_{L2}}{R_{i2}} = \frac{-50\times3}{6.2} = -24.2$; $R_{o2} = \infty$

Stage I : $R_{L1} = 8K \| 6.2K = 3.49K$; $(R_{L1}+R_{e1})h_{oe} = 3.99\times10^3\times\frac{1}{40\times10^3} = 0.0999 < 0.1$

$\therefore$ Use approximate formulas: $A_{I_1} = -50$; $R_{i1} = 1.1K + 51\times0.5K = 26.6K$

(12-4) (Cont'd) $A_{V_1} = -\frac{50\times 3.49K}{26.6K} = -6.56$; $R_{o1} = \infty$

Entire amplifier: $A_V = A_{V_1}A_{V_2} = (-6.56)(-24.2) = 159$; $R_i = R_{i_1} = 26.6K$; $R_o' = 3K$ ⟵

$A_I = \frac{A_V R_i}{R_{L2}} = \frac{159\times 26.6K}{3K} = 1410$; $A_{VS} = A_V\frac{R_i}{R_i+R_s} = \frac{159\times 26.6}{36.6} = 115$ ⟵

(12-5) a) Exact analysis: $A_I = -\frac{h_{fb}}{1+h_{ob}R_L} = -\frac{-0.98}{1+0.49\times10^{-6}\times4\times10^{3}} = \frac{0.98}{1.002} \doteq 0.98$ ⟵

$R_i = h_{ib}+h_{rb}A_IR_L = 21.6 + 2.9\times10^{-4}\times0.98\times4\times10^{3} = 21.6+1.1 = 22.7\,\Omega$ ⟵

$A_V = A_I\frac{R_L}{R_i} = 0.98\frac{4000}{22.7} = 173$; $Y_o = h_{ob} - \frac{h_{fb}h_{re}}{h_{ib}+R_s} = 0.49\times10^{-6} - \frac{(-0.98)(2.9\times10^{-4})}{21.6+2000} = 0.627\times10^{-6}$

$R_o = \frac{1}{Y_o} = 1.59M$; $R_o' = 1.59M\|4K \approx 4K$; $A_{VS} = A_V\frac{R_i}{R_i+R_s} = 173\times\frac{22.7}{2022.7} = 1.94$ ⟵

Approximate analysis: $A_I = -h_{fb} = 0.98$; $R_i = h_{ib} = 21.6\,\Omega$ ⟵

$A_V = -h_{fb}\frac{R_L}{h_{ib}} = 0.98\times\frac{4000}{21.6} = 182$; $R_o = \infty$; $R_o' = 4K$; $A_{VS} = \frac{182\times21.6}{2021.6} = 1.94$

b) CE : Exact : $A_I = -\frac{h_{fe}}{1+h_{oe}R_L} = \frac{-50}{1+4/40} = -45.5$; $R_i = 1.1K + 2.5\times10^{-4}(-45.5)(4K) = 1.055K$

$A_V = -45.5\frac{4K}{1.055K} = -172$; $Y_o = 2.5\times10^{-4} - \frac{50\times2.5\times10^{-4}}{1100+2000} = (2.5-0.4)\times10^{-4} = 2.1\times10^{-4}$

$R_o = \frac{1}{2.1\times10^{-4}} = 47.6K$; $R_o' = 47.6K\|4K = 3.7K$; $A_{VS} = \frac{-172\times1055}{3055} = -59.5$ ⟵

Approximate: $A_I = -50$; $R_i = 1.1K$; $A_V = -50\times\frac{4K}{1.1K} = -182$; ⟵

$A_{VS} = -182\times\frac{1.1K}{3.1K} = -64.5$; $R_o = \infty$; $R_o' = 4K$ ⟵

c) CC : Exact : $A_I = -\frac{h_{fc}}{1+h_{oc}R_L} = \frac{51}{1+4/40} = 46.3$; $R_i = 1.1K + 1\times46.3\times4K = 186.3K$

$A_V = 1-\frac{h_{ie}}{R_i} = 1-\frac{1.1}{186.3} \doteq 1-0.006 = 0.994$; $Y_o = 2.5\times10^{-5} + \frac{51\times1}{3100} = 1.67\times10^{-2}$

$R_o = 1/1.67\times10^{-2} = 60\,\Omega$; $R_o' = 60\,\Omega\|4000\,\Omega \approx 60\,\Omega$; $A_{VS} = 0.994\frac{186.3}{188.3} = 0.984$ ⟵

Approximate: $A_I = (1+h_{fe}) = 51$; $R_i = 1.1K + 51\times4K = 205K$ ⟵

$A_V = 1-\frac{1.1}{205} \doteq 0.995$; $R_o = \frac{R_s+h_{ie}}{1+h_{fe}} = \frac{2+1.1}{51}K = 60.8\,\Omega$; ⟵

$A_{VS} = 0.995\frac{205}{207} = 0.985$; $R_o' = 60.8\,\Omega\|4000\,\Omega \approx 60\,\Omega$ ⟵

(12-6) Stage II : Since $h_{oe}R_{L2} < 0.1$, we use the approximate analysis.

$A_{I2} = -50$; $R_{i2} = 1.1K$; $A_{V2} = -h_{fe}\frac{R_{L2}}{R_{i2}} = -50\times\frac{0.7K}{1.1K} = -31.9$; $R_{o2} = \infty$

Stage I : $R_{L1} = (6K\|16K\|6.2K\|1.1K) = 0.773K$. Use approximate analysis.

$A_{I1} = -50$; $R_{i1} = 1.1K$; $A_{V1} = -50\frac{0.773K}{1.1K} = -35$; $R_{o1} = \infty$

Overall : $A_V = A_{V1}A_{V2} = (-35)\times(-31.9) = 1115$; $R_i = (1.1K\|3.3K\|22K) = 0.795K$ ⟵

$A_{VS} = A_V\frac{R_i}{R_i+R_s} = 1115\frac{795}{795+1100} = 468$; $R_o' = 0.7K$ ⟵

(12-7) Stage III : $h_{oc}R_{L3} = 2.5\times10^{-5}\times10^{5} = 2.5 > 0.1$ ∴ Use exact analysis.

$A_{I3} = \frac{-(-51)}{1+100/40} = \frac{51}{3.5} = 14.6$; $R_{i3} = (1.1+1\times14.6\times100) = 1461K = 1.46M$ ⟵

$A_{V3} = 1 - 1.1/1461 \doteq 0.999$

(12-7) (Cont'd) Stage II : $R_{L2} = 27K \| 1.46M \approx 27K$. Again use exact analysis.

$$A_{I2} = \frac{-h_{fe}}{1+h_{oe}R_{L2}} = \frac{-50}{1+27/40} = \frac{-50}{1.675} = -29.8 \; ; \; R_{i2} = 1100+(2.5\times10^{-4})(-29.8)(27{,}000)$$
$$= 1100 - 200 = 900\Omega$$

$$A_{V2} = A_{I2}\frac{R_{L2}}{R_{i2}} = -29.8\times\frac{27K}{0.9K} = -895 \quad \longleftarrow$$

Stage I : $R_{L1} = 10K\|0.9K = 0.825K$. Here $h_{oe}R_{L1} < 0.1$ $\therefore$ Use approximate analysis. Thus, $A_{I1} = (1+h_{fe}) = 51$; $R_{i1} = 1.1K + 51\times0.825K = 43.1K$

$$A_{V1} = 1 - \frac{1.1}{43.1} = 1 - 0.025 = 0.975 \quad \longleftarrow$$

Overall : $A_V = A_{V1}A_{V2}A_{V3} = (0.975)\times(-895)\times(0.999) = -870$

$R_i = 43.1K\|50K = 23.2K$ $\quad \therefore A_{Vs} = \frac{V_o}{V_s} = A_V\frac{R_i}{R_i+R_s} = -870\frac{23.2}{23.2+5} = -717 \quad \longleftarrow$

(12-8) Stage II : $h_{oe}R_{L2} = 10^{-8}\times5\times10^{3} = 5\times10^{-5} < 0.1$ $\therefore$ Use approximate analysis.

$R_{i2} = h_{ie} + (1+h_{fe})R_{L2} = 1K + 51\times5K = 256K$; $A_{V2} = 1 - \frac{1K}{256K} = 1 - 0.0039 \doteq 0.996$

Stage I : $R_{L1} = 10K\|256K = 9.63K$. Again $h_{oe}R_{L1} < 0.1$.

Hence, $A_{I1} = -50$, $R_{i1} = h_{ie} = 1K$, $A_{V1} = -h_{fe}\frac{R_{L1}}{h_{ie}} = -50\times\frac{9.63K}{1K} = -482$

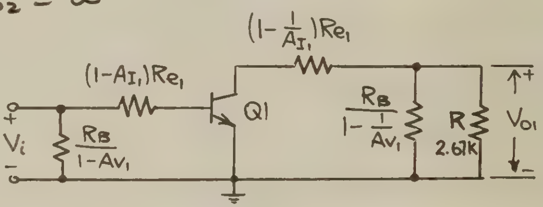
Overall : $A_V = A_{V1}A_{V2} = -482\times0.996 = -480$; $A_{Vs} = -480\frac{1K}{2K} = -240 \quad \longleftarrow$

(12-9) Stage II : $h_{oe}(R_{L2}+R_{e2}) = 2.5\times10^{-5}(2050) = 0.051 < 0.1$ $\therefore$ Use approximate analysis.

$A_{I2} = -h_{fe} = -50$; $R_{i2} = h_{ie} + (1+h_{fe})R_{e2} = 1.1K + 51(0.05K) = 1.1 + 2.55 = 3.65K$

$A_{V2} = A_{I2}\frac{R_{L2}}{R_{i2}} = -50\frac{2K}{3.65K} = -27.4$; $R_{o2} = \infty$

Stage I : Using Fig. 12-12 b and Miller's theorem, we can redraw the first stage as shown: where $R = 10K\|R_{i2} = 10K\|3.65K = 2.67K$



Since the effective load is less than 2.67K, we can use the approximate analysis. Thus, $A_{I1} = -h_{fe} = -50$, $(1-\frac{1}{A_{I1}})R_{e1} = (1+\frac{1}{50})(500) = 510\Omega$ and $(1-A_{I1})R_{e1} = (1+50)(0.5K) = 25.5K$; $R_{i1} = h_{ie} + (1-A_{I1})R_{e1} = 26.6K$

Then, $A_{V1} = A_{I1}\frac{R_{L1}}{R_{i1}} = \frac{-50}{26.6K}\frac{267/(1-\frac{1}{A_{V1}})}{(2.67+\frac{100}{1-\frac{1}{A_{V1}}})}$ or $26.6A_{V1}(2.67+\frac{100}{1-\frac{1}{A_{V1}}}) = \frac{-50\times267}{1-\frac{1}{A_{V1}}}$

or $71A_{V1} + 2660A_{V1} - 71 = -13{,}350$ $\quad \therefore A_{V1} = -\frac{13{,}279}{2731} = -4.86$

Thus, $\frac{R_B}{1-A_{V1}} = \frac{100K}{1+4.86} = 17K$ and $R_i = 17K\|26.6K = 10.4K$; $R_o' = 2K \quad \longleftarrow$

$A_V = A_{V1}A_{V2} = (-4.86)(-27.4) = 133$; $A_I = A_V\frac{R_i}{R_{L2}} = 133\frac{10.4K}{2K} = 690 \quad \longleftarrow$

(12-10) The gain of the first stage must be small, because the Miller resistance $\frac{R_B}{1-A_{V1}}$ shunts the input, thus reducing the input resistance, R_i.

(12-10) (Cont'd) Choose $R_{c2} = 2.7K$ and $R_{e2} = 150\Omega$. This assures that $R_o' < 3K$. Then, using the approximate analysis, we have:

$A_{I2} = -h_{fe} = -50$; $R_{i2} = h_{ie} + (1+h_{fe})R_{e2} = 1.1K + (51)(0.15K) = 1.1K + 7.65K = 8.75K$

$A_{V2} = A_{I2}\frac{R_{L2}}{R_{i2}} = -50\frac{2.7K}{8.75K} = -15.4$. Thus, the voltage gain of the first stage must be at least 6.5. Let $A_{V1} = -7$.

If we let $R_{e1} = 0.5K$, then for $A_{V1} = -7 = -50\frac{R_{L1}}{R_{i1}}$ where $R_{i1} = 1.1K + 51(0.5K) = 26.6K$ we have: $R_{L1} = (7 \times 26.6K)/50 = 3.72K$. But $R_{L1} = R_{c1} \| R_{i2} = (R_{c1} \times 8.75)/(R_{c1} + 8.75)$

Solving for R_{c1} we get: $R_{c1} = 6.5K$. Use $R_{c1} = 6.8K$ which is commercially available. Then, $R_{L1} = 6.8 \| 8.75 = 3.88K$ and $A_{V1} = -50\frac{3.88}{26.6} = -7.28$

The Miller resistance shunting the input is, then, $R_B' = (100K)/(1+7.28)$ or $R_B' = 12.1K$. Hence $R_i = R_{i1} \| R_B' = (26.6K \| 12.1K) = 8.3K$ ... OK ←

$A_V = A_{V1}A_{V2} = (-7.28)(-15.4) = 112$ ... OK and $R_o' = 2.7K$ ... OK ←

Note that the Miller resistance $R_B'' = R_B/(1 - \frac{1}{A_{V1}}) = 100K/(1-0.137) = 86.3K$ appears between the collector of Q1 and ground. Its value, however, is too high to affect any of the previous calculations seriously.

(12-11) Stage II : $h_{oc}R_{L2} = \frac{4K}{40K} = 0.1$ ∴ Use approximate analysis. Thus, we have:

$A_{I2} = (1+h_{fe}) = 51$; $R_{i2} = h_{ie} + (1+h_{fe})R_{L2} = 1.1K + 51(4K) = 205K$; $A_{V2} = 1 - \frac{1.1}{205} = 0.995$

Stage I : $R_{L1} = R_{c1} \| R_{i2} \| R_M$ where $R_M = (100K)/(1 - \frac{1}{A_{V1}}) \approx 100K$ assuming that $|A_{V1}| \gg 1$. Thus, $R_{L1} = (2K \| 205K \| 100K) \approx 2K$ and $h_{oe}(R_{L1} + R_{e1}) = \frac{2.1K}{40K} = 0.05 < 0.1$

∴ Use approximate analysis: $A_{I1} = -h_{fe} = -50$; $R_{i1} = 1.1K + 51(0.1K) = 6.2K$

$A_{V1} = -50\frac{2K}{6.2K} = -16.1$; Note that the assumption made above that $|A_{V1}| \gg 1$ is satisfied and $R_M = (100K)/(1 + \frac{1}{16.1}) \approx 100K$.

$R_{o1} = \infty$; $R_{o1}' \doteq 2K = R_{s2}$. Then, $R_{o2} = \frac{R_{s2} + h_{ie}}{1+h_{fe}} = \frac{2K + 1.1K}{51} = 60.8\Omega$

Overall : $R_i = R_{i1} \| R_M'$ where $R_M' = \frac{R_B}{1 - A_{V1}} = (100K)/(1+16.1) = 5.85K$. Thus,

$R_i = 5.85K \| 6.2K = 3K$; $A_V = A_{V1}A_{V2} = (-16.1)(0.995) = -16$ ←

$A_I = -\frac{I_o}{I_i} = A_V\frac{R_i}{R_{L2}} = -16 \times \frac{3K}{4K} = -12$; $R_o' = R_{o2} \| 4K = 60.8\Omega \| 4K = 60\,\Omega$ ←

(12-12) Refer to the configuration of Prob. 12-11 in the text.
Since the second stage is an emitter follower with voltage gain less than unity, then the specified voltage gain must be provided by the first stage. There is an upper limit on the gain of the first stage, however, imposed by the Miller resistance $R_B/(1-A_{V1})$ which shunts the input terminals and thus reduces the input impedance.

Conservatively, let us design the first stage to provide $|A_{V1}| = 18$. Let us, also, choose $R_{e1} = 150\Omega$ and $R_{e2} = 3.9K$. The latter choice is based on intuition. Then, using approximate analysis we have:

(12-12) (Cont'd) $A_{V_1} = -h_{fe}\dfrac{R_{L_1}}{R_{i_1}} = -18$ where $R_{i_1} = 1.1K + 51(0.15K) = 1.1K + 7.65K = 8.75K$

Thus, $R_{L_1} = (18\times 8.75)/50 = 3.15K$. Since $h_{oe}(R_{L_1}+R_{e_1}) = 3.3K/40K = 0.082 < 0.1$ and $h_{oe}R_{e_2} = 3.9K/40K = 0.0975 < 0.1$, the approximate analysis can be used indeed for both stages.

Stage II : $R_{i_2} = 1.1K + 51(3.9K) = 1.1K + 198.9K = 200K$;

$$A_{V_2} = 1 - \frac{1.1K}{200K} = 1 - 0.005 = 0.995\ ;\quad R_{o_2} = \frac{3.15K + 1.1K}{51} = \frac{4.25K}{51} = 83.2\Omega$$

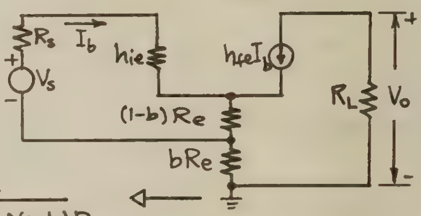
Stage I : The effective load resistance, R_{L_1}, found above is equal to the parallel combination of R_{c_1}, $R_{i_2} = 200K$ and $R_B/(1-\frac{1}{A_{V_1}}) \approx 100K$ or $R_{L_1} = 3.15K = R_{c_1}\|(200K\|100K) = (R_{c_1}\|66.7K)$ $\therefore$ $R_{c_1} \doteq 3.3K$

Overall : $A_V = A_{V_1}A_{V_2} = (-18)(0.995) = -17.9$ . . . . OK ⟵

$R_i = 8.75K\|(\frac{100K}{19}) = 8.75K\|5.27K \doteq 3.3K$ . . . OK ⟵

$R_o' = R_{o_2}\|3.9K = 83.3\Omega\|3.9K = 81.5\,\Omega$ . . . OK ⟵

(12-13) Since $h_{oe}(R_L+R_e) \leq 0.1$, we can use the approximate model of the transistor. The resulting small-signal equivalent circuit is shown. From this circuit we have:



$$A_V = \frac{V_o}{V_s} = \frac{-h_{fe}I_bR_L}{I_b(R_s+h_{ie})+(1+h_{fe})I_b(1-b)R_e} = \frac{-h_{fe}R_L}{R_s+h_{ie}+(1+h_{fe})(1-b)R_e} \longleftarrow$$

$$R_i = \frac{V_s}{I_b} = \frac{I_b(R_s+h_{ie})+(1+h_{fe})(1-b)R_eI_b}{I_b} = R_s+h_{ie}+(1+h_{fe})(1-b)R_e \longleftarrow$$

(12-14) $R_o = \dfrac{V_{oc}}{I_{sc}}$ where $V_{oc} = \lim\limits_{R_L\to\infty}(A_{V_s}V_s)$; $A_{V_s} = A_I\dfrac{R_L}{R_i+R_s} = \dfrac{\frac{h_{oe}R_e-h_{fe}}{1+h_{oe}(R_L+R_e)}R_L}{R_i+R_s}$

Using Eqs. (12-49) and (12-45), we have:

$$V_{oc} = \lim_{R_L\to\infty}\left\{\frac{\frac{h_{oe}R_e-h_{fe}}{1+h_{oe}(R_L+R_e)}R_L\,V_s}{R_s+h_{ie}+(1-h_{re})R_e+\frac{(h_{oe}R_e-h_{fe})[h_{re}(R_L+R_e)-R_e]}{1+h_{oe}(R_L+R_e)}}\right\} \quad \text{or}$$

$$V_{oc} = \frac{(h_{oe}R_e-h_{fe})V_s}{h_{oe}\left[R_s+R_e+h_{ie}+(1-h_{re})R_e-\frac{h_{fe}h_{re}}{h_{oe}}\right]}\ ;\quad I_{sc} = A_I'I_i \ \text{ where } I_i = \frac{V_s}{R_s+R_i'}$$

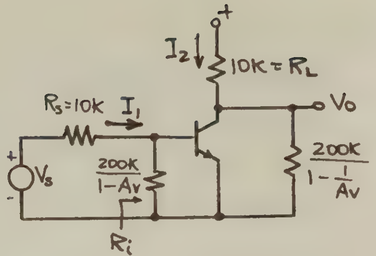
$$A_I' = A_I\Big|_{R_L=0} = \frac{h_{oe}R_e-h_{fe}}{1+h_{oe}R_e} \ \text{ and } R_i' = R_i\Big)_{R_L=0} = h_{ie}+(1-h_{re})R_e\left[1-\frac{(h_{oe}R_e-h_{fe})}{1+h_{oe}R_e}\right]$$

Thus, $I_{sc} = \dfrac{(h_{oe}R_e-h_{fe})V_s}{(R_s+h_{ie})(1+h_{oe}R_e)+(1-h_{re})R_e[(1+h_{fe})]}$; Substituting, we get after simplification:

$$R_o = \frac{1}{h_{oe}}\,\frac{(R_s+h_{ie})(1+h_{oe}R_e)+(1+h_{fe})R_e(1-h_{re})}{R_e+R_s+h_{ie}-\frac{h_{fe}h_{re}}{h_{oe}}}$$

. This is the exact expression for R_o. It differs from Eq. (12-51) by the fact that the last term in the numerator is multiplied by $(1-h_{re})$. Since $h_{re} \ll 1$, then the above expression equals that of Eq (12-51).

(12-15) Assume that $-A_V \gg 1$, then $\dfrac{200K}{1-\frac{1}{A_V}} \approx 200K$ and $R_L' = 10K \| 200K = \dfrac{10 \times 200}{210} = 9.5K$



$$\therefore A_I' = \frac{-h_{fe}}{1+h_{oe}R_L'} = \frac{-50}{1+9.5/40} = \frac{-50}{1.24} = -40.2$$

$$R_i' = h_{ie} + h_{re} A_I R_L' = 1.1K + (2.5 \times 10^{-4})(-40.2)(9.5K) \doteq 1K$$

$$A_V = \frac{A_I R_L'}{R_i'} = \frac{-40.2 \times 9.5K}{1K} = -382 \,. \quad ◁——$$

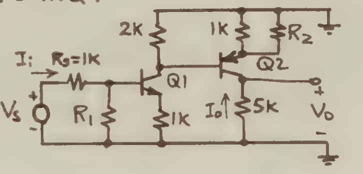
$$\frac{200K}{1-A_V} = \frac{200K}{383} = 0.522K \qquad \therefore R_i = R_i' \| 0.522K = \frac{1000 \times 522}{1522} = 343\ \Omega \quad ◁——$$

$$A_{Vs} = A_V \frac{R_i}{R_i+R_s} = -382 \frac{0.343K}{0.343K+10K} = -12.7 \quad ◁——$$

$$A_I = \frac{-I_2}{I_1} = A_{Vs} \frac{(R_i+R_s)}{R_L} = -\frac{12.7\,(10.343)}{10} = -13.1 \quad ◁——$$

(12-16) a) Switch open: Note that $(1+h_{fe})R_e = 101 \times 1K = 101K \gg h_{ie} = 1K$ applies for both stages. Thus, using Eq. (12-41) we have: $A_{V2} = -\dfrac{R_{L2}}{R_{e2}} = -5$ and $A_{V1} = -2$ $\quad \therefore A_V = \dfrac{V_o}{V_i} = A_{V1} A_{V2} = (-2)(-5) = 10$

b) Switch closed: Define A_V' as the voltage gain from the base of the first transistor to the emitter of the second. Then using Miller's theorem: $R_1 = 3K/(1-A_V')$ between the base of Q1 and ground and $R_2 = 3K/(1-\frac{1}{A_V'})$ between the emitter of Q2 and ground.



Stage II: Since h_{fe} is large, then the voltage gain between base and emitter of Q2 can be assumed to be approximately unity, and the input inpedance, $R_{i2} \gg 2K$

Stage I: Thus, $A_{V1} = -\dfrac{R_{L1}}{R_{e1}} = -2$ and $A_V' = -2$. Then, $R_1 = \dfrac{3K}{1+2} = 1K$, $R_2 = 2K$.

Effective $R_{e2} = 1K \| R_2 = 1K \| 2K = \frac{2}{3}K$ $\quad \therefore A_{V2} = -\dfrac{5K}{2/3K} = -7.5$

Note that $R_{i2} = 1 + 101(\frac{2}{3}K) = 68.4K \gg 2K$ as assumed above

$R_{i1} = 1K + 101 \times 1K = 102K$

Overall: $A_V = A_{V1} A_{V2} = (-2)(-7.5) = 15$; $R_i = R_1 \| R_{i1} = 1K \| 102K \approx 1K$ ◁——

$$A_{Vs} = A_V \frac{R_i}{R_i+R_s} = 15 \frac{1K}{1K+1K} = 7.5 \; ; \; A_I = -\frac{I_o}{I_i} = -A_{Vs}\frac{R_i}{R_{L2}} = -7.5\frac{1K}{5K} = -1.5 \quad ◁——$$

(12-17) The effective emitter resistance is $R_e' = 2K \| 4K \| 4K = 1K$. Since $h_{oe}(R_c + R_e') = 4/40 = 0.1$, we may use the approximate analysis.

Thus, $A_I' = \dfrac{I_o}{I_b} = -h_{fe} = -100$; $R_i' = \dfrac{V_i}{I_b} = h_{ie} + (1+h_{fe})R_e' = 2K + 101 \times 1K = 103K$

The voltage gain from base to emitter, $A_V' = 1 - \dfrac{h_{ie}}{R_i'} = 1 - \dfrac{2}{103} = 1 - 0.0194 = 0.9806$

Applying Miller's theorem to 10K we find that $R_1 = \dfrac{10K}{1-0.9806} = 515K$, $R_2 = \dfrac{10K}{1-\frac{1}{0.9806}}$ or $R_2 = -500K$. R_2 in parallel with 2K was neglected above.

$$\therefore R_i = 103K \| 515K = 85.6K \; ; \; A_I = \frac{I_o}{I_i} = \frac{I_o}{I_b} \cdot \frac{I_b}{I_i} = -100\left(\frac{515}{515+103}\right) = -83.2 \text{ and } A_V = -\frac{R_L}{R_e'} = -3 \quad ◁——$$

(12-18) Since $h_{oe} = 0$ for all transistors, then we can use the approximate analysis throughout. Thus, regardless what the value of R_e of each transistor is, $h_{oe}R_e < 0.1$ is satisfied. This is not realistic.
Starting with the last transistor, QN, we have: $R_{iN} = (1+h_{fe})R_e$. The emitter resistance of $Q(N-1)$ is R_{iN} $\therefore$ $R_{i(N-1)} = (1+h_{fe})R_{iN} = (1+h_{fe})^2 R_e$.

Hence, for N transistors, $R_i = (1+h_{fe})^N R_e$ $\longleftarrow$

(12-19) Effective emitter resistance of Q2 is $R'_{e2} = 0.1K \| 10K \| 82K \approx 0.1K$.
$h_{oe}(R_{c2}+R'_{e2}) = \frac{1.1K}{40K} = 0.0275 < 0.1$ $\therefore$ use approximate analysis: Thus, we have:
$R_{i2} = h_{ie} + (1+h_{fe})R'_{e2} = 1K + 101(0.1K) = 11.1K$ and $A'_{V2} = \frac{V_{o2}}{V_{i2}} = 1 - \frac{1K}{11.1K} = 0.91$
$A_{I2} = -h_{fe} = -100$ and $A_{V2} = \frac{V_{o1}}{V_{i2}} = A_{I2}\frac{R_{c2}}{R_{i2}} = -100\frac{\times 1K}{11.1} = -9$
The effective emitter resistance of Q1 is $R_{e1} = R_{i2} = 11.1K$. Thus, $h_{oe}R_{e1} > 0.1$ and the exact analysis must be used for Q1.

$$A'_{I1} = -\frac{I_{e1}}{I_{b1}} = \frac{1+h_{fe}}{1+h_{oe}R_{e1}} = \frac{101}{1+11.1/40} = \frac{101}{1.28} = 79 \quad \therefore \frac{I_{e1}}{I_{b1}} = -79 \longleftarrow$$

$$R_{i1} = h_{ie} + A'_{I1}R_{e1} = 1K + 79 \times 11.1K = 877K \; ; \; A_{V1} = 1 - \frac{1K}{877K} = 1 - 0.0010 = 0.999$$

Hence, $\frac{V_{o2}}{V_i} = A_{V1}A'_{V2} = (0.999)(0.91) = 0.909$ and $\frac{V_{o1}}{V_i} = A_{V1}A_{V2} = 0.999 \times (-9) = -8.99$ $\longleftarrow$

$$R_i = R_{i1} \| \left(\frac{100K}{1-\frac{V_{o2}}{V_i}}\right) = 877K \| \left(\frac{100K}{0.091}\right) = 877K \| 1.1M \doteq 490K \longleftarrow$$

(12-20) Effective emitter resistance of Q2 is $R'_{e2} = 1K \| 40K \| 100K \approx 1K$. Since $h_{oe}R'_{e2} = 1K/40K = 0.025 < 0.1$, we may use the approximate analysis. Thus,
$A_{I2} = (1+h_{fe}) = 51$; $R_{i2} = h_{ie} + (1+h_{fe})R'_{e2} = 1.1K + 51(1K) = 52.1K$
$A_{V2} = 1 - \frac{h_{ie}}{R_{i2}} = 1 - \frac{1.1K}{52.1K} = 1 - 0.021 = 0.979$
Effective emitter resistance of Q1 is $R_{e1} = R_{i2} = 52.1K$. Thus, $h_{oe}R_{e1} > 0.1$ and the exact analysis must be used for Q1.

$$A_{I1} = \frac{1+h_{fe}}{1+h_{oe}R_{e1}} = \frac{51}{1+52.1/40} = \frac{51}{2.3} = 22.2 \; ; \; R_{i1} = 1.1K + 22.2 \times 52.1 = 1.15M$$

$$A_{V1} = 1 - \frac{1.1K}{1150K} = 0.9999$$

Overall amplifier: $A_V = A_{V1}A_{V2} = 0.9999 \times 0.979 \doteq 0.979$; $\longleftarrow$
$470K/(1-A_V) = \frac{470K}{0.021} = 22.4M$; $R_i = R_{i1} \| 22.4M = 1.15M \| 22.4M = 1.09M$

$$A_{Vs} = A_V\frac{R_i}{R_i+R_s} = 0.979\frac{1.09}{1.09+0.1} = 0.897; \; A_I = \frac{I_o}{I_i} = A_{Vs}\frac{(R_i+R_s)}{R_{e2}} = 0.979\frac{1.19M}{1K} = 1068 \longleftarrow$$

$$R_{o2} = \frac{R_s+h_{ie}}{(1+h_{fe})^2} + \frac{h_{ie}}{1+h_{fe}} = \frac{101.1K}{(51)^2} + \frac{1.1K}{51} = 60.4\Omega \quad \therefore R'_o = 60.4 \| 1000 = 57\Omega \longleftarrow$$

(12-21) Refer to Fig. 12-18b. Since $h_{ie} \ll \frac{1}{h_{oe}}$, we may neglect h_{oe1} in parallel with h_{ie2}. Then, $V_i = I_{b1}h_{ie1} + h_{re1}(-I_{b2}h_{ie2}) + I_{b2}h_{ie2} + IR_e$ where
$I_{b2} = (1+h_{fe1})I_{b1}$ and $I = (1+h_{fe2})I_{b2} - h_{fe1}I_{b1} = [(1+h_{fe1})(1+h_{fe2}) - h_{fe1}]I_{b1}$

(12-21) (Cont'd) Substituting, we obtain:

$$V_i = \{h_{ie1} - h_{re1}(1+h_{fe1})h_{ie2} + (1+h_{fe1})h_{ie2} + [(1+h_{fe1})(1+h_{fe2}) - h_{fe1}]R_e\} I_{b1}$$

Note that $(1+h_{fe1})h_{ie2} >> h_{ie1}$, $h_{re1} << 1$, $(1+h_{fe1})(1+h_{fe2}) >> h_{fe1}$ and $(1+h_{fe1})(1+h_{fe2})R_e >> (1+h_{fe1})h_{ie2}$, if $h_{fe1} >> 1$ and $h_{fe2} >> 1$.

Hence, $R_i = \dfrac{V_i}{I_{b1}} \approx h_{fe1} h_{fe2} R_e$ QED.

(12-22) Refer to Fig. 12-18a. Stage II: Effective emitter resistance, $R'_{e2} = R_{e2} \| R_{c1}$ or $R'_{e2} = 1K \| 100K \approx 1K$; $h_{oe} R'_{e2} = \dfrac{1K}{40K} = 0.025 < 0.1$ $\therefore$ use approximate analysis. Thus, $A_{I_2} = 1 + h_{fe} = 51$; $R_{i2} = 1.1K + 51 \times 1K = 52.1K$; $A_{V2} = 1 - \dfrac{1.1K}{52.1K} = 0.979$

Stage I: $R'_{e1} = R_{i2} = 52.1K$. Note that because of emitter-to-collector bootstrapping the effective h_{oe} of Q1 is reduced to $h'_{oe} = h_{oe}(1 - A_V)$ where $A_V = A_{V1} A_{V2} \approx A_{V2} = 0.979$, since $A_{V1} \approx 1$. Thus, $h'_{oe} R'_{e1} = 2.5 \times 10^{-5} (0.021) 52.1 \times 10^{3} = 0.0273 < 0.1$ $\therefore$ we may use approximate analysis here too.

Hence, $A_{I_1} = 1 + h_{fe} = 51$; $R_{i1} = 1.1K + 51 \times 52.1K = 2.68M$; $A_{V1} = 1 - \dfrac{1.1K}{2680K} \approx 1$

Overall: $A_V = A_{V1} A_{V2} = 1 \times 0.979 = 0.979$; $R_i = R_{i1} = 2.68M$ $\longleftarrow$

(12-23) The effective emitter resistance of Q2 is $R'_{e2} = 1K \| 100K \| 100K \| 40K \approx 1K$

Then, from Prob. 12-22 we have: $A_{I_2} = 51$, $R_{i2} = 52.1K$, $A_{V2} = 0.979$;

$A_{I_1} = 51$, $R_{i1} = 2.68M$, $A_{V1} \approx 1$. Hence, $A_V = A_{V1} A_{V2} = 0.979$ $\longleftarrow$

Applying Miller's Theorem to the 470K resistor, we have: $R_1 = \dfrac{470K}{1 - A_V} = \dfrac{470K}{0.021} = 22.4M$ and $R_2 = 470K/(1 - \frac{1}{A_V}) = 470K/(1 - \frac{1}{.979}) = -21.9M$

Hence, $R_i = R_{i1} \| R_1 = 2.68M \| 22.4M \doteq 2.4M$ $\longleftarrow$

Then, $A_{VS} = A_V \dfrac{R_i}{R_i + R_s} = 0.979 \dfrac{2.4}{2.4 + 0.1} = 0.94$; $A_I = 0.94 \dfrac{2500K}{1K} = 2350$ $\longleftarrow$

and $R_{o2} = \dfrac{R'_s + h_{ie}}{(1+h_{fe})^2} + \dfrac{h_{ie}}{1 + h_{fe}} \approx \dfrac{101.1K}{(51)^2} + \dfrac{1.1K}{51} = 38.8 + 21.6 = 60.4\,\Omega$

$\therefore R'_o = R_{o2} \| R_{e2} = 60.4\,\Omega \| 1000\,\Omega = 57\,\Omega$ $\longleftarrow$

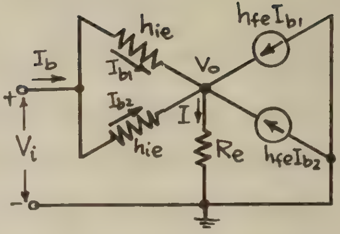
(12-24) From Eqs. (12-82) we have for the Q1-Q2 combination: $h_{ie} = 1.1K$, $h_{fe} = 49$, $h_{oe} = 0.49 \times 10^{-6}$ and $h_{re} = 7.25 \times 10^{-8}$. Thus, we note that, $h_{oe}(R_e + R_c) = 0.49 \times 10^{-6} \times 25.1 \times 10^{3} = 0.0123 < 0.1$ $\therefore$ we may use the approximate analysis: $A'_I = \dfrac{I_o}{I_i} = -h_{fe} = -49$; $R_{i1} = h_{ie} + (1 + h_{fe})R_e = 1.1K + 50 \times 0.1K = 6.1K$

$R_o = \infty$. Then $A_V = A'_I \dfrac{R_c}{R_{i1}} = -49 \times \dfrac{25K}{6.1K} = -201$ $\longleftarrow$

$R_i = R_{i1} \| 10K \| 200K = 6.1K \| 9.52K = 3.62K$; $R'_o = 25K$; $A_{VS} = -201 \dfrac{3.62}{4.62} = -157$ $\longleftarrow$

and $A_I = \dfrac{I_o}{I_i} = A_{VS} \dfrac{R_i + R_s}{R_c} = -157 \dfrac{3.62 + 1}{25} = -29$ $\longleftarrow$

(12-25) $A_V = \frac{V_o}{V_i}$ where $V_o = IR_e = (I_{b1}+I_{b2}+h_{fe}I_{b1}+h_{fe}I_{b2})R_e$



Because of symmetry of the circuit $I_{b1} = I_{b2} = \frac{I_b}{2}$

Thus, $V_o = 2(1+h_{fe})I_{b1}R_e$. Also, $V_i = I_{b1}h_{ie} + IR_e$

or $V_i = [h_{ie} + 2(1+h_{fe})R_e]I_{b1}$

$$\text{Hence, } A_V = \frac{2(1+h_{fe})R_e}{h_{ie}+2(1+h_{fe})R_e} \qquad \longleftarrow$$

$$R_i = \frac{V_i}{I_b} = \frac{1}{2}h_{ie} + (1+h_{fe})R_e \qquad \longleftarrow$$

(12-26) Since $h_{oe}R_c = 4K/40K = 0.1 \ll 1$, we may use Eq. (12-91), namely,

$$A_c = \frac{(2h_{oe}R_e - h_{fe})R_c}{2R_e[1+h_{fe}+h_{oe}(R_s+h_{ie})]+R_s+h_{ie}} = \frac{(2\times 2.5\times 10^{-5}\times 20\times 10^3 - 50)4\times 10^3}{2\times 20\times 10^3(51+2.5\times 10^{-5}\times 2.1\times 10^3)+2.1\times 10^3}$$

$$= \frac{-196\times 10^3}{2.04\times 10^6} = -0.096 \qquad \longleftarrow$$

$$\text{From Eq. (12-92), } A_d = \frac{1}{2}\frac{h_{fe}R_c}{R_s+h_{ie}} = \frac{1}{2}\frac{50\times 4\times 10^3}{2.1\times 10^3} = 47.6 \qquad \longleftarrow$$

$$\text{Hence, } \rho = \left|\frac{A_d}{A_c}\right| = \frac{47.6}{0.096} = 497 \qquad \longleftarrow$$

From Eqs. (12-88), $A_d = \frac{1}{2}(A_1 - A_2)$ and $A_c = A_1 + A_2$. Solving these we get:

$$A_1 = \frac{1}{2}(2A_d + A_c) = \frac{1}{2}(2\times 47.6 - 0.096) = 47.552 \qquad \longleftarrow$$

$$A_2 = \frac{1}{2}(A_c - 2A_d) = \frac{1}{2}(-0.096 - 2\times 47.6) = -47.648 \qquad \longleftarrow$$

(12-27) The equivalent impedance seen at the collector of Q3 may be found from Eq. (12-51) where $R_e = R_3 = 1K$ and $R_s = R_1 \| R_2 = 10K \| 100K = 9.1K$

$$\text{Thus, } R_o = \frac{1}{h_{oe}}\frac{(1+h_{fe})R_e + (R_s+h_{ie})(1+h_{oe}R_e)}{R_e+R_s+h_{ie} - h_{re}h_{fe}/h_{oe}} = (40K)\frac{51\times 10^3 + (9.1+1.1)10^3(1+1K/40K)}{(1+9.1+1.1)10^3 - 2.5\times 10^{-4}\times 50\times 40\times 10^3}$$

or $R_o = 229K$. We now use this as the value of R_e in Eq. (12-91)

$$A_c = \frac{(2\times 2.5\times 10^{-5}\times 2.29\times 10^5 - 50)\times 4\times 10^3}{2\times 2.29\times 10^5(51+2.5\times 10^{-5}\times 2.1\times 10^3)+2.1\times 10^3} = \frac{-154\times 10^3}{233\times 10^5} = -6.61\times 10^{-3} \qquad \longleftarrow$$

From Prob. 12-26, $A_d = 47.6$ (since it is independent of R_e) $\longleftarrow$

Then, $\rho = \left|\frac{47.6}{-6.61\times 10^{-3}}\right| = 7200$; $A_1 = \frac{1}{2}(2\times 47.6 - 6.61\times 10^{-3}) \approx 47.6$; $A_2 \approx -47.6$ $\longleftarrow$

(12-28) For the circuit of Fig. 12-22a, $A_I = \frac{2h_{oe}R_e - h_{fe}}{1+h_{oe}(R_c+2R_e)}$ .... (12-48) and $R_i = (1-A_I)(2R_e) + h_{ie} + h_{re}A_I(R_c+2R_e)$ .... (12-49. The the voltage gain is given by Eq. (12-50),

$$A_V = A_c = \frac{A_I R_c}{R_s+R_i} = \frac{(2h_{oe}R_e - h_{fe})R_c/[1+h_{oe}(R_c+2R_e)]}{R_s + \frac{2h_{oe}R_e - h_{fe}}{1+h_{oe}(R_c+2R_e)}(h_{re}R_c + 2h_{re}R_e - 2R_e) + 2R_e + h_{ie}}$$

If $h_{re}(R_c+2R_e) \ll 2R_e$ and $h_{oe}R_c \ll 1$,

$$A_c = \frac{(2h_{oe}R_e - h_{fe})R_c}{(1+2R_e)(R_s+2R_e+h_{ie})+(2h_{oe}R_e-h_{fe})(-2R_e)} = \frac{(2h_{oe}R_e - h_{fe})R_c}{2R_e(1+h_{fe})+(R_s+h_{ie})(1+2h_{oe}R_e)} \qquad \text{QED}$$

For the circuit of Fig. 12-22b, $A_I = -h_{fe}/(1+h_{oe}R_c)$ and $R_i = h_{ie} + h_{re}A_I R_c$;

Then, $A_V' = V_o/(V_s/2) = -2A_d = A_I R_c/(R_i+R_s) = -\frac{h_{fe}R_c}{1+h_{oe}R_c} \Big/ \left(R_s + h_{ie} - \frac{h_{re}h_{fe}R_c}{1+h_{oe}R_c}\right) \doteq \frac{-h_{fe}R_c}{(1+h_{oe}R_c)(h_{ie}+R_s)}$ provided that $h_{re}h_{fe}R_c \ll (h_{ie}+R_s)$. Moreover, if $h_{oe}R_c \ll 1$, then $A_d = \frac{1}{2}\frac{h_{fe}R_c}{h_{ie}+R_s}$ QED

CHAPTER 13

(13-1) From Eq. (9-29), the transport factor at $\omega=0$ is $\beta^* = \operatorname{sech}\frac{W}{L_B}$. At the bottom of p. 112 we find that β^* for $\omega \neq 0$ may be obtained from the dc β^* simply by replacing L_B by $L_B(1+j\omega\tau_B)^{-1/2}$.

Hence, $\alpha(\omega) = \beta^*(\omega) = \operatorname{sech}\frac{W}{L_B}(1+j\omega\tau_B)^{1/2}$ QED.

a) If $x \ll 1$, then $\operatorname{sech} x = (\cosh x)^{-1} \doteq \left(1+\frac{x^2}{2}+\cdots\right)^{-1} = \frac{1}{1+\frac{1}{2}x^2}$

$x = \frac{W}{L_B}(1+j\omega\tau_B)^{1/2}$. For frequencies such that $\omega\tau_B < 1$, and for $\frac{W}{L_B} \ll 1$,

$$x \ll 1, \quad \therefore \alpha(\omega) \approx \frac{1}{1+\frac{1}{2}\frac{W^2}{L_B^2}(1+j\omega\tau_B)} = \frac{\frac{1}{1+\frac{1}{2}W^2/L_B^2}}{1+j\frac{\omega\tau_B W^2}{2L_B^2\left(1+\frac{1}{2}\frac{W^2}{L_B^2}\right)}} = \frac{\alpha_0}{1+j\,f/f_\alpha}$$

where $\alpha_0 = \frac{1}{1+\frac{1}{2}\frac{W^2}{L_B^2}}$ and $\omega_\alpha = 2\pi f_\alpha = \frac{2L_B^2}{W^2\tau_B\alpha_0}$ QED

b) From Eq. (5-51), $L_B^2 = D_B\tau_B$. Hence, $\omega_\alpha = \frac{2D_B}{W^2\alpha_0}$ where α_0 is given in part $\underline{a}$ above. Note that for $W/L_B \ll 1$, $\alpha_0 \approx 1$. Thus, ω_α given by Eq. (13-13) is approximately equal to the value obtained here.

(13-2) a) From Prob. 13-1, we have that @ $\omega=\omega_\alpha$, $\alpha(\omega_\alpha) = \operatorname{sech}\left[\frac{W}{L_B}(1+j\omega_\alpha\tau_B)^{1/2}\right]$

$= \operatorname{sech}\left[\frac{W}{L_B}\left(1+j\frac{2L_B^2}{W^2\alpha_0}\right)^{1/2}\right]$. If $\alpha_0 = 1$ and $\frac{W^2}{L_B^2} \ll 1$ or $\frac{L_B^2}{W^2} \gg 1$, then

$\alpha(\omega_\alpha) \approx \operatorname{sech}\left[\frac{W}{L_B}\left(2j\frac{L_B^2}{W^2}\right)^{1/2}\right] = \operatorname{sech}(\sqrt{2j}) = \frac{1}{\cosh(\sqrt{2j})}$ where

$\sqrt{2j} = \sqrt{2}\left(e^{j\frac{\pi}{2}}\right)^{1/2} = \sqrt{2}\,e^{j\frac{\pi}{4}} = \sqrt{2}\left(\cos\frac{\pi}{4}+j\sin\frac{\pi}{4}\right) = (1+j)$. Thus, we have:

$\operatorname{sech}(\sqrt{2j}) = \frac{1}{\cosh(1+j)} = \frac{1}{\cosh 1\cos 1 + j\sinh 1\sin 1}$; but $\cosh 1 = 1.543$

$\sinh 1 = 1.175$, $\sin 1 = 0.8415$ and $\cos 1 = 0.5403$

$$|\alpha(\omega_\alpha)|^2 = \frac{1}{(\cosh 1\cos 1)^2 + (\sinh 1\sin 1)^2} = \frac{1}{(1.543\times 0.5403)^2 + (1.175\times 0.8415)^2} = \frac{1}{1.67} = 0.60$$

$$\therefore |\alpha(\omega_\alpha)| = \sqrt{0.6} = 0.775$$

$$\text{Phase}(\omega_\alpha) = -\tan^{-1}\frac{\sinh 1\sin 1}{\cosh 1\cos 1} = -\tan^{-1}\left(\frac{1.175\times 0.8415}{1.543\times 0.5403}\right) = -\tan^{-1}(1.185) \approx -50^\circ$$

b) From Eq. (13-4), $\alpha(\omega) = \frac{\alpha_0}{1+j\omega/\omega_\alpha}$. By inspection we see that at $\omega = \omega_\alpha$, $|\alpha(\omega_\alpha)| = 0.707\alpha_0 \approx 0.707$; Phase $(\omega_\alpha) = -45^\circ$ ←

(13-3) a) $\alpha(\omega_\alpha) = \operatorname{sech}\left[\frac{W}{L_B}(1+j\omega_\alpha\tau_B)^{1/2}\right]$. At $\omega_\alpha = 2.43D_B/W^2\alpha_0$ we have:

$$\alpha(\omega_\alpha) = \operatorname{sech}\left[\frac{W}{L_B}\left(1+j2.43\frac{D_B\tau_B}{W^2\alpha_0}\right)^{1/2}\right] = \operatorname{sech}\left[\frac{W}{L_B}\left(1+j2.43\frac{L_B^2}{W^2\alpha_0}\right)^{1/2}\right]$$

If $\alpha_0 = 1$ and $\frac{W}{L_B} \ll 1$ or $\frac{L_B^2}{W^2} \gg 1$, then $\alpha(\omega_\alpha) \approx \operatorname{sech}(\sqrt{2.43j})$

From Prob. 13-2$\underline{a}$, $\sqrt{j} = \frac{1}{\sqrt{2}}(1+j)$, thus, $\alpha(\omega_\alpha) = \operatorname{sech}\left[\sqrt{\frac{2.43}{2}}(1+j)\right]$

or $\alpha(\omega_\alpha) = \operatorname{sech}[1.1(1+j)] = [\cosh 1.1(1+j)]^{-1} = [\cosh 1.1\cos 1.1 + j\sinh 1.1\sin 1.1]^{-1}$

(13-3) (Cont'd) where $\cosh 1.1 = 1.67$, $\sinh 1.1 = 1.336$, $\cos 1.1 = 0.454$ & $\sin 1.1 = 0.891$

Then, $|\alpha(\omega_\alpha)|^2 = \dfrac{1}{(\cosh 1.1 \cos 1.1)^2 + (\sinh 1.1 \sin 1.1)^2} = \dfrac{1}{0.574 + 1.42} = 0.50$

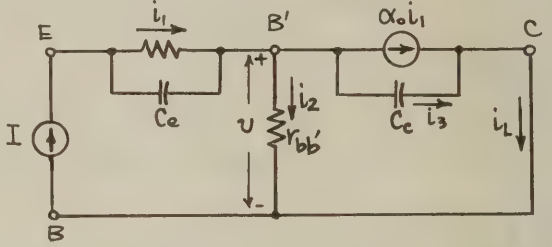
$$\therefore |\alpha(\omega_\alpha)| = \sqrt{0.5} = 0.707$$

$\text{Phase}(\omega_\alpha) = -\tan^{-1} \dfrac{1.336 \times 0.891}{1.67 \times 0.454} = -\tan^{-1}(1.57) = -57.5^\circ$

b) From Eq. (13-4), $\alpha = \dfrac{\alpha_o}{1 + j\omega/\omega_\alpha}$. At $\omega = \omega_\alpha$, $|\alpha| = 0.707\alpha_o \approx 0.707$ and phase $= -45^\circ$.

c) Phase of α = phase of $\alpha(\omega_\alpha) - m$ or $m = 57.5^\circ - 45^\circ = 12.5^\circ$

in radians, $m = \dfrac{12.5^\circ}{57.2^\circ} = 0.218$ $\longleftarrow$



(13-4) With $R_L = 0$ in Fig. 13-2, r_c' is in parallel with $r_{bb'}$ and since $r_{bb'} \ll r_c'$, we neglected r_c' in the circuit shown. From this circuit we have:

$I = \alpha_o i_1 + i_2 + i_3 = \alpha_o i_1 + \dfrac{v}{r_{bb'}} + C_c \dfrac{dv}{dt}$

where $i_1 = I(1 - e^{-t/r_e' C_e}) = I(1 - e^{-\omega_\alpha t})$ and $\omega_\alpha \equiv \dfrac{1}{r_e' C_e}$.

Thus, $\dfrac{dv}{dt} + \omega' v = \dfrac{I}{C_c}\left[1 - \alpha_o(1 - e^{-\omega_\alpha t})\right]$ where $\omega' \equiv \dfrac{1}{r_{bb'} C_c}$. The solution to the homogeneous eq.n is $v_h = A e^{-\omega' t}$. The particular solution is obtained as follows: $\dfrac{dv}{dt} + \omega' v = \dfrac{I}{C_c}(1 - \alpha_o + \alpha_o e^{-\omega_\alpha t})$ (1)

Let $v_p = K_1 + K_2 e^{-\omega_\alpha t}$ (2). Then, $\dfrac{dv_p}{dt} = -\omega_\alpha K_2 e^{-\omega_\alpha t}$ (3)

Substituting (2) & (3) into (1) we have:

$$-\omega_\alpha K_2 e^{-\omega_\alpha t} + \omega' K_1 + \omega' K_2 e^{-\omega_\alpha t} = \omega' K_1 + (\omega' - \omega_\alpha) K_2 e^{-\omega_\alpha t} \equiv \frac{I}{C_c}(1 - \alpha_o) + \frac{I}{C_c}\alpha_o e^{-\omega_\alpha t}$$

Thus, $K_1 = \dfrac{I}{C_c}\left(\dfrac{1 - \alpha_o}{\omega'}\right)$ and $K_2 = \dfrac{I}{C_c}\left(\dfrac{\alpha_o}{\omega' - \omega_\alpha}\right)$. The total solution is:

$$v = A e^{-\omega' t} + \frac{I}{C_c}\left[\left(\frac{1 - \alpha_o}{\omega'}\right) + \left(\frac{\alpha_o}{\omega' - \omega_\alpha}\right) e^{-\omega_\alpha t}\right]$$

The initial condition is that at $t = 0$, $v = 0$, since both capacitors are initially uncharged. $\therefore A = -\dfrac{I}{C_c}\left(\dfrac{1 - \alpha_o}{\omega'} + \dfrac{\alpha_o}{\omega' - \omega_\alpha}\right) = -\dfrac{I}{\omega' C_c}\left[1 + \dfrac{\alpha_o \omega_\alpha}{(\omega' - \omega_\alpha)}\right]$

$i_3 = C_c \dfrac{dv}{dt} = I\left(1 + \dfrac{\alpha_o \omega_\alpha}{\omega' - \omega_\alpha}\right) e^{-\omega' t} - I\left(\dfrac{\alpha_o \omega_\alpha}{\omega' - \omega_\alpha}\right) e^{-\omega_\alpha t}$. From the circuit we have:

$$i_L = \alpha_o i_1 + i_3 = \alpha_o I(1 - e^{-\omega_\alpha t}) + I\left(1 + \frac{\alpha_o \omega_\alpha}{\omega' - \omega_\alpha}\right) e^{-\omega' t} - I\left(\frac{\alpha_o \omega_\alpha}{\omega' - \omega_\alpha}\right) e^{-\omega_\alpha t}$$

or $\dfrac{i_L}{I} = \alpha_o + \left(1 + \dfrac{\alpha_o \omega_\alpha}{\omega' - \omega_\alpha}\right) e^{-\omega' t} - \dfrac{\alpha_o \omega'}{\omega' - \omega_\alpha} e^{-\omega_\alpha t}$ QED

(13-5) If $\omega' \gg \omega_\alpha$, $\dfrac{i_L}{I} \approx \alpha_o + \left(1 + \dfrac{\alpha_o \omega_\alpha}{\omega'}\right) e^{-\omega' t} - \alpha_o e^{-\omega_\alpha t}$. Since $\alpha_o \approx 1$ and $\dfrac{\omega_\alpha}{\omega'} \ll 1$, then $1 + \dfrac{\alpha_o \omega_\alpha}{\omega'} \approx 1$. Also, $e^{-\omega' t} \rightarrow 0$ much faster than $e^{-\omega_\alpha t}$ $\therefore \dfrac{i_L}{I} \approx \alpha_o(1 - e^{-\omega_\alpha t})$ QED

(13-6) a) $r'_e = \frac{V_T}{I_E} = \frac{26\,mV}{2\,mA} = 13\ \Omega$ ←

b) From Eq. (13-13), $2\pi f_\alpha = \frac{2D_B}{W^2}$; for a p-n-p transistor, $D_B = D_p = 47\ cm^2/sec$

Hence, $f_\alpha = \frac{2\times 47}{2\pi\times 25\times 10^{-8}} = 60\ MHz$ ←

c) From Eq. (13-12), $C_{De} = \frac{W^2 I_E}{2D_B V_T} = \frac{25\times10^{-8}\times2\times10^{-3}}{2\times47\times26\times10^{-3}} = 205\ pF$ ←

d) From Eqs. (13-14) and (13-15) we have : $t_B = \frac{1}{\omega_\alpha} = \frac{10^{-6}}{2\pi\times60} = 2.66\ nsec$ ←

(13-7) From Eq. (13-13), $W = \left(\frac{D_B}{\pi f_\alpha}\right)^{1/2}$ where for a p-n-p Si. transistor (from Table 5-1), $D_B = D_p = 13\ cm^2/sec$. Thus, $W = \left(\frac{13}{\pi\times10^5}\right)^{1/2} = (41.4)^{1/2}\times10^{-3} = 6.44\times10^{-3}\ cm$ ←

(13-8) a) From Eq. (13-13), $f_\alpha = \frac{D_B}{\pi W^2}$ where for an n-p-n Ge transistor (from Table 5-1) $D_B = D_n = 99\ cm^2/sec$. Also, $W = 1\ mil = 2.54\times10^{-3}\ cm$

Hence, $f_\alpha = \frac{99}{\pi\times(2.54\times10^{-3})^2} = 4.88\ MHz$ ←

b) From Eq. (13-19), $f_\beta = f_\alpha(1-\alpha_o)$. From Prob. 13-1 we have that

$\alpha_o = \frac{1}{1+\frac{1}{2}\frac{W^2}{L_n^2}} = \frac{1}{1+\frac{1}{2}\frac{(2.54\times10^{-3})^2}{99\times4\times10^{-6}}} = \frac{1}{1.00814}$; $1-\alpha_o = 1-\frac{1}{1.00814}$

or $1-\alpha_o = \frac{0.00814}{1.00814} \approx 0.008$ $\therefore f_\beta = 4.88\times10^{6}\times8\times10^{-3} = 39\ kHz$ ←

(13-9) From Eq. (13-13), $f_\alpha = \frac{D_B}{\pi W^2}$ and from Eq. (13-19), $f_\beta = f_\alpha(1-\alpha_o) = \frac{D_B}{\pi W^2}(1-\alpha_o)$

From Prob. 13-1, $\alpha_o = \frac{1}{1+\frac{1}{2}\frac{W^2}{L_B^2}}$ and $\frac{W}{L_B} \ll 1$.

Hence, $f_\beta = \frac{D_B}{\pi W^2}\left(1-\frac{1}{1+\frac{1}{2}\frac{W^2}{L_B^2}}\right) = \frac{D_B}{\pi W^2}\,\frac{\frac{1}{2}(W^2/L_B^2)}{1+\frac{1}{2}(W^2/L_B^2)} \cong \frac{D_B}{2\pi L_B^2} = \frac{1}{2\pi\tau_B}$ since $L_B^2 = D_B\tau_B$

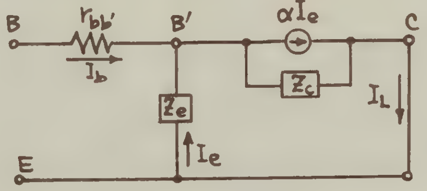
QED

(13-10) a) $A_{ie} = \frac{I_L}{I_b}$ where $I_L = \alpha I_e + \frac{V_{B'E}}{Z_c} = \alpha I_e - \frac{I_e Z_e}{Z_c}$

or $I_L = \left(\alpha - \frac{Z_e}{Z_c}\right)I_e$; Also, $I_b = I_L - I_e$

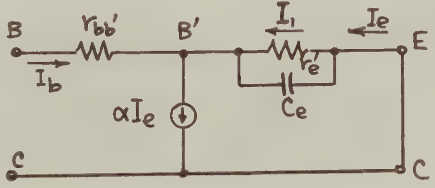
or $I_b = -\left(1-\alpha+\frac{Z_e}{Z_c}\right)I_e$

Hence, $A_{ie} = \frac{\alpha - Z_e/Z_c}{-(1-\alpha+Z_e/Z_c)} = \frac{-\alpha Z_c + Z_e}{(1-\alpha)Z_c + Z_e}$ QED



b) Since $|Z_e| \ll |(1-\alpha)Z_c|$, then $|-\alpha Z_c| \gg |Z_e|$, thus we have:

$A_{ie} \approx \frac{-\alpha Z_c}{(1-\alpha)Z_c} = \frac{-\alpha(\omega)}{1-\alpha(\omega)}$ QED



(13-11) Since $|Z_e| \ll |Z_c|$, we neglect Z_c in parallel with Z_e. From the circuit, $A_{ic} = -\frac{I_e}{I_b}$ where $I_b = -(1-\alpha)I_e$ $\therefore A_{ic} = \frac{1}{1-\alpha}$

(13-11) (Cont'd) From Fig. 13-4, $\alpha I_e = \alpha_o I_1$ where $I_1 = \frac{Z_e}{r_e'} I_e = \frac{I_e}{1+j\omega r_e' C_e}$

Thus, $\frac{I_e}{I_1} = \frac{\alpha_o}{\alpha} = (1+j\omega r_e' C_e)$ or $\alpha = \frac{\alpha_o}{1+j\omega r_e' C_e}$. Substituting, we get:

$$A_{ic} = \frac{1}{1-\alpha} = \frac{1}{1 - \frac{\alpha_o}{1+j\omega r_e' C_e}} = \frac{1+j\omega r_e' C_e}{1-\alpha_o + j\omega r_e' C_e} \doteq \frac{1+j\,f/f_\alpha}{(1-\alpha_o)+jf/f_\alpha}$$

Dividing by $(1-\alpha_o)$, we get: $A_{ic} = \frac{\frac{1}{1-\alpha_o} + j\frac{f}{(1-\alpha_o)f_\alpha}}{1+j\frac{f}{(1-\alpha_o)f_\alpha}} = \frac{\alpha_{co} + j\,f/f_{\alpha c}}{1+jf/f_{\alpha c}}$ where $\alpha_{co} = \frac{1}{1-\alpha_o}$

and $f_{\alpha c} = (1-\alpha_o) f_\alpha$. QED

Since α_o is close to unity, $\alpha_{co} = \frac{1}{1-\alpha_o} >> |j\,f/f_{\alpha c}|$ $\therefore A_{ic} \approx \frac{\alpha_{co}}{1+jf/f_{\alpha c}}$

QED

(13-12) a) Eq. (13-44) for hybrid π : $f_\beta = \frac{1}{h_{fe}} \frac{g_m}{2\pi(C_e+C_c)}$

From Eq. (13-29), $\frac{h_{fe}}{g_m} = \frac{r_e'}{1-\alpha_o}$ $\therefore f_\beta = \frac{1-\alpha_o}{2\pi r_e'(C_e+C_c)}$ QED

b) From Eqs. (13-19) and (13-13) for the high frequency T model

$f_\beta = (1-\alpha_o) f_\alpha = \frac{1-\alpha_o}{2\pi r_e' C_e}$. Since $C_e >> C_c$, then the two expressions for f_β are almost alike.

(13-13) From Eqs. (13-39): $g_m = \frac{|I_c|}{V_T} = \frac{10\text{ mA}}{26\text{ mV}} = 385$ mA/V $\longleftarrow$

$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{100}{0.385} = 260\ \Omega$; $r_{bb'} = h_{ie} - r_{b'e} = 500 - 260 = 240\Omega$ $\longleftarrow$

$r_{b'c} = \frac{r_{b'e}}{h_{re}} = 260 \times 10^4 \Omega = 2.6\text{M}$; $g_{ce} = h_{oe} - (1+h_{fe}) g_{b'c} = 4\times 10^{-5*} - \frac{101}{2.6\times 10^6} = 0.12 \times 10^{-5}$

or $r_{ce} = 1/g_{ce} = 833\text{K}$; $C_e = \frac{g_m}{2\pi f_T} = \frac{385\times 10^{-3}}{2\pi \times 50 \times 10^6} = 1224$ pF ; $C_c = 3$ pF $\longleftarrow$

(13-14) From Eq. (13-43), $|A_{ie}| = \left|\frac{-h_{fe}}{1+jf/f_\beta}\right| = \frac{h_{fe}}{[1+(f/f_\beta)^2]^{1/2}} = \frac{100}{[1+(f/f_\beta)^2]^{1/2}} = 10$

or $1 + (f/f_\beta)^2 = 100$ and $(f/f_\beta)^2 = 99$, or $f/f_\beta = 9.95$ where $f = 10$ MHz

Thus, $f_\beta = f/9.95 = 10/9.95 = 1.005$ MHz and $f_T = h_{fe} f_\beta = 100.5$ MHz $\longleftarrow$

From Eq. (13-45), $C_e = \frac{g_m}{2\pi f_T} = \frac{|I_c|}{V_T} \frac{1}{2\pi f_T} = \frac{5}{26} \frac{10^{-6}}{2\pi \times 100.5} = 304$ pF $\longleftarrow$

$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{100}{5/26} = 520\Omega$; $r_{bb'} = h_{ie} - r_{b'e} = 600 - 520 = 80\Omega$ $\longleftarrow$

(13-15) If $e^{V_C/V_T} << 1$, Eq. (9-52) becomes: $I_c \approx \frac{\alpha_N I_{EO}}{1-\alpha_N\alpha_I}\left(e^{V_E/V_T} - 1\right) + \frac{I_{co}}{1-\alpha_N\alpha_I}$

$$g_m \equiv \left.\frac{dI_c}{dV_E}\right|_{V_c} \approx \frac{\alpha_N I_{EO}}{1-\alpha_N\alpha_I} \frac{1}{V_T} e^{V_E/V_T} = \frac{1}{V_T}\left[I_c - \frac{I_{co} - \alpha_N I_{EO}}{(1-\alpha_N\alpha_I)}\right] = \frac{1}{V_T}\left[I_c - \frac{I_{co}(1-\alpha_I)}{(1-\alpha_N\alpha_I)}\right]$$

since $\alpha_N I_{EO} = \alpha_I I_{co}$ ... Eq. (9-53)

QED

(13-15) (Cont'd) Furthermore, since $I_C \gg I_{CO}$ and $\alpha_N \approx 1$, $\dfrac{1-\alpha_I}{1-\alpha_N\alpha_I} \approx 1$ and

$$g_m \approx \frac{I_C}{V_T}\left(1 - \frac{I_{CO}}{I_C}\right) \approx \frac{I_C}{V_T} \qquad \text{QED}$$

(13-16) a) $\alpha = \dfrac{\beta}{1+\beta} = \dfrac{-A_i}{1-A_i} = \dfrac{\dfrac{h_{fe}}{1+jf/f_\beta}}{1+\dfrac{h_{fe}}{1+jf/f_\beta}} = \dfrac{h_{fe}}{1+h_{fe}+jf/f_\beta} = \dfrac{\dfrac{h_{fe}}{1+h_{fe}}}{1+j\dfrac{f}{f_\beta(1+h_{fe})}} = \dfrac{\alpha_o}{1+jf/f_\alpha}$

where $\alpha_o = h_{fe}/(1+h_{fe})$ and $f_\alpha = f_\beta(1+h_{fe}) = \dfrac{f_\beta}{1-\alpha_o}$ QED

b) $\alpha_o = \dfrac{h_{fe}}{1+h_{fe}} \approx 1$ since $h_{fe} \gg 1$. Then, $f_\alpha \approx f_\beta h_{fe}$

c) From the results above: $h_{fe} = \dfrac{\alpha_o}{1-\alpha_o}$ and $f_\beta = f_\alpha(1-\alpha_o)$. Thus, we have

$$A_i = \frac{-h_{fe}}{1+jf/f_\beta} = \frac{-\alpha_o/(1-\alpha_o)}{1+jf/f_\alpha(1-\alpha_o)} = \frac{-\alpha_o}{1-\alpha_o+jf/f_\alpha} \qquad \text{QED}$$

d) Using the expression of part $\underline{c}$, $A_i = \dfrac{-\alpha_o e^{-jmf/f_\alpha}}{1-\alpha_o e^{-jmf/f_\alpha}+jf/f_\alpha}$

$= \dfrac{-\alpha_o\cos(mf/f_\alpha)+j\alpha_o\sin(mf/f_\alpha)}{1-\alpha_o\cos(mf/f_\alpha)+j\alpha_o\sin(mf/f_\alpha)+jf/f_\alpha}$. Note that when $f=f_T$, $|A_i|=1$.

Thus, $|A_i|^2 = 1 = \dfrac{\alpha_o^2\cos^2(mf_T/f_\alpha)+\alpha_o^2\sin^2(mf_T/f_\alpha)}{[1-\alpha_o\cos(mf_T/f_\alpha)]^2+[\alpha_o\sin(mf_T/f_\alpha)+f_T/f_\alpha]^2}$. Let $x = f_T/f_\alpha$. Then

$$\alpha_o^2(\cos^2 mx+\sin^2 mx) = 1-2\alpha_o\cos mx+\alpha_o^2\cos^2 mx+\alpha_o^2\sin^2 mx+2\alpha_o x\sin mx+x^2$$

or $1+x^2 = 2\alpha_o(\cos mx - x\sin mx)$ QED

e) If $mx \ll 1$, then $\cos mx \approx 1-\frac{1}{2}m^2x^2$ and $\sin mx \approx mx$

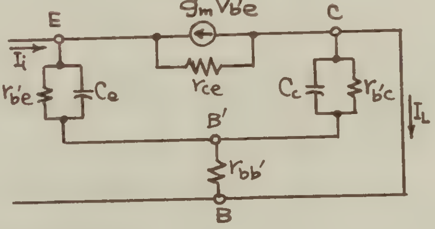
Thus, $1+x^2 \approx 2\alpha_o(1-\frac{1}{2}m^2x^2-mx^2)$ or $x^2 = \left(\dfrac{f_T}{f_\alpha}\right)^2 \approx \dfrac{2\alpha_o-1}{1+2\alpha_o m+\alpha_o m^2}$

or $f_T \approx f_\alpha\dfrac{(2\alpha_o-1)^{1/2}}{[1+2\alpha_o(m+\frac{m^2}{2})]^{1/2}}$. Now, since $\alpha_o \approx 1$, then $(2\alpha_o-1)^{1/2} \approx \alpha_o$

Hence, $f_T \approx \dfrac{\alpha_o f_\alpha}{[1+2\alpha_o(m+\frac{m^2}{2})]^{1/2}}$ QED

f) If $\alpha_o = 1$ and $m = 0.2$, then $f_T = \dfrac{1\times f_\alpha}{[1+2\times1\times(0.2+0.02)]^{1/2}} = \dfrac{f_\alpha}{(1.44)^{1/2}} = \dfrac{f_\alpha}{1.2}$

(13-17) a) The impedance of $C_c = 3\,pF$ is $-j100$ at 530 MHz which is far above the frequency range this circuit is used. Thus, C_c may be neglected. $r_{b'c} = 4M$ is in parallel with $r_{bb'} = 100\Omega$ and thus may be omitted. $r_{ce} = 80K$ is in parallel with $(r_{b'e}+r_{bb'}) = 1.1K$ and may also be neglected.



b) With C_c, $r_{b'c}$ and r_{ce} left out in the circuit of part $\underline{a}$, we have:

$A_{ib} = \dfrac{I_L}{I_i}$ where $I_L = -g_m V_{b'e}$ and $V_{b'e} = -(I_i+g_mV_{b'e})Z_e = -\dfrac{(I_i+g_mV_{b'e})}{g_{b'e}+j\omega C_e}$

or $V_{b'e} = \dfrac{-I_i}{g_{b'e}+g_m+j\omega C_e}$ $\therefore\ A_{ib} = \dfrac{g_m}{g_{b'e}+g_m+j\omega C_e}$ QED

Using Eqs. (13-39), $A_{ib} = \dfrac{h_{fe}/r_{b'e}}{\frac{1}{r_{b'e}}+\frac{h_{fe}}{r_{b'e}}+j\omega C_e} = \dfrac{h_{fe}}{1+h_{fe}+j\omega r_{b'e}C_e} = \dfrac{\frac{h_{fe}}{1+h_{fe}}}{1+j2\pi fC_e\frac{r_{b'e}}{1+h_{fe}}}$

(13-17) (Cont'd) or $A_{ib} = \frac{\alpha_o}{1+jf/f_\alpha}$ where $\alpha_o = \frac{h_{fe}}{1+h_{fe}}$ and $f_\alpha = \frac{1+h_{fe}}{2\pi r_{b'e} C_e}$

or $f_\alpha = \frac{1}{2\pi \frac{r_{b'e}}{h_{fe}}\left(\frac{h_{fe}}{1+h_{fe}}\right) C_e} = \frac{g_m}{2\pi C_e \alpha_o}$ QED

Also, $f_\alpha = \frac{\frac{1}{2\pi r_{b'e} C_e}}{\frac{1}{1+h_{fe}}} \approx \frac{\frac{1}{2\pi r_{b'e}(C_e+C_c)}}{1-\frac{h_{fe}}{1+h_{fe}}} = \frac{f_\beta}{1-\alpha_o}$ using Eq. (13-44).

(13-18) From Eqs. (13-51) & (13-52), $f_2 = \frac{1}{2\pi RC} = \frac{1}{2\pi C}\frac{R_s+h_{ie}}{(R_s+r_{bb'})r_{b'e}}$. From Eq. (13-53)
$|A_{Vso}| = \frac{h_{fe}R_L}{R_s+h_{ie}}$ $\therefore |A_{Vso}f_2| = \frac{1}{2\pi C}\frac{R_L}{R_s+r_{bb'}}\frac{h_{fe}}{r_{b'e}} = \frac{g_m}{2\pi C}\frac{R_L}{R_s+r_{bb'}}$ QED

From Eq. (13-45), $g_m \approx 2\pi C_e f_T$ and from Eq. (13-49), $C = C_e + C_c(1+g_mR_L)$
or $C \approx C_e + C_c g_m R_L = C_e + C_c(2\pi C_e f_T)R_L = C_e(1+2\pi f_T C_c R_L)$

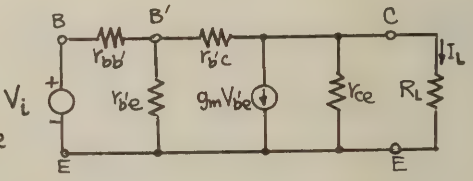
$\therefore |A_{Vso}f_2| = \frac{2\pi C_e f_T}{2\pi(1+2\pi f_T C_c R_L)C_e}\frac{R_L}{R_s+r_{bb'}} = \frac{f_T}{(1+2\pi f_T C_c R_L)}\frac{R_L}{R_s+r_{bb'}}$ QED

(13-19) From Eqs. (13-39), $h_{ie} = r_{bb'} + r_{b'e} = 1100$; $h_{fe} = g_m r_{b'e} = 50\times10^{-3}\times10^{3} = 50$
From Eq. (13-52), $R = \frac{(R_s+r_{bb'})r_{b'e}}{R_s+h_{ie}} = \left(\frac{1000+100}{1000+1100}\right)(1000) = 524\,\Omega$; From Eq. (13-49) : $C = C_e + C_c(1+g_mR_L) = 100+3(1+0.05R_L)$ pF.

a) If $R_L = 0$, $C = 103\times10^{-12}$ and $f_2 = \frac{1}{2\pi RC} = \frac{10^{12}}{2\pi(524)(103)} = 2.95$ MHz ←
$A_{Vs} = \frac{-h_{fe}R_L}{R_s+h_{ie}} = 0$ and $A_{Iso} = \frac{-h_{fe}R_s}{R_s+h_{ie}} = -\frac{50\times1K}{1K+1.1K} = -23.8$ ←

b) If $R_L = 1K$, $C = 100+3(1+0.05\times1000) = 253$ pF. Then $f_2 = \frac{10^{12}}{2\pi(524)(253)} = 1.2$ MHz
$A_{Vs} = \frac{-50\times1}{1+1.1} = -23.8$ and $A_{Iso} = -23.8$ ←

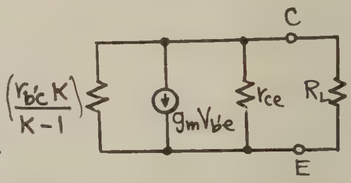
c) If $R_L = 2K$, $C = 100+3(1+0.05\times2000) = 403$ pF. Then $f_2 = \frac{10^{12}}{2\pi(524)(403)} = 755$ kHz
$A_{Vs} = \frac{-50\times2}{2.1} = -47.6$ and $A_{Iso} = -23.8$. Note that A_{Iso} is independent of R_L since Fig. 13-13 neglects r_{ce}. Thus, this is valid for $R_L \ll r_{ce}$.



(13-20) a) $K = \frac{V_{ce}}{V_{b'e}}$ where $V_{ce} = I_{sc}Z_{ce}$
with $R_L = 0$, $I_L = I_{sc} = V_{b'e}g_{b'c} - g_mV_{b'e} = (g_{b'c}-g_m)V_{b'e}$
Z_{ce} is the impedance seen between C-E.
Thus, $Z_{ce} = (r_{b'c}\|r_{ce}\|R_L)$ or $Z_{ce} = \frac{1}{g_{b'c}+g_{ce}+g_L}$

Hence, $\frac{V_{ce}}{V_{b'e}} = K = \frac{-g_m+g_{b'c}}{g_{b'c}+g_{ce}+g_L}$ QED

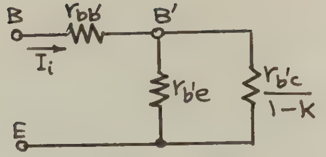
b) From the circuit we obtain :
$V_{ce} = -g_mV_{b'e}\frac{1}{g_L+g_{ce}+\frac{K-1}{K}g_{b'c}} = KV_{b'e} = \frac{-Kg_m}{K(g_L+g_{ce}+g_{b'c})-g_{b'c}}$



or $K(g_{ce}+g_{b'c}+g_L) - g_{b'c} = -g_m$ $\therefore K = \frac{-g_m+g_{b'c}}{g_{b'c}+g_{ce}+g_L}$ QED

(13-20) (Cont'd) c) $A_I = \frac{I_L}{I_i}$ where $I_L = g_L V_{ce} = g_L K V_{b'e}$

$= \frac{g_L K I_i}{g_{b'e} + (1-K) g_{b'c}}$



Hence, $A_I = \frac{g_L K}{g_{b'e} + g_{b'c} - K g_{b'c}}$

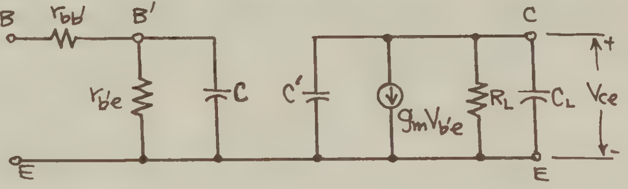
or $A_I = \frac{g_L}{(g_{b'c} + g_{b'e})/K - g_{b'c}}$ QED

d) From p. 370 $g_m >> g_{b'c}$ and $g_{b'e} >> g_{b'c}$. Thus from parts a and c we have: $K \approx \frac{-g_m}{g_{b'c} + g_{ce} + g_L}$ and $A_I \approx \frac{g_L K}{g_{b'e} - K g_{b'c}} = \frac{-g_m g_L}{g_{b'e}(g_{b'c} + g_{ce} + g_L) + g_m g_{b'c}}$

or $A_I \approx \frac{-g_m/g_{b'e}}{1 + \frac{1}{g_L}\left(g_{b'c} + g_{ce} + \frac{g_m}{g_{b'e}} g_{b'c}\right)} = \frac{-h_{fe}}{1 + h_{oe} R_L}$ using Eqs. (13-28) & (13-26).

QED

(13-21) Applying Miller's theorem we get:
where $C = C_e + C_c(1-K)$ and
$C' \approx C_c$.



a) $K = \frac{V_{ce}}{V_{b'e}} = \frac{1}{V_{b'e}}\left[\frac{-g_m V_{b'e} R_L}{1 + j\omega R_L (C_c + C_L)}\right]$

or $K = \frac{-g_m R_L}{1 + j\omega (C_c + C_L) R_L}$ QED

b) From the circuit we see that there are two time constants: $R_L(C_c + C_L)$, and $r_{b'e}[C_e + C_c(1-K)]$. If $R_L(C_c + C_L) >> \frac{1}{g_{b'e}}[C_e + C_c(1-K)] = r_{b'e}[C_e + C_c(1 + g_m R_L)]$ (since $K = K_o = -g_m R_L$), then f_2 is determined by the larger time constant. From part a we get: $f_2 = \frac{\omega_2}{2\pi} \approx \frac{1}{2\pi} \frac{1}{(C_c + C_L) R_L}$

(13-22) From Eq. (13-51), $f_2 = \frac{1}{2\pi RC}$ where $R = \frac{R_s + r_{bb'}}{R_s + h_{ie}} r_{b'e}$ ... Eq. (13-52), and $C = C_e + C_c(1 + g_m R_L)$ ... Eq. (13-49).

a) If $R_s = 0$, $f_2 = \frac{1}{2\pi} \frac{h_{ie}}{r_{bb'} r_{b'e}} \frac{1}{C_e + C_c(1 + g_m R_L)}$. Now find R_s such that $f_2' = \frac{1}{2} f_2$

$f_2' = \frac{1}{2\pi} \frac{R_s + h_{ie}}{(R_s + r_{bb'}) r_{b'e}} \frac{1}{C_e + C_c(1 + g_m R_L)} = \frac{1}{2}\left[\frac{1}{2\pi} \frac{h_{ie}}{r_{bb'} r_{b'e}} \frac{1}{C_e + C_c(1 + g_m R_L)}\right]$ where $h_{ie} = r_{bb'} + r_{b'e}$

or $\frac{R_s + h_{ie}}{(R_s + r_{bb'}) r_{b'e}} = \frac{1}{2} \frac{h_{ie}}{r_{bb'} r_{b'e}}$ $\therefore R_s = \frac{h_{ie} r_{bb'}}{h_{ie} - 2 r_{bb'}} = \frac{(r_{bb'} + r_{b'e}) r_{bb'}}{r_{b'e} - r_{bb'}} = \frac{1100 \times 100}{900} = 122\,\Omega$

←

b) If $R_s = \infty$, $f_2 = \frac{1}{2\pi} \frac{1}{r_{b'e}} \frac{1}{C_e + C_c(1 + g_m R_L)}$. To find R_s for $f_2' = 2 f_2$,

$\frac{f_2'}{f_2} = 2 = \frac{(R_s + h_{ie}) \cancel{r_{b'e}}}{(R_s + r_{bb'}) \cancel{r_{b'e}}}$ or $2R_s + 2r_{bb'} = R_s + h_{ie} = R_s + r_{bb'} + r_{b'e}$

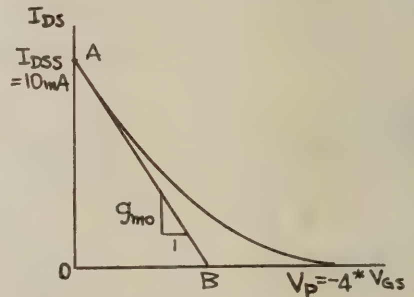
or $R_s = r_{b'e} - r_{bb'} = 1000 - 100 = 900\,\Omega$ ←

Note that the above values of R_s do not depend on R_L.

CHAPTER 14

(14-1) a) For a p-channel FET, Eq. (14-2) becomes : $V_P = \frac{eN_A}{2\epsilon} a^2$ where from Table 5-1 for Si : $\epsilon = 12\epsilon_o$ and $\mu_p = 500\ cm^2/V\text{-}sec$. For a p-type semiconductor $\sigma = \frac{1}{\rho} \approx p\mu_p e \approx N_A \mu_p e$ or $eN_A = \frac{1}{\rho\mu_p} = \frac{1}{10 \times 500}$ or $eN_A = 2 \times 10^{-4}$. Thus, $V_p = \frac{2 \times 10^{-4}(2 \times 10^{-4})^2}{2 \times 12(36\pi \times 10^{11})^{-1}} = 3.78\ V$ ←

b) For Ge $\epsilon = 16\epsilon_o$ and $\mu_p = 1800\ cm^2/V\text{-}sec$.
Hence, $V_P = 3.78 \frac{10 \times 12 \times 500}{2 \times 16 \times 1800} = 3.94\ V$ ←



(14-2) a) See plot

b) The magnitude of the slope of $\overline{AB}$ is

$g_{mo} = \frac{2I_{DSS}}{|V_P|}$ . . . Eq. (14-19)

Then from geometry, $\overline{OB} = \frac{I_{DSS}}{g_{mo}} = \frac{I_{DSS}}{\frac{2I_{DSS}}{|V_P|}} = \frac{|V_P|}{2}$ QED

(14-3) a) From Eqs. (14-18) and (14-19)

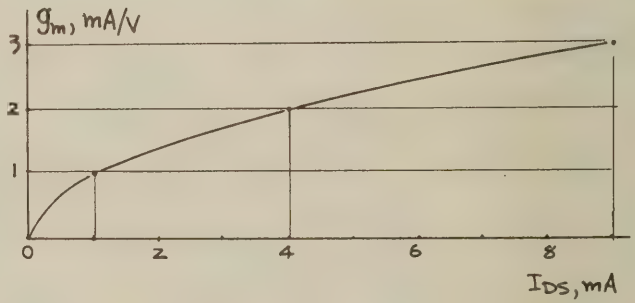
$$g_m = g_{mo}\left(1 - \frac{V_{GS}}{V_P}\right) = -\frac{2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right)$$

. From Eq. (14-8) we obtain : $\left(1 - \frac{V_{GS}}{V_P}\right) = \sqrt{\frac{I_{DS}}{I_{DSS}}}$. Thus, $g_m = \frac{-2I_{DSS}}{V_P}\sqrt{\frac{I_{DS}}{I_{DSS}}} = \frac{2}{|V_P|}\sqrt{I_{DSS} I_{DS}}$

b) $g_m = \frac{2}{4}\sqrt{4 \times 10^{-3} \times I_{DS} \times 10^{-3}} = \sqrt{I_{DS}}$ mA/V

where I_{DS} is in mA.

I_{DS}	g_m
0	0
1	1
4	2
9	3



(14-4) From Eq. (14-8), $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = I_{DSS}\left(1 - \frac{2V_{GS}}{V_P} + \frac{V_{GS}^2}{V_P^2}\right) \approx I_{DSS}\left(1 - \frac{2V_{GS}}{V_P}\right)$ provided that $V_{GS} \ll V_P$.

Thus, $I_D \approx I_{DSS} - \left(\frac{2I_{DSS}}{V_P}\right)V_{GS} = I_{DSS} + g_{mo}V_{GS}$ using Eq. (14-19) QED

(14-5) a) At $V_{DS} = 10V$ and $V_{GS} = -1.5V$, $r_d = \left.\frac{\Delta V_{DS}}{\Delta I_D}\right|_{V_{GS}} = \frac{25V}{0.6 \times 10^{-3}} \approx 40K$ ←

$g_m = \left.\frac{\Delta I_D}{\Delta V_{GS}}\right|_{V_{DS}} = \frac{0.66 - 0.41}{1} = 0.25$ mA/V $\therefore\ \mu = g_m r_d = 0.25 \times 40 = 10$ ←

b) The slope of the $V_{GS} = 0$ characteristic at the origin is
$r_d(ON) = \frac{4}{12}K = \frac{1}{3}K = 330\ \Omega$ ←

(14-6) Using Fig. 14-16b we have:

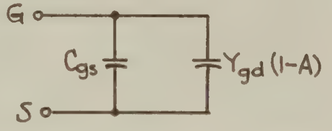
a) $A_V = \frac{V_o}{V_i}$ where $V_o = I_{sc} Z$; $I_{sc} = -g_m V_i + Y_{gd} V_i = (-g_m + Y_{gd}) V_i$

Z is the impedance seen at the output terminals with $V_i = 0$. Thus,

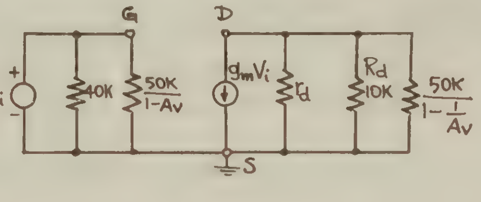
$Y = \frac{1}{Z} = Y_L + Y_{ds} + g_d + Y_{gd}$ and $\frac{V_o}{V_i} = \frac{-g_m + Y_{gd}}{Y_L + Y_{ds} + g_d + Y_{gd}}$ QED

b) Applying Miller's theorem to Y_{gd} in Fig. 14-16b we obtain the circuit shown for the input terminals. Thus, by inspection, we have

$$Y_i = Y_{gs} + (1 - A_V) Y_{gd}$$



(14-7) Applying Miller's theorem to the 50K resistor we obtain the equivalent circuit shown:



$A_V = \frac{-g_m}{\frac{1}{r_d} + \frac{1}{R_d} + \frac{(1-\frac{1}{A_V})}{R}}$ or $A_V\left[\frac{1}{r_d} + \frac{1}{R_d} + \frac{1}{R} - \frac{1}{RA_V}\right] = -g_m$

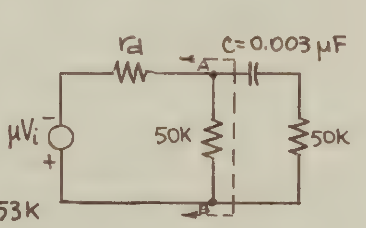
or $A_V\left(\frac{1}{r_d} + \frac{1}{R_d} + \frac{1}{R}\right) = -g_m + \frac{1}{R}$ or $A_V\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{50}\right) = -\frac{30}{5} + \frac{1}{50} = -\frac{299}{50}$

Thus, $A_V\left(\frac{16}{50}\right) = -\frac{299}{50}$ $\therefore$ $A_V = -\frac{299}{16} = -18.7$ ←

(14-8) From Prob. 14-7, $A_V = -18.7$. If the signal is impressed in series with the 40K resistor, $A_{Vs} = A_V \frac{R_i}{R_i + 40K}$ where $R_i = \frac{50K}{1+18.7} = 2.54K$

Hence, $A_{Vs} = (-18.7)\frac{2.54}{2.54+40} = -1.11$ ←

(14-9) The equivalent circuit seen at the drain is shown:



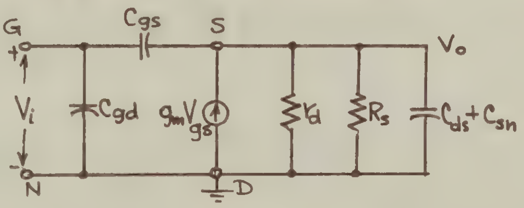
Then the Thévenin's equivalent to the left of AB consists of $R_{Th} = r_d \| 50K = 10K \| 50K = 8.33K$ and

$V_{Th} = \mu V_i \frac{50K}{50K+10K} = \frac{20 \times 50}{60} V_i = 16.67 V_i$; at 1kHz, $Z_c = -j53K$

Then, $\frac{V_o}{V_i} = \frac{-50K(16.67)}{50K+8.33K-j53K} = \frac{-833}{58.33-j53} = \frac{-833}{78.8}\angle 42.3^\circ = 10.6\angle -137.7^\circ$ ←

(14-10) a) $A_V = \frac{V_o}{V_i}$ where $V_o = I_{sc} Z$; $I_{sc} = (g_m + j\omega C_{gs}) V_i$

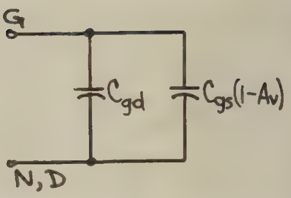
Z is the impedance seen at the output terminals with $V_i = 0$.



Thus, $Y = \frac{1}{Z} = \frac{1}{R_s} + g_d + j\omega(C_{gs} + C_{ds} + C_{sn}) + g_m$

and $A_V = \frac{I_{sc}}{V_i Y} = \frac{(g_m + j\omega C_{gs}) R_s}{1 + (g_m + g_d + j\omega C_T) R_s}$ where $C_T = C_{gs} + C_{ds} + C_{sn}$ QED

(14-10) (Cont'd) b) Applying Miller's theorem to C_{gs} we obtain the circuit shown for the input terminals.

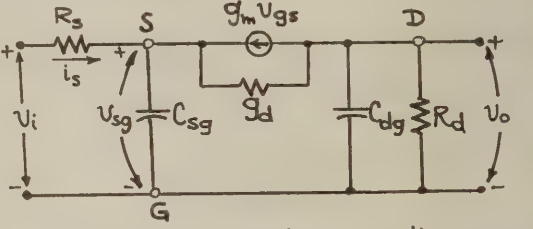


By inspection, $Y_i = j\omega C_{gd} + j\omega C_{gs}(1-A_V)$

c) The output admittance, Y_o, may be obtained from the circuit of part a by setting $V_i = 0$ and feeding a signal, V, at the output terminals. Note that $V_{gs} = -V$. Then the current drawn is $I = g_m V + (g_d + \frac{1}{R_s} + j\omega C_T)V$. Consider R_s external.

Hence, $Y_o = \frac{I}{V} = g_m + g_d + j\omega C_T$ QED.

(14-11) For a CG configuration Fig. 14-9 may be redrawn as shown:



a) $R_s = 0$, then $v_i = v_{sg} = -v_{gs}$.

$$A_V = \frac{v_o}{v_i} = \frac{v_o}{v_{sg}} \quad \text{where } v_o = i_{sc} Z$$

$i_{sc} = -g_m v_{gs} + g_d v_{sg} = (g_m + g_d)v_{sg}$ and Z = output impedance with $v_{sg} = 0$.

$$Y = \frac{1}{Z} = \frac{1}{R_d} + g_d + j\omega C_{dg} \qquad \therefore A_V = \frac{g_m + g_d}{\frac{1}{R_d} + g_d + j\omega C_{dg}} = \frac{(g_m + g_d)R_d}{1 + R_d(g_d + j\omega C_{dg})}$$

b) KCL at node S gives: $i_s = v_{sg}(j\omega C_{sg}) - g_m v_{gs} + g_d(v_{sg} - v_o)$

$$\therefore Y_i = \frac{i_s}{v_{sg}} = j\omega C_{sg} + g_m + g_d(1 - A_V) \qquad \text{QED}$$

c) $A_V' = \frac{v_o}{v_i}$. With $R_s \neq 0$, $v_{sg} = v_i - i_s R_s$ where $i_s = (g_m + g_d + j\omega C_{sg})v_{sg} - g_d v_o$

Thus, $v_{sg} = v_i - R_s(g_m + g_d + j\omega C_{sg})v_{sg} + R_s g_d v_o$ or $v_{sg} = \frac{v_i + R_s g_d v_o}{1 + (g_m + g_d + j\omega C_{sg})R_s}$

Now, KCL at node D gives: $g_m v_{gs} + v_o\left(\frac{1}{R_d} + g_d + j\omega C_{dg}\right) = g_d v_{sg}$

but $v_{gs} = -v_{sg}$, so $v_o\left(\frac{1}{R_d} + g_d + j\omega C_{dg}\right) = (g_m + g_d)v_{sg}$

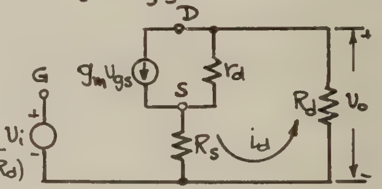
$$\text{or} \quad v_o\left(\frac{1}{R_d} + g_d + j\omega C_{dg}\right) = \frac{(g_m + g_d)(v_i + R_s g_d v_o)}{1 + (g_m + g_d + j\omega C_{sg})R_s} \qquad \text{or}$$

$$v_o\left[\frac{1}{R_d} + g_d + j\omega C_{dg} - \frac{(g_m + g_d)R_s g_d}{1 + (g_m + g_d + j\omega C_{sg})R_s}\right] = \frac{(g_m + g_d)v_i}{1 + (g_m + g_d + j\omega C_{sg})R_s}$$

$$\text{Hence, } A_V' = \frac{v_o}{v_i} = \frac{(g_m + g_d)}{\left(\frac{1}{R_d} + g_d + j\omega C_{dg}\right)\left[1 + (g_m + g_d + j\omega C_{sg})R_s\right] - (g_m + g_d)R_s g_d} \qquad \longleftarrow$$

$$\text{d) With } R_s \neq 0,\ Y_i' = \frac{1}{R_s + \frac{1}{Y_i}} = \frac{Y_i}{1 + R_s Y_i} = \frac{g_m + g_d(1 - A_V) + j\omega C_{sg}}{1 + R_s\left[g_m + g_d(1 - A_V) + j\omega C_{sg}\right]} \qquad \longleftarrow$$

(14-12) a) From the circuit we have: $v_o = -i_d R_d$ where



$i_d = g_m v_{gs} + g_d v_{ds}$; $v_{ds} = -i_d(R_s + R_d)$; $v_{gs} = v_i - i_d R_s$

or $i_d = g_m(v_i - i_d R_s) - g_d(R_s + R_d)i_d$ or $i_d = \frac{g_m v_i}{1 + g_m R_s + g_d(R_s + R_d)}$

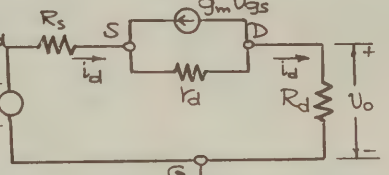
(14-12) (Cont'd) $A_V = \frac{v_o}{v_i} = -\frac{i_d R_d}{v_i} = \frac{-g_m R_d}{1+g_m R_s + g_d(R_s+R_d)} \; \dots$

Since $g_d = \frac{1}{r_d}$, then $A_V = \frac{-r_d g_m R_d}{r_d + (r_d g_m + 1)R_s + R_d} = \frac{-\mu R_d}{r_d + (\mu+1)R_s + R_d}$ } Eq.(14-30)

b) In the circuit of part a, set $v_i = 0$ and apply a voltage v across R_d. Consider R_d external to the amplifier and assume that the current drawn is i. Then, $R_o = \frac{v}{i}$. But from the circuit, we find that $i = g_m v_{gs} + (v - iR_s)g_d$ and $v_{gs} = -iR_s$. Thus, $i = -g_m R_s i + \frac{v}{r_d} - \frac{R_s}{r_d} i$ or $i\left(1 + g_m R_s + \frac{R_s}{r_d}\right) = \frac{v}{r_d}$ $\therefore R_o = r_d\left(1 + g_m R_s + \frac{R_s}{r_d}\right) = r_d + (\mu+1)R_s$

Now taking into account R_d we have: $R_o' = [r_d + (\mu+1)R_s] \| R_d$ …Eq.(14-31)

(14-13) a) $A_V = \frac{v_o}{v_s}$ where $v_o = i_d R_d$; $i_d = -g_m v_{gs} + [v_s - i_d(R_s+R_d)]g_d$



where $v_{gs} = -v_s + i_d R_s$. Thus,

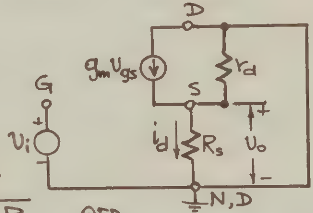
$i_d = g_m v_s - g_m R_s i_d + g_d v_s - (R_s+R_d) g_d i_d$

or $i_d = \frac{(g_m + g_d)v_s}{1 + g_m R_s + g_d(R_s+R_d)}$

$\therefore A_V = \frac{i_d R_d}{v_s} = \frac{(g_m+g_d)R_d}{1+g_m R_s + g_d(R_s+R_d)} = \frac{r_d(g_m + 1/r_d)R_d}{r_d + r_d g_m R_s + R_s + R_d} = \frac{(\mu+1)R_d}{r_d + (\mu+1)R_s + R_d}$

b) Eq.(14-33) gives the impedance seen looking between S and G, with $v_s = 0$. Consider R_s external to the amplifier and apply a voltage v between S and G. Assume that the current drawn is i. From the circuit, $i = -g_m v_{gs} + \left(\frac{v - iR_d}{r_d}\right)$ where $v_{gs} = -v$. Thus, $i\left(1 + \frac{R_d}{r_d}\right) = \left(g_m + \frac{1}{r_d}\right)v = \left(\frac{\mu+1}{r_d}\right)v$ $\therefore R_i = \frac{v}{i} = \frac{r_d + R_d}{\mu+1}$ and $R_i' = \left(\frac{r_d+R_d}{\mu+1}\right) \| R_s$.

(14-14) a) $A_V = \frac{v_o}{v_i}$ where $v_o = i_d R_s$, $i_d = g_m v_{gs} - g_d R_s i_d$ and $v_{gs} = v_i - i_d R_s$; Thus, $i_d = g_m v_i - (g_m + g_d)R_s i_d$



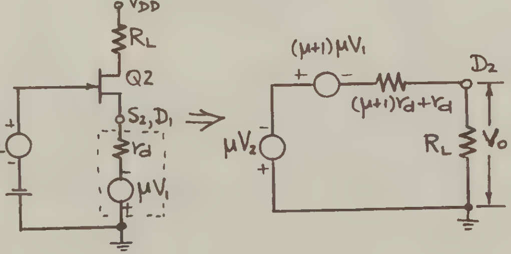
or $i_d = \frac{g_m v_i}{1 + (g_m+g_d)R_s}$ $\therefore A_V = \frac{g_m R_s}{1 + (g_m + g_d)R_s}$

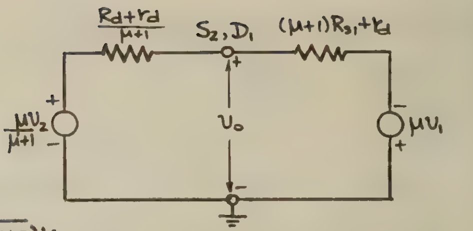
Since, $g_d = \frac{1}{r_d}$, then $A_V = \frac{r_d g_m R_s}{r_d + (r_d g_m + 1)R_s} = \frac{\mu R_s}{r_d + (\mu+1)R_s} = \frac{\frac{\mu R_s}{\mu+1}}{\frac{r_d}{\mu+1} + R_s}$ QED.

b) In the circuit of part a set $v_i = 0$ and apply a voltage v between S and N (D). Consider R_s external to the amplifier and assume that the current drawn is i. From the circuit $i = -g_m v_{gs} + g_d v = (g_m + g_d)v$ since $v_{gs} = -v$. Hence, $R_o = \frac{v}{i} = \frac{1}{g_m + g_d} = \frac{1}{g_m + 1/r_d} = \frac{r_d}{\mu+1}$ QED.

(14-15) From the small-signal equivalent circuit on the right we get:

$$V_o = \frac{-[(\mu+1)\mu V_1 + \mu V_2]R_L}{(\mu+2)r_d + R_L} \longleftarrow$$





(14-16) Using superposition we obtain:

$$v_o = \frac{\frac{R_d+r_d}{\mu+1}(-\mu v_1) + [(\mu+1)R_{s1}+r_d]\frac{\mu}{\mu+1}v_2}{\frac{R_d+r_d}{\mu+1} + (\mu+1)R_{s1}+r_d}$$

a) If $v_2=0$, $$\frac{v_o}{v_1} = \frac{-\mu(R_d+r_d)}{R_d+(\mu+1)^2R_{s1}+(\mu+2)r_d} = \frac{-20(11K)}{(1+21^2\times 0.5+22\times 10)K}$$
$$= \frac{-220}{441.5} = -0.49 \quad \longleftarrow$$

b) If $v_1=0$, $$\frac{v_o}{v_2} = \frac{\frac{\mu}{\mu+1}[(\mu+1)R_{s1}+r_d]}{\frac{R_d+r_d}{\mu+1}+(\mu+1)R_{s1}+r_d} = \frac{20\times 0.5+\frac{20\times 10}{21}}{\frac{11}{21}+21(0.5)+10} = \frac{19.53}{21.02} = 0.93 \quad \longleftarrow$$

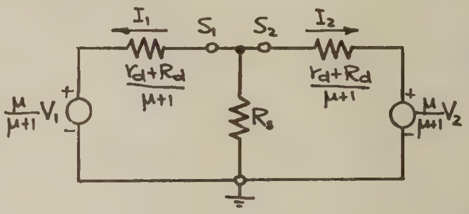
(14-17) a) The equivalent circuit of the given amplifier stage is that of Fig. 14-19 a with $v_i = 0 = v_s$. From this circuit we obtain

$$v_{DN} = -\frac{r_d+(\mu+1)R_s}{r_d+(\mu+1)R_s+R_d} v_a \quad \longleftarrow$$

b) Using Fig. 14-19 b with $v_i=0=v_s$ we obtain: $v_{SN} = -\frac{R_s v_a}{r_d+(\mu+1)R_s+R_d}$ $\longleftarrow$

c) (i) From Fig. 14-19 a with $v_i=0=v_a$, $v_{DN} = \frac{(\mu+1)R_d v_s}{r_d+(\mu+1)R_s+R_d}$ $\longleftarrow$

(ii) From Fig. 14-19 b with $v_i=0=v_a$, $v_{SN} = \frac{(r_d+R_d)v_s}{r_d+(\mu+1)R_s+R_d}$ $\longleftarrow$

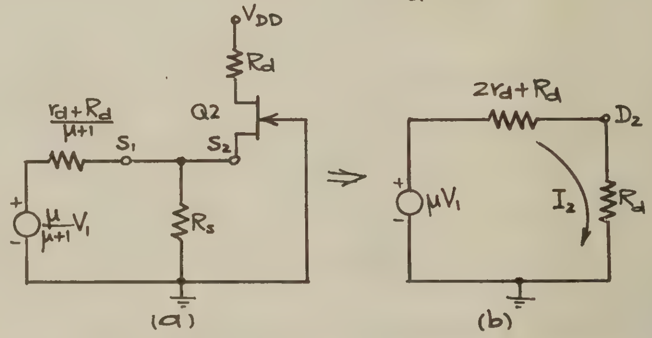


(14-18) a) The equivalent circuit seen looking into the sources of the FETs is as shown: If R_s is arbitrarily large, then the current through it can be neglected and $I_2=-I_1$

From the circuit $I_2 = \frac{\frac{\mu}{\mu+1}(V_1-V_2)}{2\frac{(r_d+R_d)}{\mu+1}} = \frac{\mu}{2}\frac{(V_1-V_2)}{(r_d+R_d)}$

Hence, $V_{o2} = I_2R_d = \frac{\mu R_d}{2(r_d+R_d)}(V_1-V_2) \equiv A_d(V_1-V_2)$ where $A_d = \frac{\mu R_d}{2(r_d+R_d)}$ $\longleftarrow$

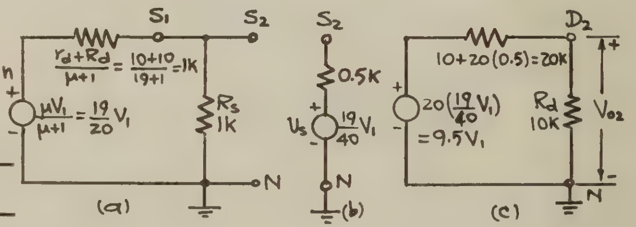
b) $V_{o1} = I_1R_d = -I_2R_d = -V_{o2}$ QED



(a) (b)

(14-19) Taking R_s arbitrarily large and looking into the drain of Q2 in (a), we obtain the circuit shown in (b). From this circuit we have:

$$I_2 = \frac{\mu V_1}{2(r_d+R_d)} \quad \longleftarrow$$

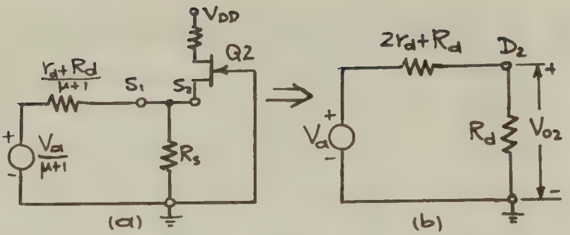

S_1
S_2
$\frac{r_d+R_d}{\mu+1} = \frac{10+10}{19+1} = 1K$
$\frac{\mu V_1}{\mu+1} = \frac{19}{20}V_1$
R_s 1K
N
S_2
0.5K
v_s
$\frac{19}{40}V_1$
N
D_2
10+20(0.5)=20K
$20(\frac{19}{40}V_1)$ =9.5V_1
R_d 10K
V_{o2}
N
+
−

(a) (b) (c)

(14-20) a) Looking into the source S_1 we see the equivalent circuit shown in (a). The Thévenin equivalent at S_2 is indicated in (b). Looking into D_2 we see the circuit in (c), from which $A = \frac{V_{o2}}{V_1} = \frac{9.5\times 10}{10+20} = 3.17$ $\longleftarrow$

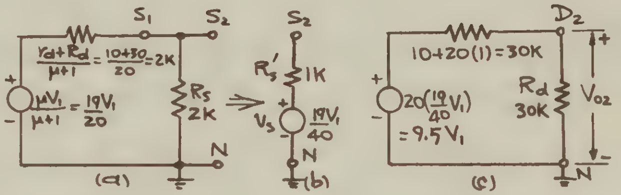
b) From (c), $R_o' = 10K \| 20K = \frac{20\times 10}{30} = 6.67K$ $\longleftarrow$

(14-21) a) Looking into S_1 we obtain the equivalent circuit shown in (a). Taking $R_s \rightarrow \infty$ and looking into D_2 we obtain the circuit in (b), from which $V_{o2} = \dfrac{R_d V_a}{2(r_d+R_d)}$ QED



b) From (b), $R_o = R_d \| (2r_d+R_d) = \dfrac{R_d(2r_d+R_d)}{2(r_d+R_d)}$ QED

(14-22) a) Looking into the source S_1 we see the equivalent circuit shown in (a). The Thévenin's equivalent at S_2 is indicated in (b). Looking into the drain of Q2 we see a resistance $r_d+(\mu+1)R_s' = 10+20\times1 = 30K$ and a generator $(\mu+1)v_s = 20(\frac{19V_1}{40}) = 9.5V_1$. The equivalent circuit is shown in (c), from which $A_V = \dfrac{V_{o2}}{V_1} = \dfrac{9.5\times30}{30+30} = 4.75$ ⟵



b) The output impedance between D_2 and N is $R_o = 30K\|30K = 15K$ ⟵

(14-23) a) From Eq. (14-8), $I_D = I_{DSS}(1-\frac{V_{GS}}{V_P})^2$ where from Fig. 14-20 $V_{GS} = -I_D R_s$ or $V_{GS} = -I_{DSS}R_s(1-\frac{V_{GS}}{V_P})^2 = -3\times0.47(1+\frac{V_{GS}}{2.4})^2 = -1.41(1+\frac{2V_{GS}}{2.4}+\frac{V_{GS}^2}{2.4^2})$. Solving, we get $V_{GS} = -0.7V$ ⟵

b) Substituting, we have $I_D = 3(1-\frac{0.7}{2.4})^2 = 3\times(\frac{1.7}{2.4})^2 = 1.5$ mA ⟵

c) $V_{DS} = V_{DD} - I_D(R_d+R_s) = 30 - 1.5\times12.47 = 11.25$ V ⟵

d) Since $r_d \gg R_d$, then $A_V = -g_m R_d$ where from Eqs (14-18) & (14-19)

$$g_m = -\frac{2I_{DSS}}{V_P}(1-\frac{V_{GS}}{V_P}) = \frac{2\times3\times10^{-3}}{2.4}(1-\frac{0.7}{2.4}) = 1770\ \mu\mho = 1.77\ mA/V$$

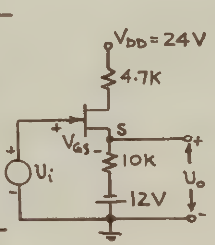
Hence; $A_V = -1.77\times12 = -21.2$ ⟵

(14-24) From the given circuit, $I_D = \dfrac{V_{DD}-V_{DN}}{R_d} = \dfrac{24-10}{56} = 0.25\ mA = I_{DSS}(1-\frac{V_{GS}}{V_P})^2 = 1(1+V_{GS})^2$

or $1+V_{GS} = (\frac{1}{4})^{1/2} = \frac{1}{2}$ $\therefore V_{GS} = -0.5$ V.

But $V_{GS} = -I_D R_1$ or $R_1 = -\dfrac{V_{GS}}{I_D} = \dfrac{0.5}{0.25\times10^{-3}} = 2K$ ⟵

(14-25) KVL in the G-S loop gives: $v_i = V_{GS} + 10i_d - 12$ or $i_d = \dfrac{v_i+12-V_{GS}}{10}$



a) If $v_i = 0$, $i_d = \dfrac{12-V_{GS}}{10} = I_{DSS}(1-\frac{V_{GS}}{V_P})^2 = 5.6(1+\frac{V_{GS}}{4})^2 = 5.6(1+\frac{V_{GS}}{2}+\frac{V_{GS}^2}{16})$

Solving, we obtain $V_{GS} = -2V$ and $i_d = 1.4$ mA

$\therefore V_o = 10i_d - 12 = 14 - 12 = 4$ V ⟵

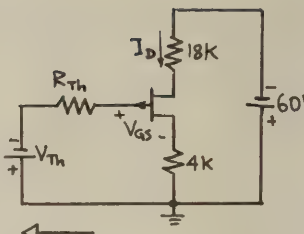
b) If $v_i = 10V$, $i_d = \dfrac{10+12-V_{GS}}{10} = 2.2 - \dfrac{V_{GS}}{10} = 5.6(1+\frac{V_{GS}}{2}+\frac{V_{GS}^2}{16})$ or $V_{GS} \approx -1.4V$, $i_d = 2.34$ mA

$\therefore v_o = 23.4 - 12 = 11.4$ V ⟵

c) If $v_o = 0$, $i_d = \frac{12V}{10K} = 1.2\ mA = 5.6(1+\frac{V_{GS}}{4})^2$ or $(1+\frac{V_{GS}}{4})^2 = \frac{1.2}{5.6} = 0.214$, $V_{GS} = -2.15V$

Then $v_i = V_{GS} = -2.15$ V ⟵

(14-26) If we find Thévenin's equivalent to the left of the gate of the given circuit, we obtain:



where $R_{Th} = 200K \| 1.3M = 173.5K$ and $-V_{Th} = \frac{200K}{1500K}(-60) = -8V$.
KVL in the G-S loop gives: $V_{GS} = -4I_D - 8$ or
$I_D = -\frac{8+V_{GS}}{4} = -2 - \frac{V_{GS}}{4} = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2 = -4\left(1-\frac{V_{GS}}{4}\right)^2$
Solving we get: $V_{GS} = 1V$ and $I_D = -2.25\ mA$ ⟵
Hence, $V_{DS} = -60 + (18+4)(2.25) = -60 + 49.5 = -10.5V$ ⟵

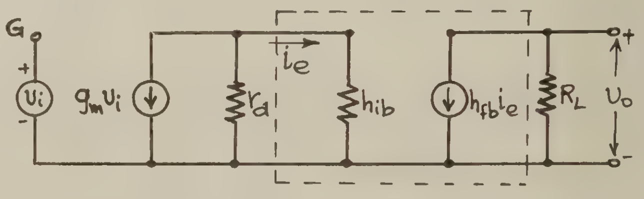
(14-27) a) Applying Eq. (14-18) to the upper curve we have: $g_m(min) = -\frac{2I_{DSS}(max)}{V_p(max)}\left(1-\frac{V'_{GS}}{V_p(max)}\right)$
where V'_{GS} corresponds to $I_D = I_{DSS}(min)$ in Eq. (14-8), or
$I_D = I_{DSS}(min) = I_{DSS}(max)\left(1-\frac{V'_{GS}}{V_p(max)}\right)$ from which $V'_{GS} = V_p(max)\left(1-\sqrt{\frac{I_{DSS}(min)}{I_{DSS}(max)}}\right)$
Substituting, we have: $g_m(min) = \frac{-2I_{DSS}(max)}{V_p(max)}\left(\cancel{1}-\cancel{1}+\sqrt{\frac{I_{DSS}(min)}{I_{DSS}(max)}}\right)$
or $V_p(max) = \frac{-2I_{DSS}(max)}{g_m(min)}\sqrt{\frac{I_{DSS}(min)}{I_{DSS}(max)}} = \frac{-2}{g_m(min)}\left[I_{DSS}(max)\,I_{DSS}(min)\right]^{1/2}$ QED

b) For the lower curve Eq. (14-19) becomes: $g_m(max) = \frac{-2I_{DSS}(min)}{V_p(min)}$
or $V_p(min) = \frac{-2I_{DSS}(min)}{g_m(max)}$ QED

c) $V_p(max) = \frac{-2}{1.5}(6\times 2)^{1/2} = -4.61\ V$; $V_p(min) = -\frac{2\times 2}{3} = -1.33V$ ⟵

(14-28) We are given that $I_D = 0.2(V_{GS}-3)^2$ where from the circuit we have: $V_{GS} = \frac{1}{2}(30-10I_D)$. Substituting and solving we obtain:
$I_D = 0.2(15-5I_D-3)^2 = 0.2(144-120I_D+25I_D^2)$ or $5I_D^2 - 25I_D + 28.8 = 0$
Then $I_D = \frac{25\pm\sqrt{625-576}}{10} = \frac{25\pm\sqrt{49}}{10}$ and $I_D = 1.8mA$ or $3.2mA$.
The second value of I_D is rejected, since $V_{GS} = \frac{1}{2}(30-32) = -1V$ and the FET will be cut off. ∴ $I_D = 1.8\ mA$ ⟵
Then $V_{GS} = \frac{1}{2}(30-18) = +6\ V$ and $V_{DS} = 30 - 1.8\times 10 = 12V$ ⟵

(14-29) From the equivalent circuit shown, we have:



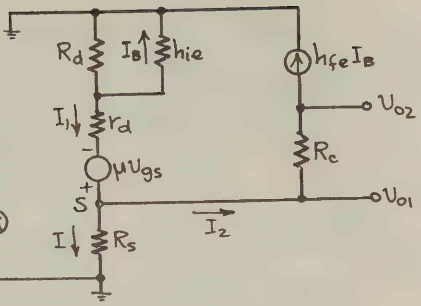
$v_o = -h_{fb} i_e R_L$ where

$i_e = -g_m v_i \frac{r_d}{r_d + hib} \simeq -g_m v_i$

since $r_d \gg hib$. Substituting, we obtain

$$A_v = \frac{v_o}{v_i} = -h_{fb}R_L(-g_m) = g_m h_{fb} R_L \qquad \text{QED}$$

(14-30) The small-signal equivalent circuit is as shown. The approximate equivalent circuit is used for Q2. From the circuit,



$v_{gs} = v_i - IR_s$ where $I = I_1 - I_2 = I_1 - h_{fe}I_B$.

Since $h_{ie} \ll R_d$, then $I_B \approx -I_1$, thus

$$I = (1+h_{fe})I_1$$

KVL in the FET loop gives:

$$[h_{ie} + r_d + R_s(1+h_{fe})]I_1 = \mu v_{gs} = \mu v_i - \mu(1+h_{fe})R_s I_1$$

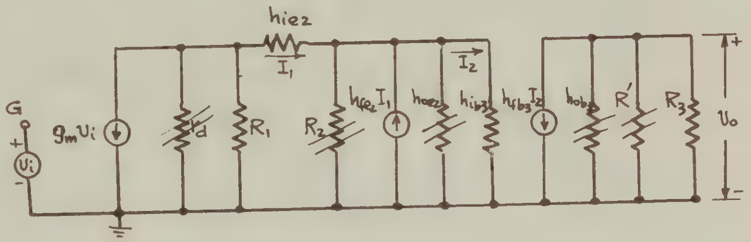
or $$I_1 = \frac{\mu v_i}{h_{ie} + r_d + (\mu+1)(1+h_{fe})R_s} \approx \frac{g_m v_i}{1 + g_m h_{fe} R_s}$$ since $r_d \gg h_{ie}$, $h_{fe} \gg 1$ and $\mu \gg 1$.

Then $v_{o1} = IR_s = (1+h_{fe})R_s I_1 \approx h_{fe}R_s I_1$ $\quad\therefore A_{v_1} = \frac{v_{o1}}{v_i} \approx \frac{g_m h_{fe} R_s}{1 + g_m h_{fe} R_s}$ QED

and $v_{o2} = v_{o1} - h_{fe}I_B R_c \approx (h_{fe}R_s + h_{fe}R_c)I_1 = h_{fe}(R_s + R_c)I_1$

Hence, $$A_{v_2} = \frac{v_{o2}}{v_i} \approx \frac{g_m h_{fe}(R_s+R_c)}{1 + g_m h_{fe} R_s}$$ QED

(14-31) The small-signal equivalent circuit of the amplifier is as shown. It is assumed that h_r is negligible for Q2 and Q3. Moreover, if $r_d \gg R_1$, $R_2 \gg h_{ib3}$, $\frac{1}{h_{oe2}} \gg h_{ib3}$, $\frac{1}{h_{ob3}} \gg R_3$ and $R' \gg R_3$, we can cross out these elements as shown.



Then, $v_o = -h_{fb3}I_2R_3$ where $I_2 = (1+h_{fe2})I_1$ and $I_1 = \frac{-g_m v_i R_1 - (1+h_{fe2})h_{ib3}I_1}{h_{ie2} + R_1}$

or $$I_1 = \frac{-g_m R_1 v_i}{R_1 + h_{ie2} + (1+h_{fe2})h_{ib3}} \qquad \therefore V_o = \frac{g_m(1+h_{fe2})R_1R_3h_{fb3}v_i}{R_1 + h_{ie2} + (1+h_{fe2})h_{ib3}}$$

Hence, $$A_o = \frac{v_o}{v_i} = g_m(1+h_{fe2})h_{fb3}\frac{R_1R_3}{R_1 + h_{ie2} + h_{ib3}(1+h_{fe2})}$$ QED

Note that A_o is negative, since h_{fb3} is a negative number. Thus, the feedback provided by R' is degenerative, stabilizing the amplifier.

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CHAPTER 15

(15-1) a) Eq. (15-3) is : $N(x,t) = N_o\left(1-\text{erf}\frac{x}{2\sqrt{Dt}}\right)$. Applying the boundary conditions, we have : BC 1 ... $N(0,t) = N_o(1-\text{erf}\,0) = N_o(1-0) = N_o$

and BC 2 ... $N(x,0) = N_o(1-\text{erf}\,\infty) = N_o(1-1) = 0$ for $x>0$ QED

b) From Eq. (15-2), $\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial x^2}$ where from Eq. (15-5), $N(x,t) = \frac{Q}{\sqrt{\pi Dt}}e^{-\frac{x^2}{4Dt}}$

Differentiating Eq. (15-5) wrt t, we obtain:

$$\frac{\partial N}{\partial t} = \left(\frac{Q}{\sqrt{\pi Dt}}e^{-\frac{x^2}{4Dt}}\right)\frac{x^2}{4Dt^2} + \left(Qe^{-\frac{x^2}{4Dt}}\right)\left(\frac{-\frac{1}{2}}{\sqrt{\pi Dt^3}}\right) = N(x,t)\left(\frac{x^2}{4Dt^2} - \frac{1}{2t}\right)$$

Differentiating Eq. (15-5) wrt x, we obtain:

$$\frac{\partial N}{\partial x} = \frac{Q}{\sqrt{\pi Dt}}e^{-\frac{x^2}{4Dt}}\left(\frac{-2x}{4Dt}\right) = \frac{-x}{2Dt}N(x,t)$$

$$D\frac{\partial^2 N}{\partial x^2} = -\frac{x}{2t}\left(\frac{-x}{2Dt}\right)N(x,t) - \frac{1}{2t}N(x,t) = N(x,t)\left(\frac{x^2}{4Dt^2} - \frac{1}{2t}\right) = \frac{\partial N}{\partial t} \qquad \text{QED}$$

Now checking the BC's :

BC 1 ... $\int_0^\infty N(x,t)\,dx = \frac{Q}{\sqrt{\pi Dt}}\int_0^\infty e^{-\frac{x^2}{4Dt}}dx = \frac{Q}{\sqrt{\pi Dt}}\left(\sqrt{\pi Dt}\right) = Q$ for $t>0$

BC 2 ... $N(x,0) = \frac{0}{0}$ (indeterminate!)

Let $t=y^2$, then $N(x,y) = \frac{Q}{\sqrt{\pi D}}\frac{1}{y}e^{-\frac{x^2}{4Dy^2}} = \frac{ay^{-1}}{e^{by^{-2}}}$. Now using L' Hospital's rule we have:

$$\lim_{y\to 0} N(x,y) = \lim_{y\to 0}\frac{-ay^{-2}}{e^{by^{-2}}(-2by^{-3})} = \lim_{y\to 0}\left(\frac{a}{2b}\frac{y}{e^{b/y^2}}\right) = \frac{0}{\infty} = 0 \qquad \text{QED}$$

(15-2) a) weight of 10^{15} P atoms/cm³ $= 10^{15}\frac{\text{atoms}}{\text{cm}^3} \times 31\frac{\text{AWU}}{\text{atom}} \times 1.672\times10^{-24}\frac{\text{g}}{\text{AWU}} = 5.2\times10^{-8}\frac{\text{g}}{\text{cm}^3}$

$\therefore$ % of P $= \frac{5.2\times10^{-8}}{2.33}\times100\% = 2.23\times10^{-6}\%$ ⟵

b) $\sigma = ne\mu_n \approx N_D e\mu_n = 10^{15}\times1.6\times10^{-19}\times1300 = 0.208 \quad \therefore \rho = \frac{1}{\sigma} = 4.8\ \Omega\text{-cm}$

c) Since $\sigma \approx (N_D - N_A)e\mu_n$, in order to halve σ we must add 0.5×10^{15} B atoms/cm³ ⟵

(15-3) a) From Fig. 15-8, the maximum solid solubility of arsenic in solid silicon is about 1.8×10^{21}/cm³. Hence, the maximum possible % concentration of arsenic is this value of solid solubility divided by the concentration of silicon, N, where

$$N = 6.02\times10^{23}\frac{\text{molecules}}{\text{mole}}\times\frac{1\text{ mole}}{28.1\text{ g}}\times\frac{2.33\text{ g}}{\text{cm}^3} = 5.0\times10^{22}\text{ molecules/cm}^3$$

Thus, % concentration $= \frac{1.8\times10^{21}}{5\times10^{22}}\times100\% = 3.6\%$ ⟵

b) For Au, % concentration $= \frac{1.2\times10^{17}}{5\times10^{22}}\times100\% = 2.4\times10^{-4}\%$ ⟵

(15-4) a) Using Eq. (15–5), we have: $N(0,t) = \dfrac{Q}{\sqrt{\pi D t}}$ and $N(25,t) = \dfrac{Q}{\sqrt{\pi D t}} e^{-\frac{(25\times10^{-4})^2}{4Dt}}$

Thus, $\dfrac{N(25,t)}{N(0,t)} = 0.9 = e^{-\frac{(25\times10^{-4})^2}{4Dt}}$ or $-\dfrac{(25\times10^{-4})^2}{4Dt} = \ln 0.9 = -0.105$

From Fig. 15-9 $D = 1.5\times10^{-11}\,\text{cm}^2/\text{sec}$ for P @ 1300°C

Hence, $t = \dfrac{(25\times10^{-4})^2}{4\times0.105\times1.5\times10^{-11}} = 9.9\times10^{5}\,\text{sec} = 275\,\text{hrs.}$ ←

b) Now $t = \dfrac{6.25\times10^{-6}}{4\times0.105\times1.5\times10^{-6}} = 9.9\,\text{sec}$ ←

c) Gold diffuses much faster than phosphorus. This indicates that it is practical to establish junctions by phosphorus (or boron) diffusions and then gold-dope without substantially moving the junctions.

(15-5) From Eq. (15-5), $N(x,t) = \dfrac{Q}{\sqrt{\pi D t}} e^{-\frac{x^2}{4Dt}}$. At $x = x_j$, $N = N_{BC} = \dfrac{Q}{\sqrt{\pi D t}} e^{-\frac{x_j^2}{4Dt}}$

or $\ln\left[\dfrac{N_{BC}\sqrt{\pi D t}}{Q}\right] = -\dfrac{x_j^2}{4Dt}$ or $x_j = \left(4Dt \ln \dfrac{Q}{N_{BC}\sqrt{\pi D t}}\right)^{1/2} = \left(2Dt \ln \dfrac{Q^2}{N_{BC}^2 \pi D t}\right)^{1/2}$

QED

(15-6) a) We have erfc distribution, thus $N(x,t) = N_o \,\text{erfc}\, \dfrac{x}{2\sqrt{Dt}}$

At $x = x_j = 2.7\,\mu$, $N(x,t) = N_{BC} = \dfrac{\sigma}{\mu_n e} = \dfrac{1/0.1}{1300\times1.6\times10^{-19}} = 4.8\times10^{16}/\text{cm}^3$

Thus, $\text{erfc}\,\dfrac{x_j}{2\sqrt{Dt}} = \dfrac{N_{BC}}{N_o} = \dfrac{4.8\times10^{16}}{4.8\times10^{18}} = 10^{-2}$. From Fig. 15-7 we get:

$\dfrac{x_j}{2\sqrt{Dt}} = 1.82 = \dfrac{2.7\times10^{-4}}{2\sqrt{Dt}}$ or $2\sqrt{Dt} = \dfrac{2.7\times10^{-4}}{1.82} = 1.48\times10^{-4}\,\text{cm} = 1.48\,\mu$

Hence, $N(x) = 4.8\times10^{18}\,\text{erfc}\,\dfrac{x}{1.48}$ where x is in μ. ←

b) From Fig. 15-9, D for B @ 1100°C is $\sim3.5\times10^{-13}\,\text{cm}^2/\text{sec}$. From part a

$Dt \doteq 0.55\times10^{-8}$ $\therefore\ t = \dfrac{0.55\times10^{-8}}{3.5\times10^{-13}} = 1.57\times10^{4}\,\text{sec} \approx 4$ hrs and 22 min. ←

c) At $x = 2\,\mu$, the boron concentration from part a is

$N(x) = 4.8\times10^{18}\,\text{erfc}\,\dfrac{2}{1.48} = 4.8\times10^{18}\,\text{erfc}\,1.35 = 2.8\times10^{17}/\text{cm}^3$

For P, $N_o = 10^{21}/\text{cm}^3$ and $N(2\mu) = 10^{21}\,\text{erfc}\,\dfrac{2}{2\sqrt{Dt}} = 2.8\times10^{17}$

$\therefore\ \text{erfc}\,\dfrac{1}{\sqrt{Dt}} = 2.8\times10^{-4}$ or from Fig. 15-7, $\dfrac{1}{\sqrt{Dt}} = 2.55$ and $2\sqrt{Dt} = 0.78\,\mu$

Hence, $N(x) = 10^{21}\,\text{erfc}\,\dfrac{x}{0.78}$ where x is in μ. ←

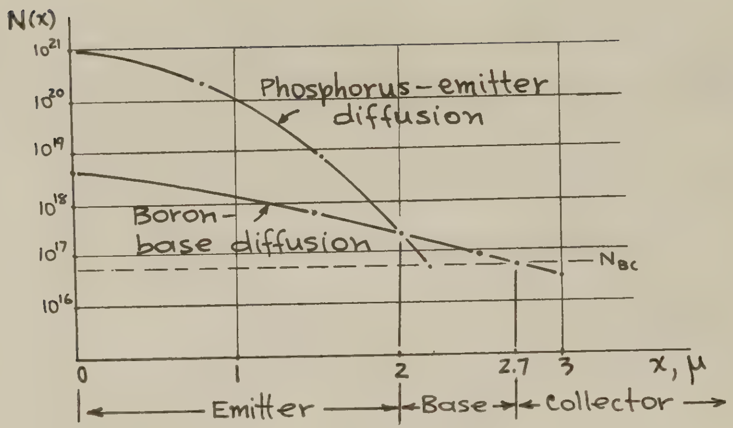
d) For part a

x	$\frac{x}{1.48}$	$\text{erfc}\,\frac{x}{1.48}$	$N(x)$
1.48	1	0.157	7.54×10^{17}
2.0	1.35	5.8×10^{-2}	2.8×10^{17}
2.5	1.69	1.8×10^{-2}	8.65×10^{16}
2.7	1.82	10^{-2}	4.8×10^{16}

For part c

x	$\frac{x}{0.78}$	$\text{erfc}\,\frac{x}{0.78}$	$N(x)$
0.78	1	0.16	1.6×10^{20}
1.5	1.92	8×10^{-3}	8×10^{18}
2	2.55	2.8×10^{-4}	2.8×10^{17}
2.2	2.8	7.5×10^{-5}	7.5×10^{16}

(15-6) (Cont'd)

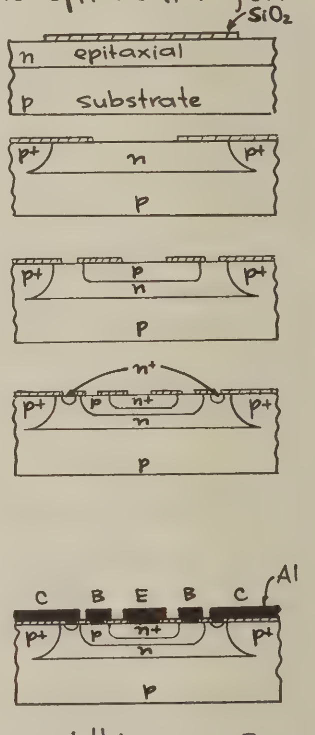


e) From part c $\sqrt{Dt} = \frac{1}{2.55}(10^{-4})$ or $Dt = \frac{10^{-8}}{6.5} = 1.54\times10^{-9}$

If $t = 30$ min $= 30\times60 = 1800$ sec, $D = \frac{1.54\times10^{-9}}{1800} = 8.55\times10^{-13}$ cm²/sec

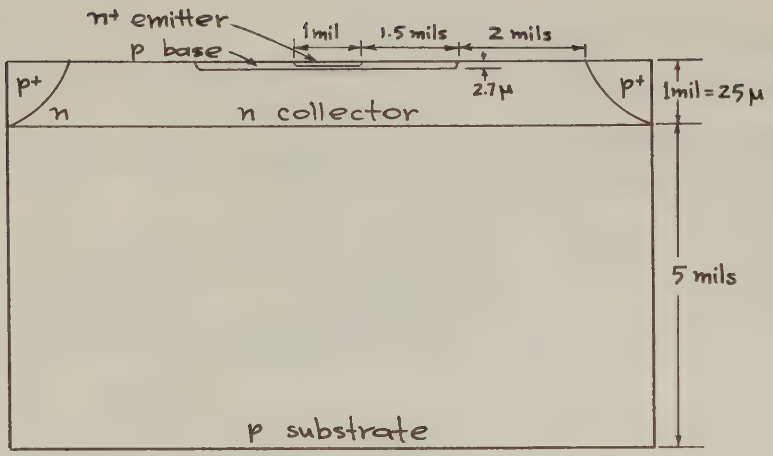
Now from Fig. 15-9, this value of D for P corresponds to a temperature of 1150°C ⟵

(15-7)
1. Start with a p-type Si wafer, 5-6 mils thick with resistivity of 10 Ω-cm.
2. Grow an epitaxial n-type layer (0.5 Ω-cm) 1 mil thick.
3. Grow a 5000 Å thick SiO_2 layer on the surface of the epitaxial layer.
4. Apply photoresist material, expose to the isolation region pattern, develop, etch SiO_2 and strip photoresist.
5. Diffuse p+ impurity (Boron)
6. Grow SiO_2
7. Photoresist etc. (as in step 4) to define the base region.
8. Diffuse p impurity (Boron) into base region.
9. Grow SiO_2
10. Photoresist etc. to define emitter stripe and collector contacts.
11. Diffuse n+ impurity
12. Grow SiO_2
13. Photoresist etc. to define "windows" where contacts to Si are to be made.
14. Vacuum evaporate Al over entire surface.
15. Photoresist etc. to define connection pattern, remove undesired Al.
16. Test
17. Scribe and break
18. Bond chip onto header
19. Bond wires from chip to header leads.
20. Encapsulate.



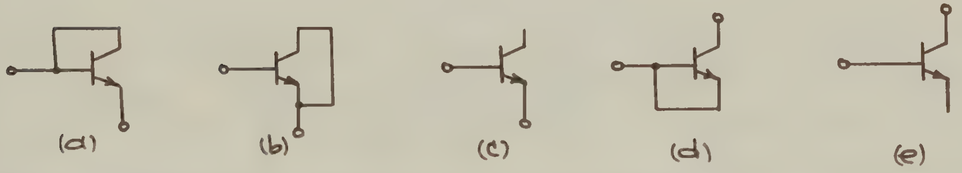
Monolithic npn Si transistor. Note that the p-substrate is not drawn to scale!

(15-8)



The emitter width is 2 μ and the base width is 0.7 μ.

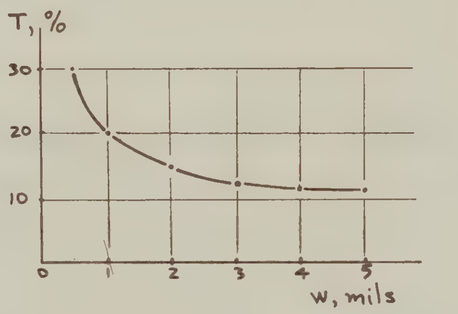
(15-9)



Connection (a) has the lowest forward voltage drop, since the transistor operates in its active region and thus the base current is very small resulting in a small base-to-emitter voltage.
Connection (e) has the highest breakdown voltage, equal to BV_{CBO} of the transistor. Refer to Sec. 9-18.

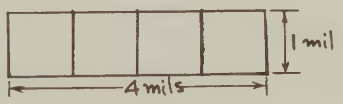
(15-10) From Eq. (15-7), we have $R = R_s \frac{l}{w}$. Hence, tolerance, $T = \pm 10\% \pm \frac{0.1}{w}(100\%)$ where w is in mils. or $T = \pm 10\% \pm \frac{10}{w}\%$

W	0.5	1	2	3	4	5
T	30	20	15	13.3	12.5	12



(15-11) From Eq. (15-6), $R_s = \frac{\rho}{y}\ \Omega/\square$ where $\rho = \frac{1}{\sigma} = \frac{1}{(N_D - N_A)\, e\, \mu_n} = \frac{10^{19}}{(10-2)10^{15} \times 1.6 \times 1300}$
or $\rho = 0.6\ \Omega\text{-cm}$. Hence, $R_s = \frac{0.6\ \Omega\text{-cm}}{25 \times 10^{-4}\text{cm}} = 240\ \Omega/\square$ ⟵

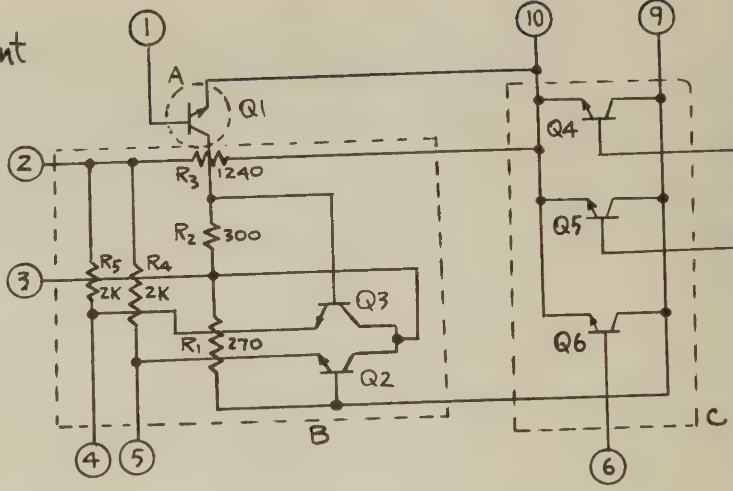
(15-12) a) Since the crossover forms 4 squares, its resistance is
$R = 2.2\ \Omega/\square \times 4\square = 8.8\ \Omega$ ⟵



b) For Al, $\rho = 2.8 \times 10^{-6}\ \Omega\text{-cm}$ ∴ $R_s = \frac{2.8 \times 10^{-6}}{0.5 \times 10^{-4}}$
or $R_s = 5.6 \times 10^{-2}\ \Omega/\square$
Hence, $R = 5.6 \times 10^{-2}\ \Omega/\square \times 4\square = 0.224\ \Omega$ ⟵

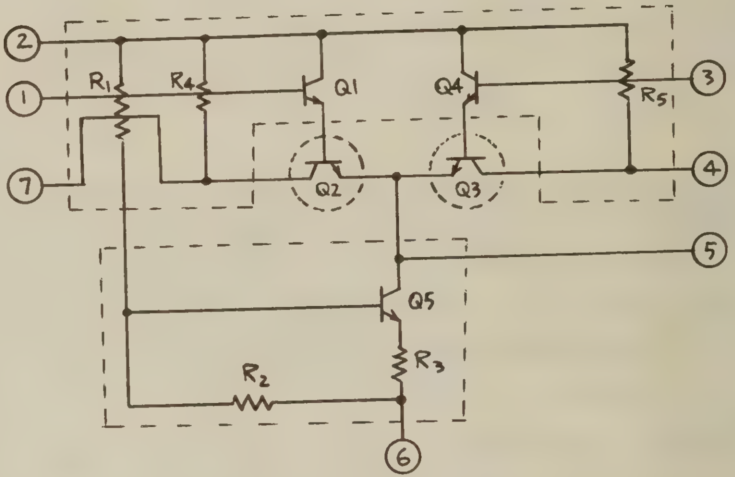
(15-13) a) There are 3 independent collectors. Hence, the minimum number of isolation regions is 3, A, B and C as shown.

b) See sketch.



(15-14) a) The minimum number of isolation regions is 4. 1) Q1 and Q4 with R_1, R_4 and R_5 2) Q2 3) Q3 4) Q5 with R_2 & R_3.

b) See sketch

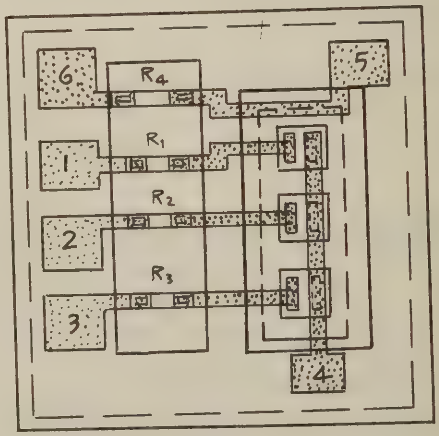


(15-15) a) The minimum number of isolation regions is 3: One containing Q1, one containing Q2 and one containing both R_1 and R_2.

b) The maximum number of isolation regions is 4, or one per component.

(15-16) a) The minimum number of isolation regions is two - one for Q1, Q2 and Q3 and one for R_1 thru R_4.

b) See sketch.



(15-17) From Eq. (6-47) in MKS units the capacitor thickness is

$$W = \left(\frac{2\epsilon V_B}{eN_A}\right)^{1/2} = \left(\frac{2\times 12\times \frac{1}{36\pi\times 109}\times 1}{1.6\times 10^{-19}\times 10^{21}}\right)^{1/2} = 1.15\times 10^{-6}\,m = 1.15\ \mu$$

From Eq. (6-49), $C = \frac{\epsilon A}{W} = \frac{12}{36\pi\times 10^{9}}\,\frac{1000\,(2.54\times 10^{-3}\times 10^{-2})^2}{1.15\times 10^{-6}} = 59.5\ pF \quad \longleftarrow$

(15-18) $W = \frac{\epsilon A}{C} = \frac{3.5}{36\pi\times 10^{9}}\,\frac{(2.54\times 10^{-3}\times 10^{-2})^2}{0.4\times 10^{-12}} = 5\times 10^{-8}\,m = 500\ \text{Å} \quad \longleftarrow$

which agrees with the value quoted in the text.

(15-19) Bottom components (Step junction):

$\sigma = \frac{1}{\rho} = \frac{1}{10} = N_A e \mu_p$ or $N_A = \frac{1}{10\times 1.6\times 10^{-19}\times 500} = 1.25\times 10^{15}\ /cm^3$

From Eq. (6-47), $W = \left(\frac{2\epsilon V_B}{eN_A}\right)^{1/2} = \left(\frac{2\times 12 \times 5}{36\pi\times 10^{9}\times 1.6\times 10^{-19}\times 1.25\times 10^{21}}\right)^{1/2} = 2.3\times 10^{-6}\,m$

From Eq. (6-49), $C = \frac{\epsilon A}{W} = \frac{12\times 1}{36\pi\times 10^{9}}\cdot\frac{(2.54\times 10^{-5})^2}{2.3\times 10^{-6}} = 0.0297\ pF/mil^2$

Sidewall components : 0.1 pF/mil²

Hence, $C_{Ts} = (0.0297\,\frac{pF}{mil^2})\times(8\times 6\ mils^2) + (0.1\,\frac{pF}{mil^2})\times(28\times 1\ mils^2) = 4.22\ pF \quad \longleftarrow$

(15-20) c

(15-21) a

(15-22) b

(15-23) c

(15-24) b

(15-25) a

(15-26) a

(15-27) b

(15-28) d

(15-29) c

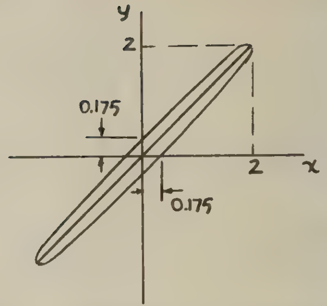
(15-30) a

(15-31) c

(15-32) a

(15-33) c

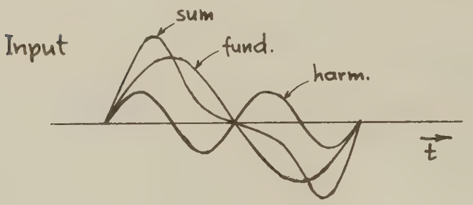
CHAPTER 16

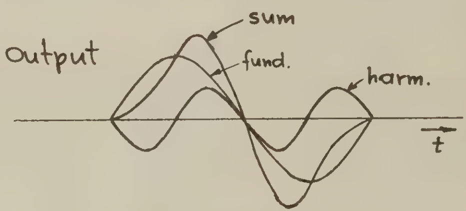


(16-1) a) $x = 2\sin\omega t$, $y = 2\sin(\omega t + 5°)$. The plot of y vs. x is a very narrow ellipse whose major axis is at 45°. For $\omega t = 0$ or π, $x = 0$, $y = \pm 2\sin 5° = \pm 0.175$. Similarly, for $y=0$, $x = \pm 0.175$. If there is zero phase shift, $y = x$, which is a straight line inclined at 45°.

b) If $x = 2\sin(\omega t + \theta)$ and $y = 2\sin(\omega t + \theta)$, then $y = x$, which is equivalent to zero phase shift. The effect of unequal phase shifts in the two amplifiers is to spread a line out into a "loop".

(16-2)





(16-3) From Fig. 16-2a we have: $V_o = \dfrac{\frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} V_i = \dfrac{1}{1 + j\omega R_2 C_2} V_i$

Hence, $A_2 = \dfrac{V_o}{V_i} = \dfrac{1}{1 + j\omega R_2 C_2}$ (1) $\quad |A_2| = \dfrac{1}{(1 + \omega^2 R_2^2 C_2^2)^{1/2}}$ (2)

At $f = f_2$, $|A_2| = \dfrac{1}{\sqrt{2}} = \dfrac{1}{(1 + \omega_2^2 R_2^2 C_2^2)^{1/2}}$ $\quad \therefore \omega_2 R_2 C_2 = 1$ or $f_2 = \dfrac{1}{2\pi R_2 C_2}$ QED

Substituting f_2 into (2) we obtain, $|A_2| = \dfrac{1}{\sqrt{1 + (f/f_2)^2}}$

From (1), $A_2 = |A_2| \angle{-\theta}$ where $\theta = \tan^{-1}(\omega R_2 C_2) = \tan^{-1}\left(\dfrac{\omega}{\omega_2}\right) = \tan^{-1}\left(\dfrac{f}{f_2}\right)$ QED

(16-4) At low frequencies we have: $\left|\dfrac{A_1}{A_o}\right| = \dfrac{1}{\sqrt{1 + (f_1/f)^2}}$. At $f = 60$ Hz, $\left|\dfrac{A_1}{A_o}\right| = 0.9$

or $\dfrac{1}{1 + (f_1/f)^2} = 0.81$, $f_1/60 = 0.484$ $\quad \therefore f_1 = 29$ Hz $= \dfrac{1}{2\pi R' C_b}$ ... Eq. (16-3)

Thus, $C_b = \dfrac{1}{2\pi \times 29 R'} = \dfrac{1}{182} \dfrac{1}{R'} = \dfrac{5.5 \times 10^{-6}}{R'} = \dfrac{5.5}{R'}$ μF if R' is in K.

(16-5) a) From Table 7-1 (p. 166), $r_p = 62K$, $g_m = 1.6$ mA/V. From Eq. (16-7), $f_1 = \dfrac{1}{2\pi(R_o' + R_i')C_b}$

where $R_o' = r_p \| R_p^* = \dfrac{62 \times 15}{62 + 15}$ K $= 12.1$K $\quad \therefore f_1 = \dfrac{1}{2\pi \times 0.02 \times 1.012} = 7.86$ Hz $\leftarrow$

b) From Eq. (16-8), $f_2 = \dfrac{1}{2\pi RC}$ where $\dfrac{1}{R} = \dfrac{1}{r_p} + \dfrac{1}{R_p} + \dfrac{1}{R_g} = \dfrac{1}{62K} + \dfrac{1}{15K} + \dfrac{1}{1000K} = 8.38 \times 10^{-5}$

$\therefore f_2 = \dfrac{8.38 \times 10^{-5}}{2\pi \times 50 \times 10^{-12}} = 2.67 \times 10^{5}$ Hz $= 267$ kHz $\leftarrow$

c) From Eq. (16-9), $|A_o| = g_m R = 1.6 \times 10^{-3} / 8.38 \times 10^{-5} = 19.1$. Then $N = 20\log_{10} 19.1 = 25.6$ dB

d) From Eq. (16-4), $\theta_1 = -\tan^{-1}(f_1/f) = -\tan^{-1}(7.86/20) = -21.45°$ $\leftarrow$

e) From Eq. (16-5), $\theta_2 = \tan^{-1}(f/f_2) = \tan^{-1}(200/267) = 36.8°$ $\leftarrow$

(16-6) a) From Eq. (16-9), $|A_o| = g_m R$ where $\frac{1}{R} = \frac{1}{r_d} + \frac{1}{R_d} + \frac{1}{R_g} = \frac{1}{44K} + \frac{1}{50K} + \frac{1}{1000K} = 4.37\times10^{-5}$

$\therefore |A_o| = \frac{\mu}{r_d}\cdot R = \left(\frac{70}{44\times10^3}\right)\frac{1}{4.37\times10^{-5}} = 36.4$. Thon, $N = 20\log_{10}36.4 = 31.2$ dB

b) From Eq. (16-8), $f_2 = \frac{1}{2\pi RC} = \frac{4.37\times10^{-5}}{2\pi\times10^{-10}} = 69.5$ kHz

c) From Eq. (16-7), $C_b = \frac{1}{2\pi f_1(R_o' + R_i')}$ where $R_o' = \frac{r_d R_d}{r_d + R_d} = \frac{44\times50}{44+50} = 23.4$K

$\therefore C_b = 1/[2\pi\times50\times(23.4+1000)\times10^3] = 0.0031\ \mu F$ ←

(16-7) We have that $N = 20\log_{10}\left|\frac{A_1}{A_o}\right| = -1$ dB where $\left|\frac{A_1}{A_o}\right| = \frac{1}{[1+(f_1/f)^2]^{1/2}}$

$\therefore 20\log_{10}[1+(f_1/f)^2]^{1/2} = +1 = 10\log_{10}[1+(f_1/f)^2]$; $1+(f_1/f)^2 = 1.26$

or $(f_1/f) = 0.509$ and $f = \frac{20}{0.509} = 39.3$ Hz.

Proceeding as above but with A_1 replaced by A_2 and (f_1/f) by (f/f_2), we obtain : $f/f_2 = 0.509$ or $f = 0.509\times20$ kHz $= 10.2$ kHz.

Thus, the 1 dB range is from 39.3 Hz to 10.2 kHz. ←

(16-8) At low frequencies, $\left|\frac{A_1}{A_o}\right| = \frac{1}{[1+(f_1/f)^2]^{1/2}} = \frac{1}{(1+0.1^2)^{1/2}} = \frac{1}{(1.01)^{1/2}} = \frac{1}{1.005} = 0.995$

At high frequencies, $\left|\frac{A_2}{A_o}\right| = \frac{1}{[1+(f/f_2)^2]^{1/2}} = \frac{1}{(1+0.1^2)^{1/2}} = 0.995$. Hence, the gain is constant within 0.5% in the frequency range $10f_1 \leq f \leq 0.1f_2$. QED

From Eq. (16-4), $\theta_1 = -\tan^{-1}(\frac{f_1}{f}) = -\tan^{-1}(0.1) \approx -0.1$ rad since for small angles $\tan\theta \approx \theta$. Similarly, $\theta_2 = \tan^{-1}(\frac{f}{f_2}) = \tan^{-1}(0.1) \approx 0.1$ rad. Hence, the phase shift is constant to within 0.1 rad $= 5.73°$ for $10f_1 \leq f \leq 0.1f_2$. QED

(16-9) a) From Eq. (16-15), $f_2 = \frac{1}{2\pi RC}$ where $C = C_e + C_c(1+g_m R_L)$ and $R = \frac{(R_c + r_{bb'})r_{b'e}}{R_c + h_{ie}}$

Thus, $f_2 = \frac{1}{2\pi}\frac{R_c + h_{ie}}{(R_c + r_{bb'})r_{b'e}}\frac{1}{C_e + C_c\left[1 + \frac{g_m R_c h_{ie}}{R_c + h_{ie}}\right]}$; f_2 will be maximum when R and C attain their minimum values. We note from above that R is minimum when $R_c = 0$. So is C. Thus, $f_2(\max) = \frac{1}{2\pi}\frac{h_{ie}}{r_{bb'}r_{b'e}(C_e + C_c)}$

From Eqs. (13-39) and (13-45), $r_{b'e} = \frac{h_{fe}}{g_m}$ and $f_T \approx \frac{g_m}{2\pi(C_e + C_c)}$.

Thus, $(f_2)_{\max} = \frac{g_m h_{ie}}{2\pi(C_c + C_e)r_{bb'}h_{fe}} = \frac{f_T h_{ie}}{r_{bb'}h_{fe}}$ ... Eq. (16-19)

b) From p. 370, $r_{bb'} = 100\Omega$, $h_{ie} = r_{bb'} + r_{b'e} = 1100\Omega$, $g_m = 50$ mA/V, $C_e + C_c = 103$ pF

$h_{fe} = g_m r_{b'e} = 50$

Hence, $(f_2)_{\max} = \frac{50\times1.1}{2\pi(103\times10^{-12})(100\times50)} = 17$ MHz ←

(16-10) From Eq. (16-11), $|A_o| = \frac{h_{fe}R_c}{R_c + h_{ie}} = 15 = \frac{50R_c}{R_c + 1.1K}$ or $15R_c + 1.1\times15 = 50R_c$

or $35R_c = 16.5$ $\therefore R_c = 0.472$ K $= 472\ \Omega$

From Eq. (16-14), $R = \frac{(472+100)1000}{472+1100} = 364\ \Omega$; From Eq. (16-12), $R_L = \frac{472\times1100}{472+1100} = 330\ \Omega$

(16-10) (Cont'd) Then, from Eq. (16-13), $C = 100 + 3(1 + 0.05 \times 330) = 153$ pF

Using Eq. (16-14), $f_2 = \dfrac{10^{12}}{2\pi(364)(153)} = 2.87$ MHz ⟵

(16-11) a) From Eq. (13-27), $g_m \approx \dfrac{|I_E|}{26} = \dfrac{10}{26} = 0.385$. Then $f_T = \dfrac{g_m}{2\pi(C_e + C_c)} = \dfrac{0.385 \times 10^{12}}{2\pi(85 + 1.5)}$

or $f_T = 710$ MHz ⟵

b) From Eq. (16-17), $(R_c)_{opt} = \dfrac{h_{ie}}{\sqrt{x} - 1}$ where $x = \dfrac{h_{fe} C_c}{C_e + C_c} \dfrac{h_{ie}}{r_{bb'}}$; $h_{fe} = g_m r_{b'e} = 38.5$

$h_{ie} = r_{bb'} + r_{b'e} = 180\ \Omega$. $\therefore x = \dfrac{(38.5 \times 1.5)(180)}{86.5 \times 80} = 1.51$. Substituting, we get:

$(R_c)_{opt} = \dfrac{180}{\sqrt{1.51} - 1} = 787\ \Omega$ ⟵

From Eq. (16-12), $R_L = \dfrac{787 \times 180}{787 + 180} = 146.5\ \Omega$. Then, from Eq. (16-16),

$|A_o f_2| = \dfrac{f_T}{(1 + 2\pi f_T C_c R_L)} \dfrac{R_c}{R_c + r_{bb'}} = \dfrac{710 \times 10^6 \times 787}{[1 + 2\pi(710 \times 10^6)(1.5 \times 10^{-12})(146.5)]} \dfrac{1}{(787 + 80)} = 326$ MHz ⟵

c) From Eq. (16-11), $10 = \dfrac{38.5 R_c}{R_c + 180}$ or $10R_c + 1800 = 38.5 R_c$ $\therefore R_c = 63.2\ \Omega$

Then $R_L = \dfrac{63.2 \times 180}{63.2 + 180} = 46.7\ \Omega$ and $C = 85 + 1.5(1 + 0.385 \times 46.7) = 113.5$ pF

From Eq. (16-14), $R = \dfrac{(63.2 + 80)(100)}{63.2 + 180} = 58.9\ \Omega$ $\therefore f_2 = \dfrac{10^{12}}{2\pi(58.9)(113.5)} = 23.9$ MHz ⟵

d) From Eq. (16-12), $R_L = \dfrac{2000 \times 180}{2180} = 165\ \Omega$. Then, $C = 85 + 1.5(1 + 0.385 \times 165) = 182$ pF

$R = \dfrac{(2000 + 80)(100)}{2180} = 95.5\ \Omega$ $\therefore f_2 = \dfrac{10^{12}}{2\pi(95.5)(182)} = 9.16$ MHz ⟵

e) From Eq. (16-19), $(f_2)_{max} = \dfrac{f_T h_{ie}}{h_{fe} r_{bb'}} = \dfrac{710 \times 180}{38.5 \times 80} = 41.5$ MHz ⟵

(16-12) From Eq. (16-21), $t_r = \dfrac{0.35}{f_2} = 20 \times 10^{-9}$ or $f_2 = \dfrac{0.35}{2 \times 10^{-8}} = 17.5$ MHz. Then, from Eq. (16-15), $RC = \dfrac{1}{2\pi f_2} = \dfrac{10^{-6}}{2\pi(17.5)} = 0.91 \times 10^{-8}$. Using Eqs. (16-13) & (16-14) and the values from Prob. 16-11, we have: $\left[\dfrac{100(R_c + 80)}{R_c + 180}\right] \cdot \left(86.5 + \dfrac{104 R_c}{R_c + 180}\right) = 9.1 \times 10^{+3}$

Solving, we obtain $R_c = 140\ \Omega$ ⟵

Then, from Eq. (16-11), $A_o = \dfrac{-h_{fe} R_c}{R_c + h_{ie}} = -\dfrac{38.5 \times 140}{140 + 180} = -16.85$ ⟵

(16-13) a) From Eq. (16-17), $(R_c)_{opt} = \dfrac{h_{ie}}{\sqrt{x} - 1}$ where $h_{ie} = r_{bb'} + \dfrac{1}{g_{b'e}} = 45 + \dfrac{10^3}{0.39} = 2605\ \Omega$

$h_{fe} = \dfrac{g_m}{g_{b'e}} = \dfrac{54}{0.39} = 138$

and $x = \dfrac{h_{fe} C_c}{C_e + C_c} \dfrac{h_{ie}}{r_{bb'}} = \dfrac{138 \times 3.5}{783.5} \dfrac{2605}{45} = 35.6$

$\therefore (R_c)_{opt} = \dfrac{2605}{\sqrt{35.6} - 1} = 525\ \Omega$ ⟵

b) From Eqs. (16-16), (16-13) & (16-12), $|A_o f_2| = \dfrac{g_m}{2\pi} \cdot \dfrac{R_c}{R_c + r_{bb'}} \cdot \dfrac{R_c + h_{ie}}{g_m h_{ie} R_c C_c + (R_c + h_{ie})(C_e + C_c)}$

or $|A_o f_2| = \dfrac{0.0086 R_c}{R_c + 45} \cdot \dfrac{R_c + 2605}{1.28 \times 10^{-9} R_c + 2.04 \times 10^{-6}}$

For $R_c = 100\ \Omega$, $|A_o f_2| = 7.4$ MHz ; For $R_c = 1$K, $|A_o f_2| = 8.9$ MHz ⟵

$R_c = 10$K, $|A_o f_2| = 7.25$ MHz ; and for $(R_c)_{opt} = 525\ \Omega$, $|A_o f_2| = 91.6$ MHz ⟵

(16-14) From Eq. (16-21), $f_2 = \frac{0.35}{t_r} = \frac{0.35}{10^{-6}} = 3.5\times10^5$. Using the parameter values of Prob. 16-13, we have: $R = (2560)\frac{R_c+45}{R_c+2605}$ ... Eq. (16-14) and from Eqs. (16-13) & (16-12), $C = (783.5 + \frac{492R_c}{R_c+2605})10^{-12}$. From Eq. (16-15), $RC = \frac{1}{2\pi f_2} = \frac{10^{-5}}{2\pi(3.5)}$ or $RC = 4.55\times10^{-7} = \left[(2560)\frac{R_c+45}{R_c+2605}\right]\cdot\left[783.5 + \frac{492R_c}{R_c+2605}\right]\times10^{-12}$. Solving, we obtain $R_c = 600\ \Omega$. Then, from Eq. (16-11), $A_o = -\frac{138\times600}{600+2605} = -25.8$ $\longleftarrow$

(16-15) As $R_c \rightarrow \infty$, $A_o = -\frac{h_{fe}R_c}{R_c+h_{ie}} \longrightarrow -h_{fe} = -\frac{g_m}{g_{b'e}} = -50$

$$R_L = \frac{R_c h_{ie}}{R_c+h_{ie}} \longrightarrow h_{ie} = r_{bb'} + r_{b'e} = 1100\,\Omega\ ; \quad C \rightarrow C_e + C_c(1+g_m h_{ie}) = 268\text{ pF}$$

$$R = \frac{(R_c+r_{bb'})r_{b'e}}{R_c+h_{ie}} \longrightarrow r_{b'e} = 1\text{K}$$

Hence, from Eq. (16-15), $f_2 \rightarrow \frac{10^{12}}{2\pi(10^3)(268)} = 595\text{ kHz}$, $|A_of_2| = (50\times595\times10^3) = 29.75\text{ MHz}$ $\longleftarrow$

(16-16) From Eqs. (16-16) & (16-12), $|A_of_2| = \frac{g_m}{2\pi}\,\frac{1}{C_e+C_c+g_mC_cR_ch_{ie}/(R_c+h_{ie})}\cdot\frac{R_c}{R_c+r_{bb'}}$

$= \frac{1}{2\pi C_c}\cdot\frac{1}{\left(\frac{C_e+C_c}{g_mC_c} + \frac{R_ch_{ie}}{R_c+h_{ie}}\right)}\cdot\frac{1}{\left(\frac{R_c+r_{bb'}}{R_c}\right)}$; Now, $|A_of_2|$ is maximum when the denominator is minimum. To find $(R_c)_{opt}$

$$\frac{d}{dR_c}\left[\left(\frac{C_e+C_c}{g_mC_c} + \frac{R_ch_{ie}}{R_c+h_{ie}}\right)\left(\frac{R_c+r_{bb'}}{R_c}\right)\right] = 0 = \left(\frac{C_e+C_c}{g_mC_c}\right)\left(-\frac{r_{bb'}}{R_c^2}\right) + \frac{(R_c+h_{ie})h_{ie} - h_{ie}(R_c+r_{bb'})}{(R_c+h_{ie})^2}$$

or $-\frac{(C_e+C_c)r_{bb'}}{g_mC_cR_c^2} + \frac{h_{ie}r_{b'e}}{(R_c+h_{ie})^2} = 0$; $\frac{(C_e+C_c)r_{bb'}}{h_{fe}C_ch_{ie}} = \frac{R_c^2}{(R_c+h_{ie})^2} \equiv \frac{1}{x}$

Then $R_c^2(1-x) + 2h_{ie}R_c + h_{ie}^2 = 0$, $(R_c)_{opt} = \frac{-2h_{ie} \pm \sqrt{4h_{ie}^2 - 4(1-x)h_{ie}^2}}{2(1-x)} = \frac{-h_{ie} \pm \sqrt{x h_{ie}^2}}{1-x}$

For $x > 1$, and taking the minus-sign value, we obtain $(R_c)_{opt} > 0$ namely, $(R_c)_{opt} = \frac{(\sqrt{x}+1)}{(x-1)}h_{ie} = \frac{(\sqrt{x}+1)}{(\sqrt{x}+1)(\sqrt{x}-1)}h_{ie} = \frac{h_{ie}}{\sqrt{x}-1}$ ... Eq. (16-17) QED

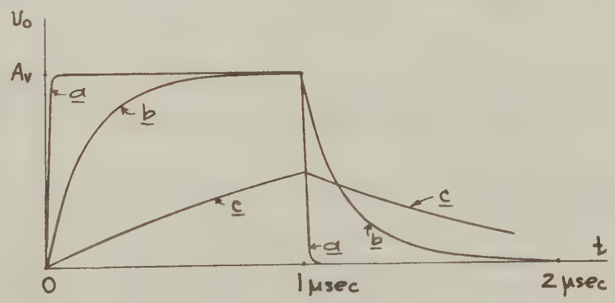
If $x < 1$, $(R_c)_{opt} < 0$ regardless which sign is taken above. This means that under this condition and for $R_c > 0$, $|A_of_2|$ starts from 0 at $R_c = 0$ and increases monotonically with R_c.

(16-17) Assuming that $f_1 = 0$, we have:

a) $f_2 = 10\text{ MHz}$; $RC = \frac{10^{-7}}{2\pi} = 0.0159\ \mu\text{sec}$

b) $f_2 = 1\text{ MHz}$; $RC = 0.159\ \mu\text{sec}$, $v_o = A_v(1 - e^{-t/0.159})$
at $t = 1\ \mu\text{sec}$, $v_o = A_v(1 - e^{-6.3}) \approx A_v$

c) $f_2 = 0.1\text{ MHz}$; $v_o = A_v(1 - e^{-0.63t})$
at $t = 1\ \mu\text{sec}$, $v_o = A_v(1 - e^{-0.63}) = 0.467A_v$



(16-18) a) From Eq. (16-25), $P = \frac{100\pi f_1}{f}$ where $f_1 = \frac{1}{2\pi(R_o' + R_i')C_b} \approx \frac{1}{2\pi(0.5\times 0.2)} = 1.59$ Hz ⊲—

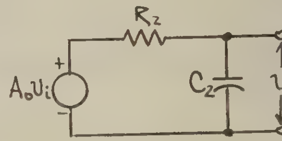
Thus, $P = \frac{100\pi\,(1.59)}{100} = 5\,\%$

b) $f_1 = \frac{1}{2\pi(3+2)10^3\times 10^{-5}} = 3.18$ Hz $\therefore$ $P = \frac{100\pi\,(3.18)}{100} = 10\,\%$ ⊲—

c) $P = 1 = \frac{100\pi f_1}{f}$ or $f = 100\pi f_1$

(i) $f = 100\pi\,(1.59) = 500$ Hz ⊲—

(ii) $f = 100\pi\,(3.18) = 1000$ Hz ⊲—



(16-19) a) The equivalent circuit for each stage is as shown. From this circuit we have: $\frac{dv_o}{dt} + \frac{v_o}{R_2C_2} = \frac{A_o}{R_2C_2} v_i$

If the input to the first stage is a unit step, i.e. $v_{i1} = u(t)$, then its output may be obtained from the above d.e. Thus, $v_{o1} = A_o(1 - e^{-t/R_2C_2})u(t)$. This is now fed to the input of the second stage, i.e. $v_{i2} = v_{o1}$. We now have: $\frac{dv_{o2}}{dt} + \frac{v_{o2}}{R_2C_2} = \frac{A_o^2}{R_2C_2}(1 - e^{-t/R_2C_2})$ whose general solution is $v_{o2} = A_o^2(1 - e^{-t/R_2C_2}) + Bt\,e^{-t/R_2C_2}$. Substituting, we find $B = -\frac{A_o^2}{R_2C_2}$ $\therefore$ $v_{o2} = A_o^2\left[1 - \left(1 + \frac{t}{R_2C_2}\right)e^{-t/R_2C_2}\right]u(t)$. Let $x \equiv \frac{t}{R_2C_2}$

Then, $v_o = A_o^2\left[1 - (1+x)e^{-x}\right]$ for $t \geq 0$ QED

b) If $t \ll R_2C_2$, then $x \ll 1$ and $e^{-x} \approx 1 - x + \frac{x^2}{2} - \dots$ Substituting, we obtain, $v_o = A_o^2\left[1 - (1+x)(1 - x + \frac{x^2}{2} - \dots)\right] = A_o^2\left[1 - (1 - \frac{x^2}{2} + \dots)\right] = A_o^2\,\frac{x^2}{2}$ QED

(16-20) From Fig. 16-11, $t_r^{(2)} = t\Big|_{\frac{v_o}{A_o^2} = 0.9} - t\Big|_{\frac{v_o}{A_o^2} = 0.1}$ where from Prob. 16-19 a

$\frac{v_o}{A_o^2} = 0.9 = 1 - (1+x)e^{-x}$ or $(1+x)e^{-x} = 0.1$. Solving by trial & error, we get $x = 3.88$. Also $\frac{v_o}{A_o^2} = 0.1 = 1 - (1+x)e^{-x}$ or $(1+x)e^{-x} = 0.9$ for which $x = 0.53$

Thus, for the two-stage amplifier, $t_r^{(2)} = (3.88 - 0.53)RC = 3.35\,RC$. From Eq. (16-15), $f_2 = \frac{1}{2\pi RC}$. Hence, $t_r^{(2)} f_2 = \frac{3.35}{2\pi} = 0.533$ QED

(16-21) For one stage, $\frac{v_{o1}}{v_{i1}} = \frac{A_{o1}}{1 + j\omega R_1C_1} = \frac{A_{o1}}{1 + j\,f/f_2^{(1)}}$ where $f_2^{(1)} = \frac{1}{2\pi R_1C_1}$

For the second stage: $\frac{v_{o2}}{v_{i2}} = \frac{A_{o2}}{1 + j\omega R_2C_2} = \frac{A_{o2}}{1 + j\,f/f_2^{(2)}}$ where $f_2^{(2)} = \frac{1}{2\pi R_2C_2}$

For the two stages cascaded: $\frac{v_{o2}}{v_{i1}} = \frac{A_{o1}A_{o2}}{1 + jf\left(\frac{1}{f_2^{(1)}} + \frac{1}{f_2^{(2)}}\right) - \frac{f^2}{f_2^{(1)}f_2^{(2)}}}$

If $f_2^{(1)} \ll f_2^{(2)}$, then $\frac{v_{o2}}{v_{i1}} \approx \frac{A_{o1}A_{o2}}{1 + j\,f/f_2^{(1)}}$; If $f_2^{(2)} \ll f_2^{(1)}$, then $\frac{v_{o2}}{v_{i1}} \approx \frac{A_{o1}A_{o2}}{1 + j\,f/f_2^{(2)}}$

Thus, the overall bandwidth is approximately that of the stage with lower f_2.

(16-22) From Eq. (16-26), $f_2^{(3)} = f_2(2^{1/3} - 1)^{1/2} = (0.258)^{1/2} f_2 = 0.508\,f_2 = 20$ kHz

$\therefore$ $f_2 = \frac{20\text{ kHz}}{0.508} = 39.4$ kHz ⊲—

From Eq. (16-27), $f_1^{(3)} = \frac{f_1}{0.508} = 20$ Hz. $\therefore$ $f_1 = 0.508\times 20 = 10.2$ Hz ⊲—

(16-23) a) $N = 20\log_{10}\left|\frac{A_1}{A_o}\right| = 20\log_{10}\frac{1}{[1+(f_1/f)^2]^{1/2}} = -1\text{ dB} = -10\log_{10}[1+(f_1/f)^2]$

or $1+(f_1/f)^2 = 1.26$, $f_1/f = 0.509$ $\quad\therefore f_1 = 0.509\times 10 = 5.09\text{ Hz}$

From Eq. (16-7), $C_b = \frac{1}{2\pi f_1(R_o'+R_i')}$ where $R_o' = r_d \| R_d = \frac{5.5K\times 10K}{15.5K} = 3.55K$

or $C_b = \frac{1}{2\pi(5.09)(503.55\times10^3)} = 0.0621\ \mu F$ ⟵

b) For the two stages, $20\log_{10}\left|\frac{A_1}{A_o}\right|^2 = -20\log_{10}[1+(f_1/f)^2] = -1\text{ dB}$

or $1+(f_1/f)^2 = 1.125$, $f_1/f = 0.352$ $\quad\therefore f_1 = 0.352\times 10 = 3.52\text{ Hz}$

Hence, $C_b = (0.0621)\frac{5.09}{3.52} = 0.09\ \mu F$ ⟵

c) Since the high-frequency gain expression is similar to that of low frequencies provided that (f_1/f) is replaced by (f/f_2), then from part b $(f/f_2) = 0.352$ where $f_2 = \frac{1}{2\pi RC}$; $\frac{1}{R} = \frac{1}{r_d}+\frac{1}{R_d}+\frac{1}{R_g} = (\frac{1}{5.5}+\frac{1}{10}+\frac{1}{500})\times10^{-3}$

or $\frac{1}{R} = 2.835\times10^{-4}$ $\quad\therefore f = 0.352 f_2 = \frac{0.352\times 2.835\times10^{-4}}{2\pi\times 50\times10^{-12}} = 317\text{ kHz}$ ⟵

d) Overall midband gain $= (g_m R)^2 = \left(\frac{10\times10^{-3}}{2.835\times10^{-4}}\right)^2 = 1240$ ⟵

(16-24) a) $|A_o| = g_m R$ where $\frac{1}{R} = (\frac{1}{7.7}+\frac{1}{10}+\frac{1}{100})\times10^{-3} = 0.24\times10^{-3}$; $|A_o| = \frac{2.6\times10^{-3}}{0.24\times10^{-3}}$

or $|A_o| = 10.8$ $\quad\therefore N = 20\log_{10}(10.8) = 20.7\text{ dB}$. Hence, the overall midband gain of three stages is $3\times 20.7 = 62.1\text{ dB}$ ⟵

b) From Eq. (16-7), $f_1 = \frac{1}{2\pi C_b(R_o'+R_i')}$ where $R_o' = r_d\|R_d = \frac{7.7\times10}{17.7}K = 4.35K$

Thus, $f_1 = \frac{1}{2\pi(5\times10^{-9})(104.35\times10^3)} = 305\text{ Hz}$ ⟵

c) From Eq. (16-27), $f_1^{(3)} = f_1/(2^{1/3}-1)^{1/2} = 305/(0.258)^{1/2} = 305/0.508 = 600\text{ Hz}$ ⟵

d) $f_2 = \frac{1}{2\pi RC} = \frac{0.24\times10^{-3}}{2\pi\times 60\times10^{-12}} = 636\text{ kHz}$ ⟵

e) From Eq. (16-26), $f_2^{(3)} = f_2(2^{1/3}-1)^{1/2} = 636\times 0.508 = 324\text{ kHz}$ ⟵

(16-25) a) From the plate characteristics (Fig. D-5) at $I_P = 10\text{ mA}$ and $V_G = -5V$, we find $V_P = 185\text{ V}$. Hence, $R_p = \frac{V_{PP}-V_P}{I_P} = \frac{300-185}{10} = 11.5K$ ⟵

b) $|A_o| = g_m R$ where $\frac{1}{R} = (\frac{1}{7.7}+\frac{1}{11.5}+\frac{1}{1000})\times10^{-3} = 2.18\times10^{-4}$; $\therefore |A_o| = \frac{2.2\times10^{-3}}{2.18\times10^{-4}} = 10.1$

Overall gain, $|A_o|^2 = (10.1)^2 = 102$ ⟵

c) From Eq. (8-44), $C_i = C_{gk} + C_{gp}(1+|A_o|) = 1.6 + 1.5\times 11.1 = 18.3\text{ pF}$

d) $f_1 = \frac{1}{2\pi C_b(R_o'+R_i')}$ where $R_o' = r_p\|R_p = \frac{7.7\times11.5}{7.7+11.5}K = 4.7K \ll R_i' = 1M$. Thus,

$f_1 = \frac{1}{2\pi(10^{-8})(10^6)} = 15.8\text{ Hz}$ ⟵

e) From Eq. (16-15), $f_2 = \frac{1}{2\pi RC}$. For the first stage $C_1 = C_{pk} + C_{stray} + C_{i2} = 38.8\text{ pF}$

Thus, $(f_2)_1 = \frac{2.18\times10^{-4}}{2\pi(38.8\times10^{-12})} = 895\text{ kHz}$ ⟵

For the second stage, the total shunt capacitance is $C_2 = C_{pk} + C_{stray} = 20.5\text{ pF}$. Since this is smaller than C_1, then the upper 3-dB frequency $(f_2)_2$ for this stage is higher than that of the first stage: $(f_2)_2 = 895\,\frac{38.8}{20.5} = 1.69\text{ MHz}$ ⟵

Since C_b is the same for both stages, then f_1 is also the same.

(16-25) (Cont'd) f) The scope impedance of 1M shunts $R_{p2} = 11.5K$ and reduces the gain of the second stage by a negligible amount. However, f_1 and f_2 of the second stage are affected. The new value of $R_i' = 0.5M$, and

$(f_1)_2 = 15.8 \frac{1M}{0.505M} = 31.3$ Hz

$C_2 = 20.5 + 20 = 40.5$ pF $\quad \therefore (f_2)_2 = 1.69 \frac{20.5}{40.5} = 855$ kHz ←

(16-26) 1. 6AK5

a) We have: $|A_o| = g_m R$ or $R = \frac{15}{5.1} K = 2.94K$. Since for a pentode $r_p \gg R_p$ and $R_g \gg R_p$, then $R_p \approx R = 2.94K$ ←

b) $f_1 = \frac{1}{2\pi C_b(R_o' + R_i')}$ where $R_o' \approx R_p = 2.94K \ll R_i' = 1M \quad \therefore f_1 = \frac{1}{2\pi(10^{-8})(10^6)} = 15.9$Hz ←

c) $f_2 = \frac{1}{2\pi RC} = \frac{10^{+12}}{2\pi(2.94\times10^3)(16.8)} = 3.22$ MHz ←

2. 6CG7

a) From Table 7-1, $r_p = 7.7K$ and $g_m = 2.6$ mA/V $\quad \therefore R = \frac{15}{2.6} = 5.76K$

Now, $\frac{1}{R} = \frac{1}{5.76} = \frac{1}{7.7} + \frac{1}{1000} + \frac{1}{R_p}$. Solving, we get $R_p = 23.3K$ ←

b) $R_o' = r_p \| R_p = (7.7\times23.3)/(7.7+23.3) = 5.8K \quad \therefore f_1 = \frac{1}{2\pi(10^{-8})(1.006)(10^6)} \approx 15.9$ Hz ←

c) $C_i = C_{gk} + (1+|A_o|)C_{gp} = 2.4 + 16\times3.9 = 64.8$ pF; $\quad C = C_i + C_o + C_{stray} = 75.5$ pF

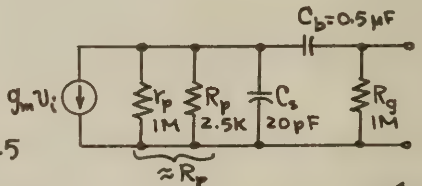
$\therefore f_2 = \frac{10^{12}}{2\pi(75.5)(5.76\times10^3)} = 366$ kHz ←

Note that for the same gain, the upper 3-dB frequency of the pentode is about 9 times that of the triode.

(16-27) From Eq. (16-26), $f_2 = \frac{f_2^{(2)}}{(2^{1/2}-1)^{1/2}} = \frac{10^7}{(0.414)^{1/2}} = 1.55\times10^7$ Hz. From Eq. (16-10),

$|A_o| f_2 = \frac{g_m}{2\pi C_{shunt}} \quad \therefore |A_o| = \frac{(10\times10^{-3})\times10^{12}}{2\pi(1.55\times10^7)(20)} = 5.13; \quad \therefore |A_o|^2 = (5.13)^2 = 26.4$ ←

(16-28) The equivalent circuit of each stage is as shown.



a) $A_o = -g_m R = -g_m(r_p \| R_p \| R_g) \approx -g_m R_p = -10\times2.5 = -25$

$\therefore$ overall gain, $A_o^2 = (-25)^2 = 625$ ←

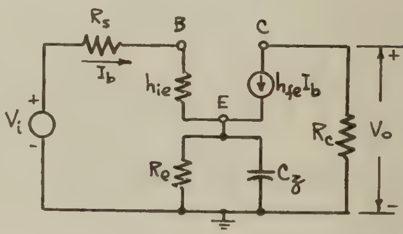
b) For one stage, $t_r = 2.2RC$ where $R \approx R_p$ and $C = C_s = 20$ pF

$\therefore t_r = 2.2\times(2.5\times10^3)(20\times10^{-12}) = 0.11$ μsec; $\quad t_r^{(2)} = \frac{0.11}{(2^{1/2}-1)^{1/2}} = 0.171$ μsec ←

c) If overall tilt is 10%, then the tilt per stage is 5% $= 100\pi\frac{f_1}{f}$% where

$f_1 = \frac{1}{2\pi C_b(R_o' + R_i')} \approx \frac{1}{2\pi(5\times10^{-7})10^6} = \frac{1}{\pi} \quad \therefore f = \frac{100}{5} = 20$ Hz ←

(16-29) The equivalent circuit is as shown (assuming that hre and hoe are negligible.)



a) $V_o = -h_{fe} I_b R_c$ where $I_b = \frac{V_i - (1+h_{fe})I_b Z_e}{R_s + hie}$

$Z_e = \frac{Re}{1 + j\omega R_e C_z}$. Then $I_b = \frac{V_i}{R_s + hie + (1+h_{fe})\frac{Re}{1+j\omega R_e C_z}}$

(16-29) (Cont'd) Let $R \equiv R_s + h_{ie}$ and $R' \equiv (1+h_{fe})R_e$. Then

$$V_o = \frac{-h_{fe}R_c V_i}{R + \frac{R'}{1+j\omega R_e C_z}} = -\frac{h_{fe}R_c(1+j\omega R_e C_z)V_i}{R' + R(1+j\omega R_e C_z)}$$

. At low frequencies where the effect of C_z is important, the gain is $A_1 = \frac{V_o}{V_i} = -\frac{h_{fe}R_c(1+j\omega R_e C_z)}{R(\frac{R'}{R}+1+j\omega R_e C_z)}$

In the midband C_z short-circuits R_e and the gain is $A_o = -\frac{h_{fe}R_c}{R}$

Thus, $\frac{A_1}{A_o} = \frac{1+j\omega R_e C_z}{(\frac{R'}{R}+1)+j\omega R_e C_z} = \frac{1+j\omega R_e C_z}{B+j\omega R_e C_z}$ where $B = 1+\frac{R'}{R}$ QED

b) Let $x \equiv \omega R_e C_z$. Then, $\frac{A_1}{A_o} = \frac{1+jx}{B+jx}$. At the lower 3-dB frequency,

$$\left|\frac{A_1}{A_o}\right| = \frac{1}{\sqrt{2}} = \sqrt{\frac{1+x_1^2}{B^2+x_1^2}} = \sqrt{\frac{1}{\left(\frac{B^2+x_1^2}{1+x_1^2}\right)}} \quad \therefore \quad \frac{B^2+x_1^2}{1+x_1^2} = 2 \quad \text{or} \quad B^2-2 = x_1^2$$

or $x_1 = \sqrt{B^2-2} = 2\pi f_1 R_e C_z$ $\quad \therefore f_1 = \frac{\sqrt{B^2-2}}{2\pi R_e C_z}$ QED

Note that if $B^2<2$, then at zero frequency (d.c.), $\left|\frac{A_1}{A_o}\right| = \frac{1}{B} > \frac{1}{\sqrt{2}}$. Hence, there exists no lower 3-dB frequency. In other words, the voltage gain never drops 3 dB from its midband value at low frequencies.

c) If $B \gg 1$, then $B^2 \gg 1$ and also $B^2 \gg 2$ $\quad \therefore f_1 \approx \frac{B}{2\pi R_e C_z} = \frac{1+R'/R}{2\pi R_e C_z}$

or $f_1 \approx f_p = \frac{1+R'/R}{2\pi R_e C_z}$ QED

(16-30) a) From Fig. 16-15, $V_{gk} = V_s - g_m V_{gk} Z_K$ or $V_{gk} = \frac{V_s}{1+g_m Z_K}$

But $V_o = -g_m V_{gk} R_L = \frac{-g_m R_L}{1+g_m Z_K}V_s$ $\quad \therefore A_V = \frac{V_o}{V_s} = -\frac{g_m R_L}{1+g_m Z_K}$ QED

b) $Z_K = \frac{R_K}{1+j\omega R_K C_K}$. Thus, as $\omega \to \infty$, $Z_K \to 0$ and from part $\underline{a}$

$A_o = -g_m R_L$ QED

c) From part $\underline{a}$, $\frac{V_o}{V_s} = -\frac{g_m R_L}{1+\frac{g_m R_K}{1+j\omega R_K C_K}} = \frac{-g_m R_L(1+j\omega R_K C_K)}{1+g_m R_K + j\omega R_K C_K} = \frac{-g_m R_L}{(1+g_m R_K)} \cdot \frac{(1+j\omega R_K C_K)}{(1+j\frac{\omega R_K C_K}{1+g_m R_K})}$

Then, $\frac{A_V}{A_o} = \frac{1}{1+g_m R_K} \cdot \frac{1+jf/f_o}{1+jf/f_p}$ where $f_o \equiv \frac{1}{2\pi C_K R_K}$ and $f_p \equiv \frac{1+g_m R_K}{2\pi C_K R_K}$

(16-31) Tilt is caused by C_b and C_K, and if the contributions are small, then the total tilt is equal to the sum of tilt due to C_b and that due to C_K. (i) Tilt due to C_b: $P_1 = \frac{100\pi f_1}{f}$ where $f_1 = \frac{1}{2\pi C_b(R_o'+R_i')} \approx \frac{1}{2\pi C_b R_i'}$,

or $f_1 = \frac{1}{2\pi(0.25)(0.5)} = \frac{4}{\pi}$ $\quad \therefore P_1 = \frac{100\times 4}{200} \doteq 2\%$

(ii) Tilt due to C_K: $P_2 = \frac{g_m t}{C_K}(100\%) = \frac{g_m}{2C_K f}(100\%)$ .... Eq. (16-40)

or $P_2 = \frac{5\times10^{-3}\times100\%}{2\times500\times10^{-6}\times200} = 2.5\%$. Since P_1 & P_2 are small, then the

Total Tilt = 2% + 2.5% = 4.5% $\leftarrow$

(16-32) From Fig. 16-13b we obtain: $v_o = -h_{fe} i_b R_c = -h_{fe} R_c \frac{V - v_{en}}{R}$ where $R = R_s + h_{ie}$

Since the voltage across a capacitor cannot change instantaneously, then $v_{en} = 0$ and $i_b = \frac{V}{R}$. Assuming next that all the emitter current flows through C_z, we obtain: $v_{en} = \frac{(1+h_{fe}) i_b}{C_z} t = \frac{(1+h_{fe}) V t}{R C_z}$

Then, $v_o = -\frac{h_{fe} R_c}{R} V \left[1 - \frac{(1+h_{fe})}{R C_z} t\right]$ QED

Since $t = \frac{1}{2f}$, then $P = \frac{1+h_{fe}}{2 R C_z f} \times 100\% = \frac{1+h_{fe}}{2 f C_z (R_s + h_{ie})} \times 100\%$ ... Eq. (16-37)

(16-33) a) Using Eq. (16-42), we have: $B = \frac{1}{|A_{vo}|^2} \int_0^\infty |A_v(f)|^2 df = \int_0^\infty \frac{df}{1 + (f/f_2)^2}$

or $B = f_2^2 \int_0^\infty \frac{df}{f_2^2 + f^2} = f_2^2 \left[\frac{1}{f_2} \tan^{-1}\left(\frac{f}{f_2}\right)\right]_0^\infty = f_2 \left(\frac{\pi}{2} - 0\right) = \frac{\pi f_2}{2}$

b) If $f_2 = 10$ kHz, $B = 5\pi$ kHz $= 15.7$ kHz

(16-34) a) The output noise voltage of the given circuit is expressed by Eq. (16-41). From this and from Eq. (16-42), we obtain:

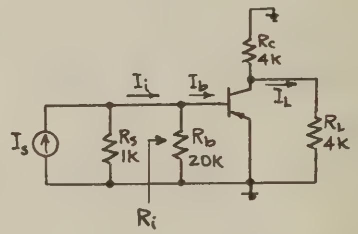
$V_o^2 = 4\bar{k} T R B = \frac{4\bar{k} T R}{|A_{vo}|^2} \int_0^\infty |A_v(f)|^2 df$ where $|A_{vo}| = 1$ and $A_v(f) = \frac{1}{1 + j\omega RC}$

$\therefore V_o^2 = 4\bar{k} T R \int_0^\infty \frac{df}{1 + (2\pi f RC)^2} = \frac{4\bar{k} T R}{(2\pi RC)} \int_0^\infty \frac{dx}{1 + x^2} = \frac{4\bar{k} T R}{2\pi RC} \frac{\pi}{2} = \frac{\bar{k} T}{C}$

b) $\frac{1}{2} C V_o^2 = \frac{1}{2} C \left(\frac{\bar{k} T}{C}\right) = \frac{1}{2} \bar{k} T$ QED

CHAPTER 17

(17-1) a) From the circuit we have: $\frac{I_L}{I_s} = \frac{I_i}{I_s} \cdot \frac{I_b}{I_i} \cdot \frac{I_L}{I_b}$



where $\frac{I_i}{I_s} = \frac{R_s}{R_s + R_i}$; $R_i \approx R_b \| h_{ie} = 20K \| 1.1K = 1.04K$

or $\frac{I_i}{I_s} = \frac{1}{2.04}$; $\frac{I_b}{I_i} = \frac{R_b}{R_b + h_{ie}} = \frac{20K}{20K + 1.1K} = 0.95$

$\frac{I_L}{I_b} = -h_{fe} \frac{R_c}{R_c + R_L} = -50 \frac{4}{8} = -25$ $\therefore \frac{I_L}{I_s} = \left(\frac{1}{2.04}\right)(0.95)(-25) = -11.65$ ⟵

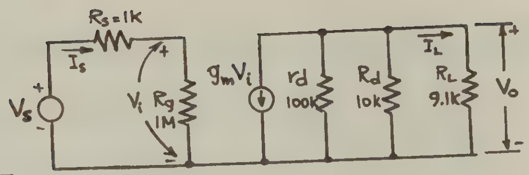
b) $\frac{V_o}{V_s} = \frac{I_L R_L}{I_s R_s} = (-11.65) \frac{4K}{1K} = -46.6$ ⟵

c) $\frac{I_L}{V_s} = \frac{V_o}{R_L} \cdot \frac{1}{V_s} = \frac{V_o}{V_s} \frac{1}{R_L} = -\frac{46.6}{4 \times 10^3} = -11.65$ mA/V ⟵

d) $\frac{V_o}{I_s} = R_s \frac{V_o}{V_s} = 10^3 (-46.6) = -46.6 \times 10^3$ V/A ⟵

e) $R_i \approx R_b \| h_{ie} = 20K \| 1.1K = 1.04K$ ⟵

f) $R_o \approx \frac{1}{h_{oe}} \| R_c = 40K \| 4K = 3.64$ K ⟵



(17-2) a) $\frac{I_L}{I_s} = R_g \frac{I_L}{V_i} = \frac{R_g}{R_L}\frac{V_o}{V_i}$ where from the equivalent circuit, $\frac{V_o}{V_i} = -g_m(r_d \| R_d \| R_L)$

$= -5\times10^{-3} \times \frac{9.1}{2}\times10^3 = -22.7$

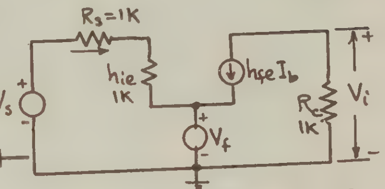
$\therefore \frac{I_L}{I_s} = \frac{10^6}{9.1\times10^3}(-22.7) = -2500$ ⟵

b) $\frac{V_o}{V_s} = \frac{I_L R_L}{I_s(R_s+R_g)} = -2500\,\frac{9.1}{1001} \approx -22.7$ Note that here $V_s = I_s(R_s+R_g)$ ⟵

c) $\frac{I_L}{V_s} = \frac{V_o}{R_L}\frac{1}{V_s} = \left(\frac{V_o}{V_s}\right)\frac{1}{R_L} = -22.7\frac{1}{9.1\times10^3} = -2.5$ mA/V ⟵

d) $\frac{V_o}{I_s} = R_s\frac{V_o}{V_i} = 10^3(-22.7) = -22.7\times10^3$ V/A $= -22.7$ V/mA ⟵

e) $R_i = \frac{V_i}{I_s} = R_g = 1M$; f) $R_o = r_d \| R_d = 100K \| 10K = 9.1K$ ⟵



(17-3) a) From the equivalent circuit shown we have:

$V_i = -h_{fe}I_bR_c$ where $I_b = \frac{V_s-V_f}{R_s+hie}$. Thus,

$V_i = -h_{fe}R_c\frac{V_s-V_f}{R_s+hie} = -100\times\frac{1K}{2K}(V_s-V_f) = -50(V_s-V_f)$ ⟵

b) With the output of the inverting* amplifier connected to the input of the β-network in Fig. 17-7c, we have: $V_f = \beta V_o$ and $V_o = AV_i = A_vV_i$

Thus, $V_o = -A_v(50)(V_s-\beta V_o) = +5\times10^4(V_s-0.01V_o)$ or $V_o(1+500) = +5\times10^4V_s$

Hence, $A_{vf} = \frac{V_o}{V_s} = \frac{5\times10^4}{501} \approx +100$ ⟵

(17-4) a) From Eq. (12-87), $V_i = A_dV_d + A_cV_c$ where $A_d = \frac{1}{2}\frac{h_{fe}R_c}{R_s+hie} = \frac{1}{2}\frac{100\times1K}{1K} = 50$

and $A_c = \frac{(2\times2.5\times10^{-5}\times10^4-100)\times10^3}{2\times10^4(101+2.5\times10^{-5}\times10^3)+10^3} = \frac{-99.5\times10^3}{20.2\times10^5} = -0.0493$

$\therefore V_i = 50(V_s-V_f) - 0.0493\left(\frac{V_s+V_f}{2}\right) \approx 50(V_s-V_f)$ ⟵

b) $V_o = A_vV_i = 50\times10^3(V_s-0.01V_o)$ or $V_o(1+500) = 5\times10^4V_s$

Hence, $A_{vf} = \frac{V_o}{V_s} = \frac{5\times10^4}{501} \approx +100$ ⟵

(17-5) From Eq. (17-5), we obtain: $\left|\frac{dA_f}{A_f}\right| \equiv |\psi_f| = \frac{1}{|1+\beta A|}\left|\frac{dA}{A}\right|$ where $A = A_1^3$

or $dA = 3A_1^2dA_1$. Then $\frac{dA}{A} = 3\frac{dA_1}{A_1} = 3\psi_1$. From Eq. (17-4), $\frac{1}{1+\beta A} = \frac{A_f}{A}$.

Substituting, we have: $|\psi_f| = \frac{A_f}{A}3|\psi_1|$ or $A = 3A_f\frac{|\psi_1|}{|\psi_f|}$ QED

(17-6) a) From Eq. (17-5), $\left|\frac{dA_f}{A_f}\right| = \frac{1}{|1+\beta A|}\left|\frac{dA}{A}\right|$ or $\frac{0.1}{100} = \frac{1}{1+\beta A}\frac{100}{1000}$ $\therefore 1+\beta A = 100$ ⟵

or $\beta A = 99$. Hence, $\beta = \frac{99}{1000} = 0.099$

b) $A_f = \frac{A}{1+\beta A} = \frac{1000}{1+99} = 10$ ⟵

(17-7) a) $A = \frac{36}{0.028} = 1285$; $\beta = 0.012$. From Eq. (17-4), $A_f = \frac{A}{1+\beta A} = \frac{1285}{1+(0.012)(1285)} = 78.2$

$\therefore V_o = A_f V_s = 78.2 \times 0.028 = 2.19\text{ V}$ ←

b) If the output remains constant at 36V, then the distortion produced within the active devices of the amplifier is unchanged. However, since the distortion at the output is less than in part a by a factor of 7, it follows that the feedback now increased by 7 and hence, the voltage gain decreased by 7. Thus, the input signal required to produce the same output (as in part a without FB) must be: $V_s = 7\,(0.028\text{V}) = 0.196\text{ V}$ ←

(17-8) a) $-40\text{ dB} = 20\log_{10}\frac{1}{|1+\beta A|} = -20\log_{10}|1+\beta A|$ $\therefore |1+\beta A| = 100$

But $|A_f| = \left|\frac{A}{1+\beta A}\right| = \frac{1000}{100} = 10$. When the amplifier delivers 10W, its output voltage is $V_o = AV_s = 1000\times(10\times10^{-3}) = 10\text{ V}$. If the output power is to remain at 10W, then the output voltage also must remain at 10V. Hence, the input signal required when FB is applied will be $V_s = \frac{V_o}{A_f} = \frac{10\text{V}}{10} = 1\text{ V}$ ←

b) The distortion of the amplifier with FB will be reduced by the FB factor i.e. by $|1+\beta A| = 100$. Hence, $D_f = \frac{10\%}{100} = 0.1\%$ ←

(17-9) a) The voltage gain at high frequencies is given by Eq. (16-5),

$A_{v2} = \frac{A_{vo}}{1+jf/f_2}$ and $|A_{v2}| = \frac{|A_{vo}|}{[1+(f/f_2)^2]^{1/2}}$

The gain with FB falls 3 dB at a frequency $f = f_{2f} = f_2(1+\beta A_{vo})$ ... Eq. (17-22)

or $f = f_2(1+0.1\times1000) = 101 f_2$ $\therefore |A_{v2}| = \frac{1000}{[1+(101)^2]^{1/2}} = \frac{1000}{101} = 9.9$ ←

Starting with $A_{v1} = \frac{A_{vo}}{1+jf_1/f}$ and $f_{1f} = f_1/(1+\beta A_{vo})$, we obtain the same result at the low-frequency end of the spectrum.

b) $f_{2f}/f_2 = 1+\beta A_{vo} = 101$ and $f_{1f}/f_1 = 1/101 = 0.0099$ ←

c) $f_{2f} = 101\times 50\text{kHz} = 5.05\text{ MHz}$ and $f_{1f} = 0.0099\times 20\text{Hz} = 0.198\text{ Hz}$ ←

(17-10) a) The gain of each stage without FB is $A_{vo} = -g_m R$ where

$\frac{1}{R} = \frac{1}{r_d} + \frac{1}{R_d} + \frac{1}{R_g} = \left(\frac{1}{10}+\frac{1}{50}+\frac{1}{1000}\right)\times10^{-3} = 1.21\times10^{-4}$ $\therefore R = 8.26\text{K} = R_o$

Thus, $A_{vo} = -6\times8.26 = -49.6$. Also, $\beta = -\frac{R_1}{R_g} = -\frac{40}{10^6} = -4\times10^{-5}$

For the 3 stages the gain without FB is $A = A_{vo}^3 = -1.22\times10^5$

$\beta A = (4\times10^{-5})(1.22\times10^5) = 4.88$ $\therefore A_f = \frac{A}{1+\beta A} = \frac{-1.22\times10^5}{1+4.88} = -2.07\times10^4$ ←

From Eq. (17-13), $R_{of} = \frac{R_o}{1+\beta A} = \frac{8.26\text{K}}{5.88} = 1.41\text{K}$ ←

(17-10) (Cont'd) b) If we take the output terminals as B & N, then the forward gain is the gain of Q1 or $A_{BN} = A_{Vo} = -49.6$. However, Q2 and Q3 must be considered part of the feedback loop. Hence, $\beta_{BN} = -A_{Vo}^2 \frac{R_1}{R_g} = -(49.6)^2 \times 4\times10^{-5}$ or $\beta_{BN} = -0.99$. Note that the loop gain $-\beta_{BN}A_{BN} = A_{Vo}^3 \frac{R_1}{R_g} = -4.88 = -\beta A$ which is the same value as in part a. (It should be clear that regardless of where the output terminals are taken, the loop gain is unchanged.)

$$\therefore A_f = \frac{A_{BN}}{1+\beta A} = \frac{-49.6}{5.88} = -8.44 \; ; \; R_f = \frac{8.26K}{5.88} = 1.41K \quad \longleftarrow$$

(17-11) Refer to Fig. 17-11. With $R_s = 0$ we have: $V_o = I_L R_L$ and $V_s = I_i R_{if}$ where $R_{if} = R_i(1+\beta A_V)$ ... Eq. (17-18). Thus, $A_{Vf} \equiv \frac{V_o}{V_s} = \frac{I_L R_L}{I_i R_{if}} = A_{If}\frac{R_L}{R_{if}} = A_{If}\frac{R_L}{R_i(1+\beta A_V)}$ where $A_{If} \equiv \frac{I_L}{I_i}$ = current gain with FB. Now using Eq. (17-4) we have:

$$A_{Vf} = \frac{A_V}{1+\beta A_V} = A_{If}\frac{R_L}{R_i(1+\beta A_V)} \quad \text{or} \quad \frac{A_V R_i}{R_L} = A_{If}\frac{(1+\beta A_V)}{(1+\beta A_V)} = A_{If}$$

Since, $A_V = A_I \frac{R_L}{R_i}$, we have $\left(A_I \frac{R_L}{R_i}\right)\frac{R_i}{R_L} = A_I = A_{If}$ QED

(17-12) a) Since $h_{oe}R_c = 10^{-5}\times10^4 = 0.1 \ll 1$ and neglecting h_{re}, then we may use Eq. (12-91) to obtain $A_c = \frac{(2\times10^{-5}\times1.2\times10^5 - 100)10^4}{2\times1.2\times10^5(101) + (2\times10^{-5}\times1.2\times10^5+1)10^3} = \frac{-97.6\times10^4}{2.42\times10^7} = -0.0403$

From Eq. (12-92), $A_d = \frac{1}{2}\frac{(100)(10^4)}{10^3} = 500$

b) Neglecting the loading of Q4, we have: $\frac{v_2}{v_1} \approx -\frac{47K}{3.9K} = -12 \quad \longleftarrow$

Q4 is an emitter follower with an effective emitter load, $R_e' = 10K \| (90+10)K$ or $R_e' = 9.1K$. Then $R_i \approx h_{ie} + (1+h_{fe})R_e' = 1K + 101\times9.1K = 920K$. From Eq. (12-34)

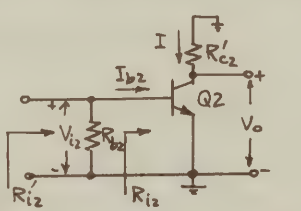
$$1 - \frac{v_o}{v_2} = \frac{h_{ie}}{R_i} = \frac{1K}{920K} \approx 0.0011 \quad \therefore \frac{v_o}{v_2} = 1 - 0.0011 = 0.9989 \quad \longleftarrow$$

$$\text{Hence, } A = \frac{v_o}{v_1} = \frac{v_o}{v_2}\cdot\frac{v_2}{v_1} = (0.9989)(-12) \approx -12 \quad \longleftarrow$$

c) $A_V = \frac{v_o}{v_s}$ where $v_o = Av_1 = -12\left[500(v_f - v_s) - \frac{0.0403}{2}(v_f + v_s)\right.$; $v_f = \frac{v_o}{10}$

or $v_o = -6000\left(\frac{v_o}{10} - v_s\right) + 0.243\left(\frac{v_o}{10} + v_s\right)$ or $v_o(1 + 600 - 0.0243) = (6000 + 0.243)v_s$

$$\text{Hence, } A_V = \frac{6000.243}{601.0243} \approx 10 \quad \longleftarrow$$



(17-13) a) Starting with the second stage we have:

Stage II : $R_{c2}' = 4.7K \| 10K = 3.2K$; $R_{i2} = h_{ie} = 1.1K$

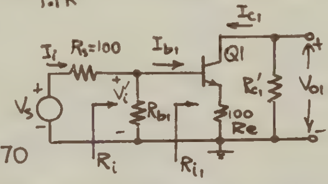
$R_{b2} = 22K \| 220K = 20K$; $R_{i2}' = 20K \| 1.1K = 1.04K$

Then, $A_{I_2} = -\frac{I}{I_{b2}} = \frac{-h_{fe}}{1+h_{oe}R_{c2}'} \approx -50$; $A_{V2} = \frac{V_o}{V_{i2}} = A_{I_2}\frac{R_{c2}'}{R_{i2}} = -50\frac{3.2K}{1.1K} = -145$; $R_o \approx 3.2K$

Stage I : $R_{c1}' = 22K \| 1.04K \approx 1K$; $A_{I_1} = -\frac{I_{c1}}{I_{b1}} \approx -h_{fe} = -50$

$R_{i1} = h_{ie} + (1+h_{fe})R_e = 1.1 + 51(100) = 6.2K$

Then, $A_{V_1} = \frac{V_{o1}}{V_i'} = -50\frac{1K}{6.2K} = -8.06$; $R_{b1} = 22K \| 220K = 20K$



overall

$R_i = R_{b1} \| R_{i1} = 20K \| 6.2K = 4.74K$; $A_V = \frac{V_o}{V_i'} = A_{V_1}A_{V2} = (-8.06)(-145) = 1170$

$A_{Vs} = A_V\frac{R_i}{R_i + R_s} = 1170\frac{4.74K}{4.84K} = 1145$; $A_I = -\frac{I}{I_i} = A_V\frac{R_i}{R_{c2}'} = 1170\frac{4.74}{3.2} = 1730$; $R_o \approx 3.2K \quad \longleftarrow$

(17-13) (Cont'd) b) With S in B, the FB amplifier becomes of the type shown in Fig. 17-17. Thus, $\beta = \frac{R_1}{R_1+R_2} = \frac{100}{100+10{,}000} \approx \frac{1}{100}$. Using Eq. (17-4), we have: $A_{Vf} = \frac{V_o}{V_i'} = \frac{A_V}{1+\beta A_V} = \frac{6600}{1+66} = 98.5$ since $A_V = (-50\frac{1K}{1.1K})(-145) = 6600.$ $\leftarrow$

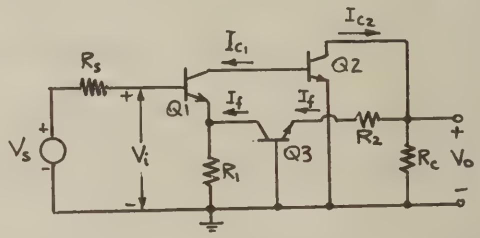
$R_{of} = \frac{R_o}{1+\beta A_V} = \frac{3.2K}{67} = 47.7\ \Omega$; $R_{i1f} = R_{i1}(1+\beta A_V) = 1.1K \times 67 = 73.8K$

Then, $R_{if} = \frac{V_i'}{I_i} = 20K \| 73.8K = 15.75K$; $A_{Vsf} = \frac{V_o}{V_s} = 98.5\,\frac{15.75}{15.85} = 98$ $\leftarrow$

and $A_{If} = A_{Vf}\frac{R_{if}}{R_{c2}'} = 98.5\,\frac{15.75K}{3.2K} = 485$ $\leftarrow$

Note that $A_{If} \neq A_I$ (see Prob. 17-11) because of the effect of the biasing resistors of the first stage.

(17-14) a) From the circuit we have:



$$V_i = \frac{I_{c1}}{h_{fe}}h_{ie} + R_1(I_{c1}+I_f) = I_{c1}\left(R_1+\frac{h_{ie}}{h_{fe}}\right)+R_1 I_f$$

or $V_i \approx (I_{c1}+I_f)R_1$ since $R_1 \gg \frac{h_{ie}}{h_{fe}}$.

But $I_f = I_{c2}\frac{R_c}{R_c+R_2+h_{ib3}} \approx I_{c2}\frac{R_c}{R_c+R_2} = h_{fe}I_{c1}\frac{R_c}{R_c+R_2}$

So $V_i \simeq R_1 I_{c1}\left(1+\frac{h_{fe}R_c}{R_c+R_2}\right) \approx \frac{h_{fe}R_1R_c}{R_c+R_2}I_{c1}$; $V_o = \left(I_{c2}\frac{R_2}{R_2+R_c}\right)R_c = \frac{h_{fe}R_cR_2}{R_2+R_c}I_{c1}$

where $I_{c1} = \left(\frac{R_c+R_2}{R_1R_c}\frac{1}{h_{fe}}\right)V_i$. Hence, $A_V = \frac{V_o}{V_i} \approx \frac{R_2}{R_1}$ QED

b) To find R_o, set $V_s=0$ and apply a source V across the output terminals. Consider R_c external to the amplifier and let the current drawn from the source V be I. Then, $R_o' = \frac{V}{I}$ and $R_o = R_c \| R_o'$.

Thus, $I = I_f - I_{c2} = \frac{V}{R_2} - h_{fe}I_{c1}$ where $I_{c1} \simeq -I_f\frac{R_1}{R_1+\frac{R_s+h_{ie}}{h_{fe}}} = -\frac{R_1}{R_2}\,\frac{V}{R_1+\frac{R_s+h_{ie}}{h_{fe}}}$

So, $I = \frac{V}{R_2}\left(1+\frac{h_{fe}R_1}{R_1+\frac{R_s+h_{ie}}{h_{fe}}}\right)$. Hence, $R_o' = \frac{R_2}{1+\frac{h_{fe}R_1}{R_1+\frac{R_s+h_{ie}}{h_{fe}}}} \approx \frac{R_2}{h_{fe}}$

since $h_{fe}R_1 \gg R_s+h_{ie}$ and $h_{fe} \gg 1$. QED

(17-15) a) With $h_{fe} \gg 1$, Eq. (17-44) becomes: $I_L \approx \frac{-h_{fe}V_s}{R_s+h_{ie}+h_{fe}R_e}$. But $V_o = I_L R_L$

and $A_f = \frac{V_o}{V_s} = \frac{-h_{fe}R_L}{R_s+h_{ie}+h_{fe}R_e}$ QED

b) From a, $\frac{1}{A_f} = \frac{R_s+h_{ie}+h_{fe}R_e}{-h_{fe}R_L} = -\frac{(R_s+h_{ie})}{h_{fe}R_L} - \frac{R_e}{R_L}$. Differentiating wrt h_{fe}

we have: $-\frac{1}{A_f^2}\frac{dA_f}{dh_{fe}} = \frac{R_s+h_{ie}}{h_{fe}^2R_L}$ or $\frac{dA_f}{A_f} = -\frac{A_f(R_s+h_{ie})}{h_{fe}R_L}\cdot\frac{dh_{fe}}{h_{fe}} \equiv \Psi_f$

or $\Psi_f = \frac{R_s+h_{ie}}{R_s+h_{ie}+h_{fe}R_e}\cdot\left(\frac{dh_{fe}}{h_{fe}}\right)$. Solving for R_e, we have:

$$h_{fe}R_e\Psi_f + (R_s+h_{ie})\Psi_f = (R_s+h_{ie})\frac{dh_{fe}}{h_{fe}} \quad \text{or} \quad R_e = \frac{R_s+h_{ie}}{h_{fe}}\left(\frac{dh_{fe}/h_{fe}}{\Psi_f}-1\right)$$

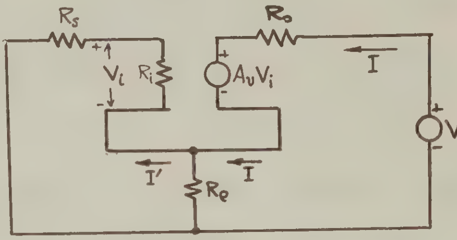
QED

(17-16) From Eq. (17-32), $R_{of} = R_o + R(1 - A_{vs}) = \frac{1}{h_{oe}} + R_e(1 - A_{vs})$. From Fig. 17-21c and Eq. (17-31), $A_{vs} = \frac{A_v R_i + R_e}{R_s + R_i + R_e} = \frac{A_v h_{ie} + R_e}{R_s + h_{ie} + R_e} = \frac{-\frac{h_{fe}}{h_{oe}} + R_e}{R_s + h_{ie} + R_e} = \frac{1}{h_{oe}} \frac{(-h_{fe} + h_{oe} R_e)}{(R_s + h_{ie} + R_e)}$

Thus, $R_{of} = \frac{1}{h_{oe}} + R_e\left[1 - \frac{1}{h_{oe}} \frac{(-h_{fe} + h_{oe}R_e)}{(R_s + h_{ie} + R_e)}\right] = \frac{R_s + h_{ie} + R_e + R_e h_{oe}(R_s + h_{ie} + R_e) + h_{fe}R_e - h_{oe}R_e^2}{h_{oe}(R_s + h_{ie} + R_e)}$

or $R_{of} = \frac{(R_s + h_{ie})(1 + h_{oe}R_e) + R_e(1 + h_{fe})}{h_{oe}(R_s + h_{ie} + R_e)}$ which is Eq. (12-51) with $h_{re} = 0$

(17-17) From the circuit, $V = A_v V_i + I(R_o + R_e) - I'R_e$ (1)



where $I' = \frac{R_e I}{R_e + R_i + R_s}$ and $V_i = -I'R_i = -\frac{R_i R_e}{R_e + R_i + R_s} I$ (2)

Substituting (2) into (1) we get:

$V = A_v\left(-\frac{R_i R_e I}{R_e + R_i + R_s}\right) + (R_o + R_e)I - \frac{R_e^2 I}{R_e + R_i + R_s}$

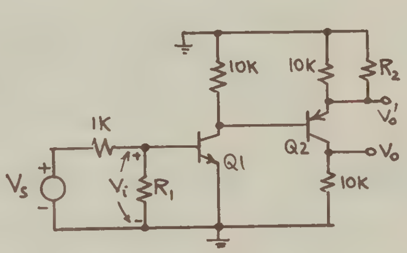
$R_{of} \equiv \frac{V}{I} = R_o + \frac{R_e[R_s + R_i(1 - A_v)]}{R_e + R_i + R_s}$

Consider $\frac{R_s + R_i(1 - A_v)}{R_s + R_i + R_e} = \frac{R_s + R_i + R_e - R_e - A_v R_i}{R_s + R_i + R_e} = 1 - \frac{A_v R_i + R_e}{R_s + R_i + R_e} = 1 - A_{vs}$

where use of Eq. (17-31) was made in the last step.

Hence, $R_{of} = R_o + R_e(1 - A_{vs})$ QED

(17-18) a) Assume that $-A_v' = -\frac{V_o'}{V_i} \gg 1$. Then, $R_2 = \frac{15K}{1 - 1/A_v'} \approx 15K$



and $R_{e2}' = 15K \| 10K = 6k$.

$R_{i2} \approx h_{ie} + (1 + h_{fe})R_{e2}' = 2k + 51 \times 6k = 308K$. The base-to-emitter voltage gain of Q2 is

$A_{v2}' = 1 - \frac{h_{ie}}{R_{i2}} = 1 - \frac{2K}{308K} = 1 - 0.0065 = 0.9935$.

The effective load of Q1 is $10K \| 308K = 9.7K = R_{L1}'$. Then $A_{v1} = -\frac{h_{fe}R_L'}{h_{ie}} = \frac{-50 \times 9.7K}{2K}$ $= -242$. Hence, $A_v' = A_{v1}A_{v2} = (-242)(0.9935) = -240$ ←

Note that $-A_v' = 240 \gg 1$ as assumed.

b) $R_1 = \frac{15k}{1 - A_v'} = \frac{15k}{241} = 62.2\ \Omega$. Thus, $R_{if} = R_1 \| h_{ie} = 62.2 \| 2000 \approx 60\ \Omega$ ←

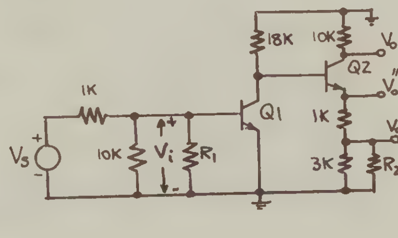
c) $A_{v2} = -\frac{h_{fe}R_{c2}}{R_{i2}} = -\frac{50 \times 10K}{308K} = -1.62$. $\therefore A_v = \frac{V_o}{V_i} = A_{v1}A_{v2} = (-242)(-1.62) = 392$ ←

d) $A_{vf} = \frac{V_o}{V_s} = A_v \frac{R_{if}}{R_{if} + R_s} = 392 \frac{60}{60 + 1000} = 22.2$ ←

e) From Eq. (17-49), $A_{vf} \approx \frac{R'}{R_e} \cdot \frac{R_{c2}}{R_s} = \frac{15K}{10K} \cdot \frac{10K}{1K} = 15$ ←

The agreement is bad, because Eq. (17-49) assumes that $R' \gg R_e$ which is not true here.

(17-19) a) Assume that $-A_v' = -\frac{V_o'}{V_i} \gg 1$. Then $R_2 = \frac{27K}{1 - 1/A_v'} \approx 27K$



and $R_{e2}'' = 27K \| 3K = 2.7K$. $\therefore R_{e2}' = 1K + 2.7K = 3.7K$

So, $R_{i2} \approx h_{ie} + (1 + h_{fe})R_{e2}' = 2K + 51(3.7K) \approx 190K$

$\frac{V_o''}{V_{i2}} = 1 - \frac{2K}{190K} = 1 - 0.010 = 0.99$ and $A_{v2}' = 0.99 \frac{2.7K}{3.7K} = 0.72$

(17-19) (Cont'd). The effective load of Q1 is: $18K \| 190K = 16.5K \quad \therefore A_{V_1} = \frac{-50 \times 16.5}{2} = -412$

Hence $A_V' = A_{V_1} A_{V_2}' = (-412)(0.72) = -297$ $\longleftarrow$

Note that $-A_V' = 297 \gg 1$ indeed as assumed.

b) $R_1 = \frac{27K}{1 - A_V'} = \frac{27K}{298} = 90.5\,\Omega \quad \therefore R_{if} = 90.5 \| 10{,}000 \| 2{,}000 \approx 86\,\Omega$ $\longleftarrow$

c) $A_{V_2} = \frac{V_o}{V_{i2}} = -\frac{50 \times 10K}{190K} = -2.63 \quad \therefore A_V = A_{V_1} A_{V_2} = (-412)(-2.63) = 1085$ $\longleftarrow$

d) $A_{Vf} = \frac{V_o}{V_s} = A_V \frac{R_{if}}{R_{if} + R_s} = 1085 \frac{86}{1086} \approx 86$ $\longleftarrow$

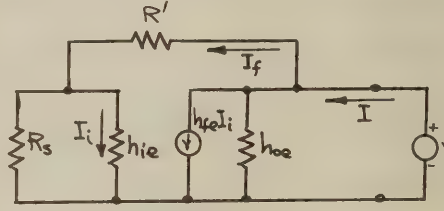
e) From Eq. (17-49), $A_{Vf} \approx \frac{R'}{R_{e_2}} \frac{R_{c2}}{R_s} = \frac{27K}{3K} \cdot \frac{10K}{1K} = 90$ $\longleftarrow$

The agreement is good here because $R' = 27K \gg R_{e2} = 3K$.

(17-20) a) $I = h_{fe} I_i + V h_{oe} + I_f \quad ; \quad I_f = \frac{V}{R' + h_{ie} \| R_s}$

and $I_i = \frac{R_s I_f}{R_s + h_{ie}}$. Hence,

$$I = \left(h_{oe} + \frac{1}{R' + h_{ie} \| R_s} + \frac{h_{fe} R_s}{R_s + h_{ie}} \frac{1}{R' + h_{ie} \| R_s} \right) V$$



$$\therefore Y_{of} = \frac{I}{V} = h_{oe} + \frac{1}{R' + h_{ie} \| R_s} \left(1 + \frac{h_{fe} R_s}{R_s + h_{ie}} \right) \quad \ldots \text{ Eq. (17-53)}$$

b) For the example of Sec. 17-9 we have:

$$Y_{of} = \frac{1}{40 \times 10^3} + \frac{1}{40 \times 10^3 + (1.1 \| 10)K} \left(1 + \frac{50 \times 10^4}{11.1 \times 10^3} \right) = 0.25 \times 10^{-4} + 11.2 \times 10^{-4} = 11.45 \times 10^{-4}$$

Hence, $R_{of} = \frac{1}{11.45 \times 10^{-4}} = 873\,\Omega$ $\longleftarrow$

(17-21) $R_e = 0$. The Miller resistance across 10K is $R_2 = \frac{100K}{1 - 1/A_V} \approx 100K$, if $-A_V \gg 1$.

a) Then, $R_L' = 10K \| 100K = 9.1K$ and $A_V = \frac{V_o}{V_i} = -h_{fe} \frac{R_L'}{h_{ie}} = -100 \times \frac{9.1K}{1K} = -910$ $\longleftarrow$

b) $A_{Vs} = A_V \frac{R_{if}}{R_{if} + R_s}$ where $R_{if} = R_1 \| h_{ie} = \frac{R'}{1 - A_V} \| h_{ie} = \frac{100K}{911} \| 1K = 100\,\Omega$

$\therefore A_{Vs} = -910 \frac{100}{1100} = -82.6$ $\longleftarrow$

c) From part b above, $R_{if} = 100\,\Omega$ $\longleftarrow$

d) From Eq. (17-53), $Y_{of} = \frac{1}{100 \times 10^3 + 0.5 \times 10^3} \left(1 + 100 \times \frac{1K}{2K} \right) \approx 10^{-5}(1 + 50) = 51 \times 10^{-5}$

Hence, $R_{of} = \frac{1}{51 \times 10^{-5}} = 1960\,\Omega$ $\longleftarrow$

$R_e = 1K$

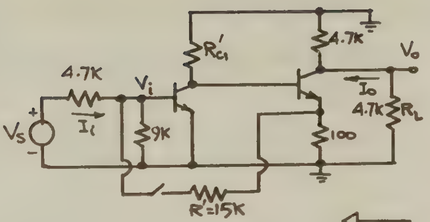
a) To the first approximation assume that $-A_V \gg 1$. Then, $R_2 \approx 100K$ and $R_L' = 10K \| 100K = 9.1K$. Also $R_i = h_{ie} + (1 + h_{fe}) R_e = 1K + 101 \times 1K = 102K$. Thus, $A_V = -h_{fe} \frac{R_L'}{R_i} = -100 \frac{9.1K}{102K} = -8.92$. Then $R_2 = \frac{100K}{1 + 1/8.92} \approx 90K$ and $R_L'' = 10K \| 90K = 9K$. For this value, $A_V = -\frac{100 \times 9K}{102K} \approx -8.8$ and $R_2 \approx 88.2K$ or $R_L''' \approx 9K$. Hence, $A_V = -8.8$ $\longleftarrow$

b) $R_{if} = \frac{100K}{1 + 8.8} \| 102K = (10.2 \| 102)K \approx 9.3K \quad \therefore A_{Vs} = -8.8 \frac{9.3K}{10.3K} = -7.95$ $\longleftarrow$

c) $R_{if} = 9.3K$ $\longleftarrow$

d) Y_{of} may be found from Eq. (17-53) by using $h_{ie}' = h_{ie} + (1 + h_{fe}) R_e = 102K$ instead of h_{ie}. Thus, $Y_{of} = \frac{1}{100 \times 10^3 + (1 \| 102) 10^3} \left(1 + 100 \frac{10^3}{(1 + 102) 10^3} \right) = 1.97 \times 10^{-5} \quad \therefore R_{of} = 51.2K$ $\longleftarrow$

(17-22) a) The simplified circuit is as shown:



Stage II : $R'_{c_2} = 4.7K \| 4.7K = 2.35K$; $A_{I_2} = -\frac{I_{c_2}}{I_{b_2}} \approx -50$

$R_{i_2} = 1.1K + 51(0.1K) = 6.2K$; $A_{V_2} = -50\,\frac{2.35}{6.2} = -19$; $R_o \approx 4.7K$

Stage I : $R'_{c_1} = (4.7\|91\|10)K = 3.1K$; $R''_{c_1} = (3.1\|6.2)K = 2.07K$

$A_{I_1} = -\frac{I_{c_1}}{I_{b_1}} \approx -50$; $R_{i_1} = h_{ie} = 1.1K$ $\therefore A_{V_1} = -50 \times \frac{2.07K}{1.1K} = -94$

Overall : $R_i = \frac{V_i}{I_i} = 9K \| 1.1K = 0.98K = 980\ \Omega$ $\longleftarrow$

$A_V = A_{V_1} A_{V_2} = (-94)(-19) = 1785$; $A_I = -\frac{I_o}{I_i} = A_V \frac{R_i}{R_L} = 1785\,\frac{0.98K}{4.7K} = 372$ $\longleftarrow$

$A_{Vs} = A_V \frac{R_i}{R_i + R_s} = 1785\,\frac{0.98}{0.98 + 4.7} = 308$; $R_o \approx 4.7K$ $\longleftarrow$

b) With the switch closed, we apply Miller's theorem to $R' = 15K$. Thus $R_1 = \frac{R'}{1 - A'_V}$ across the input terminals and $R_2 = \frac{R'}{1 - 1/A'_V}$ across the $100\ \Omega$ resistor, where A'_V is the first-base-to-second-emitter voltage gain. Assume that $-A' \gg 1$. Then, $R_2 \approx R' = 15K$ and $R_{i_2} \approx h_{ie} + (1 + h_{fe})R_{e_2} = 1.1k + 51(0.1k) = 6.2k$ since $0.1k \| 15k \approx 0.1K$; $A'_{V_2} = 1 - \frac{h_{ie}}{R_{i_2}} = 1 - \frac{1.1K}{6.2K} = \frac{5.1}{6.2} = 0.836$; $R''_{c_1} = (3.1\|6.2)K = 2.07K$ as in a.

$A_{V_1} = -94$. Thus, $A'_V = A_{V_1} A'_{V_2} = (-94)(0.836) = -78.5$; Note that $-A'_V \gg 1$.

$R_1 = \frac{15K}{1 + 78.5} = 189\ \Omega$. $\therefore R_i = 189 \| 9{,}000 \| 1100 = 159\ \Omega$; $A_V = (-94)(-19) = 1785$ $\longleftarrow$

$A_{Vs} = 1785\,\frac{159}{159 + 4700} = 58.5$; $A_I = 1785\,\frac{159}{4700} = 60.5$; $R_o \approx 4.7K$ $\longleftarrow$

(17-23) $h_{fe} = 100$ in Prob. 17-22b: $R_2 \approx 15K$ and $R_{e_2} = 0.1K \| 15K \approx 0.1K$ $\therefore R_{i_2} = 1.1K + 101(0.1K) = 11.1K$

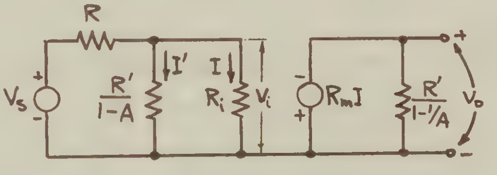
$A'_{V_2} = 1 - \frac{1.1K}{11.1K} = \frac{10}{11.1} = 0.90$; $A_{V_2} = -100\,\frac{2.35K}{11.1K} = -21.2$; $R''_{c_1} = (3.1\|11.1)K = 2.42K$

$A_{V_1} = -100\,\frac{2.42K}{1.1K} = -220K$; $A'_V = (-220)(0.9) = -198$ $\therefore R_1 = \frac{15{,}000}{199} = 75.4\ \Omega$

Hence, $R_i = 75.4 \| 9{,}000 \| 1{,}100 = 75.4 \| 980 = 74\ \Omega$; $A_V = (-220)(-21.2) = 4660$ $\longleftarrow$

$A_{Vs} = 4660\,\frac{74}{74 + 4700} = 72.2$; $A_I = 4660\,\frac{74}{4700} = 73.4$ and $R_o \approx 4.7K$ $\longleftarrow$

(17-24) Applying Miller's Theorem to R', we obtain the equivant circuit shown, where



$A = \frac{V_o}{V_i} = -\frac{R_m I}{R_i I} = -\frac{R_m}{R_i}$; $I' = \frac{R_i(1-A)}{R'} I = \frac{(R_i + R_m)}{R'} I$

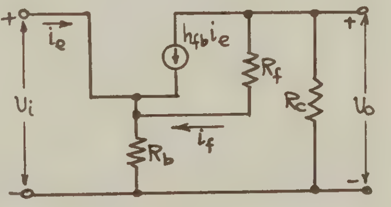
$\therefore V_s = (I' + I)R + IR_i = I\left(\frac{R_i + R_m}{R'} + 1\right)R + IR_i$

Hence,

$$A_{Vf} = \frac{V_o}{V_s} = \frac{-R_m I}{V_s} = \frac{-R_m}{\left(\frac{R_i + R_m}{R'} + 1\right)R + R_i} = \frac{-R_m R'}{(R_i + R_m + R')R + R_i R'}$$

$$= \frac{-R' R_m}{R\left(R_m + R_i + R' + \frac{R_i R'}{R}\right)} = \frac{-R'}{R\left(1 + \frac{R_i + R'}{R_m} + \frac{R_i R'}{R R_m}\right)} = -\frac{R'}{R} \cdot \frac{1}{1 + \frac{R'}{R_m}\left(\frac{R_i + R'}{R'} + \frac{R_i}{R}\right)}$$

(17-25) a) Under the specified conditions, the equivalent circuit is as shown:



$v_i = [(1 + h_{fb})i_e + i_f]R_b$; $v_o = -(h_{fb} i_e + i_f)R_c$ where $i_f = (v_o - v_i)/R_f$. Substituting, we get:

$v_o = -h_{fb} i_e R_c - \frac{R_c}{R_f}(v_o - v_i)$ or $v_o\left(1 + \frac{R_c}{R_f}\right) = \frac{R_c}{R_f} v_i - h_{fb} R_c i_e$ (1)

(17-25) (Cont'd) $v_i = \left[(1+h_{fb})i_e + \frac{v_o - v_i}{R_f}\right] R_b$ or $i_e = \frac{1}{R_b}\left(\frac{v_i}{1+h_{fb}}\right) - \frac{v_o - v_i}{(1+h_{fb})R_f}$ (2)

From (1) and (2): $v_o\left(1+\frac{R_c}{R_f}\right) = \frac{R_c}{R_f}v_i - \frac{h_{fb}R_c}{(1+h_{fb})R_b}v_i + \frac{h_{fb}R_c}{(1+h_{fb})R_f}(v_o - v_i)$

or $v_o\left[1+\frac{R_c}{R_f}\left(1-\frac{h_{fb}}{1+h_{fb}}\right)\right] = \left[\frac{R_c}{R_f}\left(1-\frac{h_{fb}}{1+h_{fb}}\right) - \frac{h_{fb}}{1+h_{fb}}\frac{R_c}{R_b}\right]v_i$

Hence, $A_{vf} = \frac{v_o}{v_i} = \frac{\frac{R_c}{R_f}\left(1-\frac{h_{fb}}{1+h_{fb}}\right) - \frac{h_{fb}}{1+h_{fb}}\frac{R_c}{R_b}}{1+\frac{R_c}{R_f}\left(1-\frac{h_{fb}}{1+h_{fb}}\right)} = \frac{\frac{R_c}{R_f} - h_{fb}\frac{R_c}{R_b}}{1+h_{fb}+\frac{R_c}{R_f}} = \frac{\frac{1}{R_f}-\frac{h_{fb}}{R_b}}{\frac{1+h_{fb}}{R_c}+\frac{1}{R_f}}$ QED

b) Using the result of part a, we obtain: $\frac{1}{R_f} - \frac{h_{fb}}{R_b} = A_{vf}\left(\frac{1+h_{fb}}{R_c}+\frac{1}{R_f}\right)$

Multiplying both sides by R_f, we have:

$1 - \frac{R_f}{R_b}h_{fb} = A_{vf}\left[(1+h_{fb})\frac{R_f}{R_c}+1\right]$ or $\frac{R_f}{R_b} = \frac{1}{h_{fb}}\left\{1 - A_{vf}\left[(1+h_{fb})\frac{R_f}{R_c}+1\right]\right\}$ QED

(17-26) a) Using superposition, we have: $V_i = \frac{Z_i \| Z'}{Z_i \| Z' + Z} V_s + \frac{Z \| Z_i}{Z \| Z_i + Z'} V_o$

$$= \frac{\frac{Z_i Z'}{Z_i + Z'} V_s}{\frac{Z_i Z'}{Z_i+Z'} + Z} + \frac{\frac{Z Z_i}{Z+Z_i} V_o}{\frac{Z Z_i}{Z+Z_i} + Z'} = \frac{Z_i Z' V_s}{Z_i Z' + Z_i Z + Z Z'} + \frac{Z Z_i V_o}{Z Z_i + Z Z' + Z_i Z'}$$

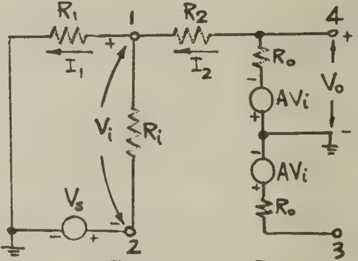
Dividing numerator and denominator of each term by $Z_i Z' Z$, we get

$$V_i = \frac{\frac{1}{Z} V_s}{\frac{1}{Z} + \frac{1}{Z'} + \frac{1}{Z_i}} + \frac{\frac{1}{Z'} V_o}{\frac{1}{Z} + \frac{1}{Z'} + \frac{1}{Z_i}} = \frac{Y V_s}{Y+Y'+Y_i} + \frac{Y' V_o}{Y+Y'+Y_i} \equiv \alpha V_s - \beta V_o$$

where $\alpha \equiv \frac{Y}{Y+Y'+Y_i}$ and $\beta \equiv \frac{-Y'}{Y+Y'+Y_i}$. From the given circuit we obtain by inspection: $V_i = \alpha V_s - \beta V_o$. Hence, Fig. 17-26b is equivalent to the given circuit.

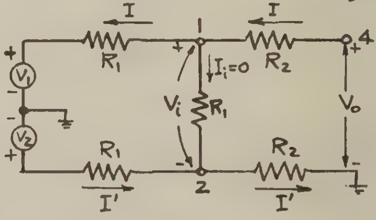
b) From Eq. (17-12) we obtain: $A_{vf} = \frac{V_o}{V_s} = \frac{\alpha A_v}{1+\beta A_v} = \frac{Y A_v/(Y+Y'+Y_i)}{1 - Y' A_v/(Y+Y'+Y_i)}$ QED

c) $A_{vf} = \frac{Y A_v}{Y+Y'+Y_i - Y' A_v} = \frac{-Y}{Y' - \frac{1}{A_v}(Y+Y'+Y_i)}$ ... Eq. (17-56)



(17-27) If the gain is infinite (i.e. $A \to -\infty$), then for a finite V_o we must have $V_i = 0$. Hence, the current in R_i is zero and $I_1 = I_2 = \frac{V_s}{R_1} = \frac{V_o - V_s}{R_2}$

$\therefore \left(\frac{R_2}{R_1}+1\right)V_s = V_o$ or $A_{vf} = \frac{V_o}{V_s} = 1 + \frac{R_2}{R_1}$ QED



(17-28) With V_o finite, $V_i = 0$ for $A = -\infty$ $\therefore I_i = 0$

Then, $I = \frac{V_o - V_1}{R_1+R_2}$ and $I' = \frac{V_2}{R_1+R_2}$

$\therefore V_i = V_1 - V_2 + IR_1 + I'R_1 = V_1 - V_2 + \frac{R_1}{R_1+R_2}(V_o - V_1) + \frac{R_1}{R_1+R_2}V_2$

or $V_i = V_1 - V_2 + \frac{R_1}{R_1+R_2}(V_o - V_1 + V_2) = 0$

or $(V_1 - V_2)\left(1 - \frac{R_1}{R_1+R_2}\right) = -\frac{R_1}{R_1+R_2}V_o$ or $(V_1-V_2)\frac{R_2}{R_1+R_2} = -\frac{R_1}{R_1+R_2}V_o$ $\therefore V_o = \frac{R_2}{R_1}(V_2 - V_1)$

QED

(17-29) We now have: $(V_4-V_1)\frac{R_1}{R_1+R_2}+V_1=(V_3-V_2)\frac{R_1}{R_1+R_2}+V_2$

or $(V_4-V_1)R_1+V_1(R_1+R_2)=(V_3-V_2)R_1+V_2(R_1+R_2)$ or $V_4R_1+V_1R_2=V_3R_1+V_2R_2$

Hence, $V_4-V_3=\frac{R_2}{R_1}(V_2-V_1)=V_o$ because $V_o=V_4-V_3$ QED

(17-30) a) $V_o=AV_{12}=A(V_1-V_2)$ where $V_1=V_s$ and $V_2=\frac{R}{R+nR}V_o=\frac{V_o}{n+1}$ since $R_i=\infty$.

Thus, $V_o=A\left(V_s-\frac{V_o}{n+1}\right)$ or $V_o\left(1+\frac{A}{n+1}\right)=AV_s$

and $A_{Vf}=\frac{V_o}{V_s}=\frac{A}{1+\frac{A}{n+1}}=\frac{n+1}{1+\frac{n+1}{A}}$ ←

b) $\lim\limits_{A\to\infty}A_{Vf}=\lim\limits_{A\to\infty}\frac{n+1}{1+\frac{n+1}{A}}=n+1$ QED

(17-31) If terminals 1 and 2 are short-circuited, the current flowing in this short is, $I_{sc}=\frac{V_s}{Z}=V_sY$. The admittance between terminals 1 and 2 is, $Y_{12}=Y+Y_i+(1-A_v)Y'$. Hence, $V_i=I_{sc}/Y_{12}=\frac{V_sY}{Y+Y_i+(1-A_v)Y'}$.

Then, $A_{Vf}=\frac{V_o}{V_s}=\frac{A_vV_i}{V_s}=\frac{A_vY}{Y+Y_i+(1-A_v)Y'}=\frac{-Y}{Y'-\frac{1}{A_v}(Y+Y_i+Y')}$ . . . . Eq. (17-56)

(17-32) From Eq. (17-55), $-\frac{Z'}{Z}=1\angle 45^\circ$. $\therefore |Z'|=|Z|$ and $-\angle\theta'-\theta=\angle 45^\circ$

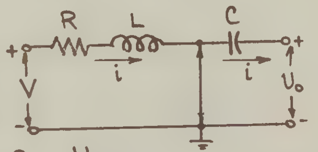
or $\theta'-\theta=45^\circ-180^\circ=-135^\circ$. Since θ' and θ are each less than 90°, then θ' must be negative and θ positive.

Hence, $Z'=R'-jX_c$ and $Z=R+jX_L$ such that $(R'^2+X_c^2)=(R^2+X_L^2)$ and $|\theta'|+|\theta|=135^\circ$. One special case is: $R'=X_c$, so that $\theta'=-45^\circ$ and $R=0$, so that $\theta=90^\circ$ (with $X_L=\sqrt{2}R'$.).

(17-33) Assuming that $Y_i=0$, we have from Eq. (17-56): $A_{Vf}=\frac{-Y}{Y'-\frac{1}{A_V}(Y'+Y)}=\frac{-Y/Y'}{1-\frac{1}{A_V}(1+Y/Y')}$.

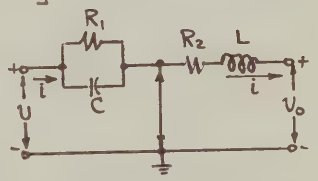
But $Y/Y'=Z'/Z=-jX_c/R=-j$ and $A_V=-100$.

Hence, $A_{Vf}=\frac{+j}{1+0.01(1-j)}=\frac{j}{1.01-0.01j}=\frac{1\angle 90^\circ}{1.01\angle -0.57^\circ}=0.99\angle 90.57^\circ$ ←



(17-34) Because of the virtual ground, V is impressed across R and L in series. Thus, $V=L\frac{di}{dt}+iR$. If $i=0$ at $t=0$, then, $i=\frac{V}{R}(1-e^{-\frac{Rt}{L}})$. Since v_o is the voltage across C, then

$$v_o=-\frac{1}{C}\int_0^t i\,dt=-\frac{V}{RC}\int_0^t(1-e^{-\frac{Rt}{L}})\,dt=-\frac{V}{RC}\left[t+\frac{L}{R}(e^{-\frac{Rt}{L}}-1)\right]$$

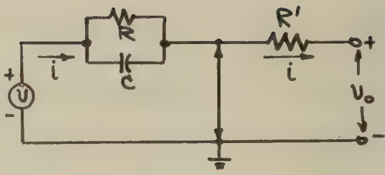


(17-35) Because of the virtual ground, v is impressed across R_1 and C in parallel.

Thus, $i=\frac{v}{R_1}+C\frac{dv}{dt}$. Since v_o is the voltage across R_2 and L in series, we have:

$$-v_o=iR_2+L\frac{di}{dt}=\frac{R_2}{R_1}v+R_2C\frac{dv}{dt}+\frac{L}{R_1}\frac{dv}{dt}+LC\frac{d^2v}{dt^2}=\frac{R_2}{R_1}v+\left(R_2C+\frac{L}{R_1}\right)\frac{dv}{dt}+LC\frac{d^2v}{dt^2}$$

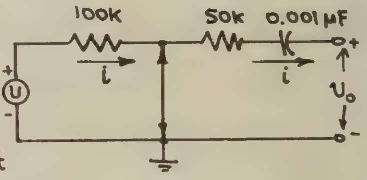
(17-36) Because of the virtual ground, v is impressed across R and C in parallel. Thus, $i = \frac{v}{R} + C\frac{dv}{dt}$. If $v = \alpha t$, then $i = \frac{\alpha t}{R} + \alpha C$.
Hence, $v_o = -iR' = -\alpha R'C - \alpha\frac{R'}{R}t$ QED



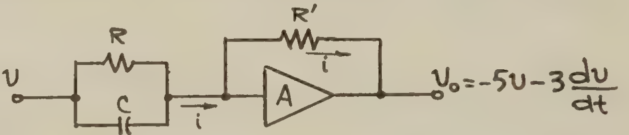
(17-37) From the equivalent circuit, we have: $v = 10e^{-2\times10^3t}$ and $i = \frac{v}{100K} = \frac{10}{10^5}e^{-2\times10^3t} = 10^{-4}e^{-2\times10^3t}$

Then, $v_o(t) = -5\times10^4 i - \frac{1}{10^{-9}}\int_0^t i\,dt = -5e^{-2\times10^3t} - 10^5\int_0^t e^{-2\times10^3t}dt$

or $v_o(t) = -5e^{-2\times10^3t} + 50e^{-2\times10^3t} - 50 = 45e^{-2\times10^3t} - 50$



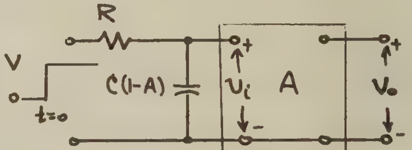
(17-38)



$i = \frac{v}{R} + C\frac{dv}{dt}$

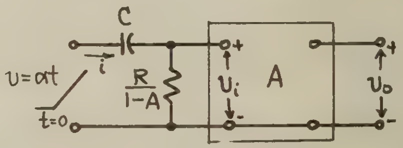
$v_o = -R'i = -\frac{R'}{R}v - R'C\frac{dv}{dt} \equiv -5v - 3\frac{dv}{dt}$ $\therefore \frac{R'}{R} = 5$ or $R = \frac{R'}{5}$ and $C = \frac{3}{R'}$

(17-39) a) The equivalent circuit corresponding to Fig. 17-30a is as shown. Note that $Z'(s)/(1-A) = \frac{1}{sC(1-A)}$ and thus $Z'/(1-A)$ represents a capacitor of value $C(1-A)$ as shown. Assume that $R_i = \infty$. Then the time constant of the input circuit is $\tau = RC(1-A)$. Hence, for $t \geq 0$, we have: $v_i = V(1-e^{-t/\tau})$ since at $t=0$, $v_i = 0$ and at $t = \infty$, $v_i = V$.
Thus, $v_o = Av_i = AV[1 - e^{-t/RC(1-A)}]$ QED



b) For the simple RC integrating circuit, $v_o = V(1-e^{-t/RC})$ which for large RC becomes: $v_o = \frac{Vt}{RC}(1 - \frac{t}{2RC} + \dots)$. For the operational integrator of part a we have: $v_o = \frac{AVt}{RC(1-A)}[1 - \frac{t}{2RC(1-A)} + \dots] \approx -\frac{Vt}{RC}[1 - \frac{t}{2RC(1-A)} + \dots]$ if $-A \gg 1$. Thus, the output voltage of both circuits varies approximately linearly with time, if RC is large, and for either circuit $\left|\frac{dv_o}{dt}\right| = \frac{V}{RC}$. Since the second term in the expressions represents the deviation from linearity, we see that the operational integrator is more linear than the simple RC circuit by a factor of $1/(1-A)$.

(17-40) a) The equivalent circuit of an operational differentiator is as shown, from which: $v = \alpha t = \frac{iR}{1-A} + \frac{1}{C}\int i\,dt$. Differentiating, we get:
$\alpha = \frac{i}{C} + \frac{R}{1-A}\frac{di}{dt}$. Let $\tau = \frac{RC}{1-A}$, then $v_i = \frac{iR}{1-A}$ or $i = \frac{(1-A)}{R}v_i$
and $\alpha = \frac{v_i}{\tau} + \frac{dv_i}{dt}$ whose solution is $v_i = \alpha\tau + Ke^{-t/\tau} = \alpha\tau(1-e^{-t/\tau})$
since at $t=0$, $v_i = 0 = v$. Now, $v_o = Av_i = \frac{A}{1-A}\alpha RC[1 - e^{-t\frac{(1-A)}{RC}}]$ QED

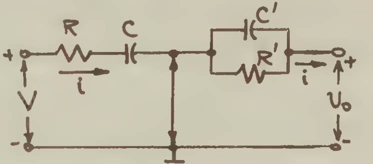


b) For the simple RC differentiating circuit we have: $v = \alpha t = iR + \frac{1}{C}\int i\,dt$

(17-40) (Cont'd). Differentiating, we get : $\alpha = \frac{i}{C} + R\frac{di}{dt}$. But $v_o = iR$ or $i = \frac{v_o}{R}$. Thus, $\alpha = \frac{v_o}{RC} + \frac{dv_o}{dt}$ whose solution is : $v_o = \alpha RC + K\bar{e}^{t/RC} = \alpha RC(1-\bar{e}^{t/RC})$ since @ $t=0$, $v_o = v = 0$.

The final value of output for (a) is $\frac{A}{1-A}\alpha RC \approx -\alpha RC$ if $-A \gg 1$, and for (b) αRC. Hence, $|v_o(\infty)|$ is approximately the same for both circuits. Since $\alpha = \frac{dv}{dt}$, then $v_o(\infty) = RC\frac{dv}{dt}$, demonstrating that both circuits do indeed perform differentiation.

If v_o reaches, say, 99.9% of its final value in time t_b for case b, then $e^{-t_b/RC} = 0.001$. If t_a is the time for v_o to reach 99.9% of its final value for case a, then $e^{-t_a(1-A)/RC} = 0.001$. Hence, $t_a = \frac{t_b}{1-A}$ and for $-A \gg 1$, t_a is very small indeed, indicating that the operational differentiator approaches its final value much more quickly than does the simple RC circuit.



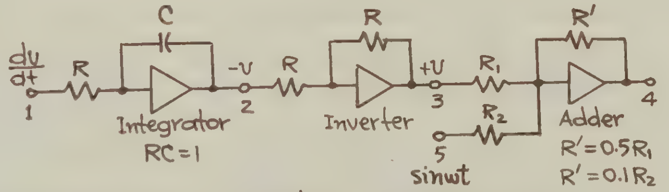
(17-41) a) Assume that the capacitors are initially uncharged. Since the voltage across C' cannot change abruptly, then v_o starts at zero. In the steady state there can be no current through C or C'. Hence, i drops to zero, the current through R' becomes zero and v_o falls to zero. Thus, v_o starts at zero, increases to some finite value and then drops to zero.

b) $V = iR + \frac{1}{C}\int i\,dt$ or $0 = R\frac{di}{dt} + \frac{i}{C}$ $\therefore$ $i = K\bar{e}^{t/RC} = \frac{V}{R}\bar{e}^{t/RC}$ since $i = \frac{V}{R}$ at $t=0$. Because of the virtual ground, R' and C' are in parallel with v_o across them. Hence, $-i = \frac{v_o}{R'} + C'\frac{dv_o}{dt} = -\frac{V}{R}\bar{e}^{t/RC}$. The solution is of the form : $v_o = K_1\bar{e}^{t/RC} + K_2\bar{e}^{t/R'C'}$, but since at $t=0$, $v_o = 0$, then $K_1 = -K_2$ and $v_o = K_1(\bar{e}^{t/RC} - \bar{e}^{t/R'C'})$. To find K_1, substitute into the differential equation: Thus, $\bar{e}^{t/RC}\left(\frac{K_1}{R'} - \frac{K_1C'}{RC}\right) - \bar{e}^{t/R'C'}\left(\frac{K_1}{R'} - \frac{K_1C'}{R'C'}\right) = \frac{-V}{R}\bar{e}^{t/RC}$

$$\frac{K_1}{R'} - \frac{K_1C'}{RC} = -\frac{V}{R} \quad \text{or} \quad K_1 = \frac{R'CV}{R'C'-RC} \quad \text{provided that } R'C' \neq RC$$

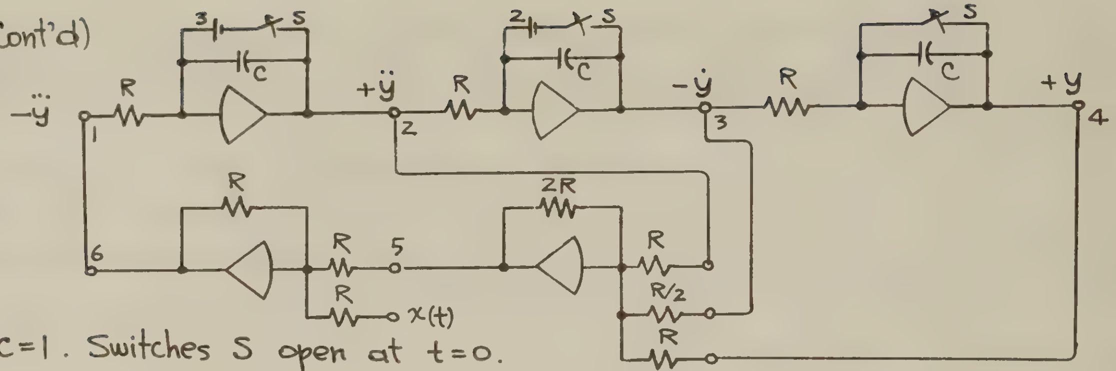
$$\therefore \quad v_o = \frac{R'CV}{R'C'-RC}\left(\bar{e}^{t/RC} - \bar{e}^{t/R'C'}\right) \qquad \text{QED}$$

(17-42) If the input at terminal 1 is $\frac{dv}{dt}$ and at 5 $\sin\omega t$, then the output at 4 is $-0.5v - 0.1\sin\omega t$. But from the given equation this equals $\frac{dv}{dt}$. Hence, terminals 4 and 1 must be connected together. The solution, $v(t)$, is taken from terminal 3.



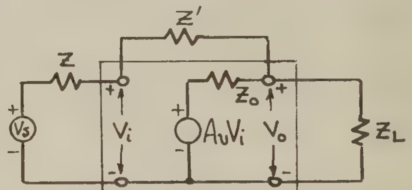
(17-43) Let $\frac{dy}{dt} \equiv \dot{y}$, $\frac{d^2y}{dt^2} \equiv \ddot{y}$ and $\frac{d^3y}{dt^3} \equiv \dddot{y}$. Then, the given equation may be written in the form: $-\dddot{y} = -x + 2\ddot{y} - 4\dot{y} + 2y$ which may be set up using operational amplifiers as follows:

(17-43) (Cont'd)



RC=1. Switches S open at t=0.

(17-44) a) See circuit



b) Evaluating V_o as the product of the short-circuit current times the impedance between the output terminals, we have:

$$V_o = \frac{\frac{V_i}{Z'} + \frac{A_vV_i}{Z_o}}{\frac{1}{Z'}+\frac{1}{Z_o}+\frac{1}{Z_L}} = \frac{V_iZ_L\left(\frac{1}{Z'}+\frac{A_v}{Z_o}\right)}{Z_L\left(\frac{Z_o+Z'}{Z_oZ'}\right)+1} \quad \text{or} \quad \frac{V_o}{V_i} = \frac{Z_L\left(\frac{Z_o+A_vZ'}{Z_oZ'}\right)\left(\frac{Z_oZ'}{Z_o+Z'}\right)}{Z_L+\frac{Z_oZ'}{Z_o+Z'}} = A_v\left(\frac{Z'+\frac{Z_o}{A_v}}{Z'+Z_o}\right)\left(\frac{Z_L}{Z_L+\frac{Z_oZ'}{Z_o+Z'}}\right)$$

c) From b we have: $\frac{V_o}{V_i} = \frac{AZ_L}{Z_L+Z_o'}$ where $A = A_v\frac{Z'+Z_o/A_v}{Z'+Z_o}$ and $Z_o' = \frac{Z_oZ'}{Z_o+Z'}$ QED

d) Evaluating V_i as the product of the short-circuit current times the impedance between the input terminals, we have:

$$V_i = \frac{\frac{V_s}{Z}+\frac{V_o}{Z'}}{\frac{1}{Z}+\frac{1}{Z'}} = \frac{Z'V_s+ZV_o}{Z'+Z}$$

. From the equations defining A and Z_o',

$$V_o = \frac{AZ_L}{Z_L+Z_o'}V_i = \left(A\frac{Z_L}{Z_L+Z_o'}\right)\left(\frac{Z'V_s}{Z+Z'}+\frac{ZV_o}{Z+Z'}\right) \quad \text{or} \quad V_o\left(1-A\frac{Z_L}{Z_L+Z_o'}\frac{Z}{Z+Z'}\right) = \frac{AZ_L}{Z_L+Z_o'}\,\frac{Z'V_s}{Z+Z'}$$

$$\text{or} \quad \frac{V_o}{V_s} = \frac{AZ'Z_L}{(Z_L+Z_o')(Z+Z')-AZZ_L} = \frac{AZ'Z_L}{Z_L(Z+Z'-AZ)+Z_o'(Z+Z')} = \frac{AZ'}{(Z+Z'-AZ)}\cdot\frac{Z_L}{Z_L+\frac{Z_o'(Z+Z')}{Z+Z'-AZ}} \equiv \frac{A_{vf}Z_L}{Z_L+Z_{of}}$$

$$\therefore A_{vf} = \frac{AZ'}{Z+Z'-AZ} = \frac{1/Z}{\frac{1}{A}\left(\frac{1}{Z'}+\frac{1}{Z}\right)-\frac{1}{Z'}} = \frac{-Y}{Y'-\frac{1}{A}(Y'+Y)}$$

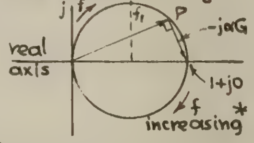
which agrees with Eq. (17-56).

$$\text{and} \quad Z_{of} = \frac{Z_o'(Z+Z')}{Z+Z'-AZ} = \frac{Z_o'}{1-AZ/(Z+Z')}$$

QED

(17-45) Assume that β is independent of frequency. At low frequencies, $A_1\beta = \frac{A_o\beta}{1-jf_1/f} = |A_o\beta|\,G$ where $G = \frac{1}{1-jf_1/f} = \frac{1}{1-j\alpha}$ where $\alpha = f_1/f$. Thus, $G - j\alpha G = 1$.

At a given frequency, α is a constant and G is a vector in the complex plane. This equation states that to vector G we must add another vector which lags G by 90° (because of the factor $-j$) and that their sum is the vector $1+j0$.



This interpretation locates the end point P of vector G on a semicircle as shown. In the midband region $f \gg f_1$ and $\alpha \to 0$, so that $G \to 1$. As $f \to 0$, $\alpha \to \infty$ and $G \to \frac{j}{\alpha}$ i.e. G approaches zero along the +j-axis. These two extremes are consistent with the semicircular plot. Note that $f = f_1$ means that $\alpha = 1$ and $G - jG = 1$ and this locates the lower 3dB frequency half way around the top semicircle as shown.

Starting with the equation of gain at high frequencies and proceeding as above we obtain the lower half of the circle diagram.

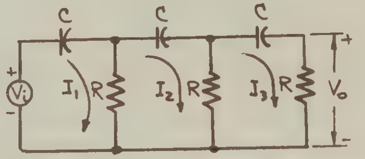
(17-46) a) The loop gain must be less than unity when the overall phase shift is 180° or when the phase shift per stage is 60°. At high frequencies, $\tan\theta = f/f_2 = \tan 60° = \sqrt{3}$ and $A_2 = \frac{A_o}{[1+(f/f_2)^2]^{1/2}} = \frac{A_o}{(1+3)^{1/2}} = \frac{A_o}{2}$

The three-stage gain is $(\frac{A_o}{2})^3 = \frac{A_o^3}{8}$ and the loop gain at this frequency which causes 180° overall phase shift is $\frac{\beta A_o^3}{8} < 1$. Hence, the midband loop gain βA_o^3 must not exceed 8.

b) $|A_o| = g_m R = 6\times10^{-3}/(\frac{1}{10}+\frac{1}{50}+\frac{1}{1000})\times10^{-3} = 49.6$ $\therefore (R_1)_{max} = \beta_{max} R_g = \frac{8}{|A_o|^3} R_g$

or $(R_1)_{max} = \frac{8\times10^6}{(49.6)^3} = 67\ \Omega$ ⟵

(17-47) Let $X \equiv \frac{1}{\omega C}$. Then, using mesh analysis, we have:



$$V_i = I_1(R-jX) - I_2 R + I_3(0)$$
$$0 = -I_1 R + I_2(2R-jX) - I_3 R$$
$$0 = I_1(0) - I_2 R + I_3(2R-jX)$$
; I_3 is found from $I_3 = \Delta_3/\Delta$ where $\alpha = \frac{X}{R} = \frac{1}{\omega RC}$

$$\Delta = \begin{vmatrix} R-jX & -R & 0 \\ -R & 2R-jX & -R \\ 0 & -R & 2R-jX \end{vmatrix} = R^3 \begin{vmatrix} 1-j\alpha & -1 & 0 \\ -1 & 2-j\alpha & -1 \\ 0 & -1 & 2-j\alpha \end{vmatrix} = R^3[(1-j\alpha)(2-j\alpha)^2 - (1-j\alpha) - (2-j\alpha)] = R^3[1-5\alpha^2 + j(\alpha^3 - 6\alpha)]$$

$$\Delta_3 = \begin{vmatrix} R-jX & -R & V_i \\ -R & 2R-jX & 0 \\ 0 & -R & 0 \end{vmatrix} = R^2 V_i \qquad \therefore I_3 = \frac{\Delta_3}{\Delta} = \frac{R^2 V_i}{R^3[1-5\alpha^2 + j(\alpha^3-6\alpha)]}$$

Hence, $-\beta = \frac{V_o}{V_i} = \frac{I_3 R}{V_i} = \frac{1}{1-5\alpha^2 + j(\alpha^3 - 6\alpha)}$ .... Eq. (17-64)

For 180° phase shift, $\alpha^3 - 6\alpha = 0$ or $\alpha^2 = 6$ and $\beta = \frac{-1}{1-5\alpha^2} = \frac{-1}{1-30} = +\frac{1}{29}$

(17-48) a) From the mesh equations derived in Prob. 17-47, $I_1 = \frac{\Delta_1}{\Delta}$ where

$$\Delta_1 = \begin{vmatrix} V_i & -R & 0 \\ 0 & 2R-jX & -R \\ 0 & -R & 2R-jX \end{vmatrix} = V_i[(2R-jX)^2 - R^2] = V_i R^2 (3-\alpha^2 - j4\alpha)$$

$\therefore Z_i = \frac{V_i}{I_i} = \frac{V_i \Delta}{\Delta_1} = R\left[\frac{1-5\alpha^2 + j(\alpha^3 - 6\alpha)}{3-\alpha^2 - j4\alpha}\right]$ QED

b) For $\alpha = \sqrt{6}$, $Z_i = R\left[\frac{1-5\times6 + j(6\sqrt{6}-6\sqrt{6})}{3-6-j4\sqrt{6}}\right] = R\,\frac{-29}{-3-j4\sqrt{6}} = R(0.83 - j2.7)$ QED

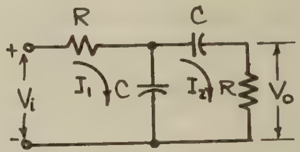
(17-49) a) Since the gain must exceed 29, a tube with $\mu < 29$ cannot be used. Thus, the 6CG7 and 12AU7 cannot be used.

b) $|A| = \frac{\mu R_p}{r_p + R_p} = 29$ or $R_p = \frac{29 r_p}{\mu - 29}$. For example: for a 12AT7, $\mu = 55$ and $r_p = 5.5\text{k}$
$\therefore R_p = \frac{29\times5.5\text{k}}{55-29} = 6.12\text{k}$

c) $\alpha = \frac{1}{2\pi f RC} = \sqrt{6}$ or $RC = 1/(2\pi\times5\times10^3\sqrt{6}) = 12.9\ \mu\text{sec}$

d) In order not to load down the amplifier, the input impedance of the phase shift network must be high compared to the output impedance of the amplifier. If R is chosen too large, however, C will be impractically small. e.g. if R = 1M, C = 12.9 pF which is of the same order of magnitude as stray wiring capacitance. This argument shows the advantage of using a tube with a low plate resistance such as the 12AT7.

(17-49) (Cont'd) If we choose this tube, the amplifier output impedance is $\frac{r_p R_p}{r_p+R_p}$ $= \frac{5.5K \times 6.12K}{5.5K + 6.12K} = 2.9\,K$. If we choose R to be, say, 10 times this value, or 30K, then the loading of the phase shift network will be negligible. Then $C = \frac{12.9 \times 10^{-6}}{3 \times 10^4} = 430\text{ pF}$



(17-50) a) Let $X = \frac{1}{\omega C}$. Then

$$V_i = I_1(R-jX) - I_2(-jX)$$
$$0 = -I_1(-jX) + I_2(R-j2X)$$

$$I_1 = \left(\frac{R-j2X}{-jX}\right) I_2 = \left(2+j\frac{R}{X}\right) I_2 \text{ so } V_i = I_2\left[(R-jX)\left(2+j\frac{R}{X}\right)+jX\right]$$
$$= I_2\left[3R + j\left(\frac{R^2}{X} - X\right)\right]$$

$$\therefore \frac{V_o}{V_i} = \frac{I_2 R}{V_i} = \frac{R}{3R+j\left(\frac{R^2}{X}-X\right)} = \frac{1}{3+j\left(\frac{R}{X}-\frac{X}{R}\right)} = \frac{1}{3+j\left(\omega RC - \frac{1}{\omega RC}\right)} \quad \text{QED}$$

b) For zero phase shift, $\left(\omega RC - \frac{1}{\omega RC}\right) = 0$ or $\omega RC = 1$ and $f = \frac{1}{2\pi RC}$ QED

Since $\beta = \frac{V_o}{V_i} = \frac{1}{3}$ and $|A\beta| > 1$, then $A > 3$. ◁—

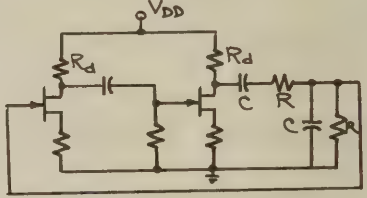
(17-51) a) Using voltage division, we have

$$\frac{V_o}{V_i} = \frac{R(-jX)}{(R-jX)} \cdot \frac{1}{R-jX+\frac{R(-jX)}{R-jX}} = \frac{-jRX}{R^2-j2RX-X^2-jRX}$$
$$= \frac{1}{3+j\left(\frac{R}{X}-\frac{X}{R}\right)} = \frac{1}{3+j\left(\omega RC - \frac{1}{\omega RC}\right)}$$

b) At the frequency at which β is real, $\beta = \frac{1}{3}$. Since $A\beta > 1$, then $A > 3$ must be positive. Hence, a two stage amplifier is necessary.

c) For zero phase shift, $\omega RC = \frac{1}{\omega RC}$ or $f = \frac{1}{2\pi RC}$

d) From part b, $A_{min} = 3$



◁—

(17-52) Using mesh analysis and letting $X = \frac{1}{\omega C}$, we have:

$$V_i = I_1(R-jX) - I_2 R$$
$$0 = -I_1 R + I_2(2R-jX)$$

or $V_i = I_2\left[\frac{(R-jX)(2R-jX)}{R} - R\right] = \frac{I_2}{R}(R^2 - X^2 - j3RX)$

$$\therefore \frac{V_o}{V_i} = \frac{I_2 R}{V_i} = \frac{1}{1-\left(\frac{X}{R}\right)^2 - j\frac{3X}{R}} = \frac{1}{1-\alpha^2-j3\alpha} \quad \text{where } \alpha = \frac{1}{\omega RC} = \frac{X}{R}$$

In order to have a phase shift of 180°, $\tan\theta = \tan 180° = 0 = \frac{3\alpha}{1-\alpha^2}$. Hence, either $\alpha = 0$ or $\alpha = \infty$. If $\alpha = 0$, then $\frac{V_o}{V_i} = +1$ and the phase shift is 0° and not 180°. Note that $\alpha = 0$ requires either $R = \infty$ or $C = \infty$, which means that the output is connected directly to the input. On the other hand, if $\alpha = \infty$, then $\frac{V_o}{V_i} = 0$ and this would require an amplifier with infinite gain. Note that $\alpha = \infty$ means either that $C = 0$ (the output separated from the input by an infinite impedance) or $R = 0$ (the output short-circuited) so the attenuation is infinite.

(17-53) a) This network would be identical with that of Prob. 17-47 if R and (-jX) were interchanged. Hence the solution may be obtained from Prob. 17-47 by replacing $\alpha = \frac{X}{R} = \frac{j(-jX)}{R}$ by $\frac{jR}{(-jX)} = \frac{R}{-X} = -\omega RC$. Let $\gamma = \omega RC$. Then, we must replace α by $-\gamma$.

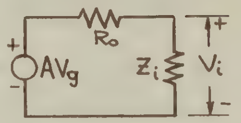
$$\therefore \frac{V_o}{V_i} = \frac{1}{1-5\alpha^2+j(\alpha^3-6\alpha)} = \frac{1}{1-5\gamma^2+j(6\gamma-\gamma^3)} \quad \text{where } \gamma = \omega RC$$ ◁—

(17-53) (Cont'd) b) The solution is obtained by replacing α by $-\gamma$ and R by $(-jX)$ in Prob. 17-48. $\therefore\ Z_i = -\frac{j}{wC}\left[\frac{1-5\gamma^2+j(6\gamma-\gamma^3)}{3-\gamma^2+j4\gamma}\right]$ where $\gamma = \omega RC$

c) For β real, $6\gamma-\gamma^3=0$ or $\gamma^2 = 6 = \omega^2R^2C^2$ $\therefore\ f = \frac{\sqrt{6}}{2\pi RC}$ ◁—

$-\beta = \frac{V_o}{V_i} = \frac{1}{1-5\times 6} = -\frac{1}{29}$, $|A\beta| > 1$ $\therefore\ |A| > 29$ ◁—

(17-54) In the equivalent circuit shown Z_i is given in Prob. 17-48, namely, $Z_i = R\,\frac{1-5\alpha^2+j(\alpha^3-6\alpha)}{3-\alpha^2-j4\alpha}$ where $\alpha = \frac{1}{\omega RC}$. Using Eq. (17-64), $V_o = -\beta V_i$ where $V_i = \frac{Z_i}{Z_i+R_o}AV_g$ and $-\beta = \frac{1}{1-5\alpha^2+j(\alpha^3-6\alpha)}$



$$\frac{Z_i}{Z_i+R_o} = \frac{R\left[\frac{1-5\alpha^2+j(\alpha^3-6\alpha)}{3-\alpha^2-j4\alpha}\right]}{R\left[\frac{1-5\alpha^2+j(\alpha^3-6\alpha)}{3-\alpha^2-j4\alpha}\right]+R_o} = \frac{1-5\alpha^2+j(\alpha^3-6\alpha)}{1-5\alpha^2-\frac{R_o}{R}(3-\alpha^2)+j(\alpha^3-6\alpha-4\alpha\frac{R_o}{R})}$$

. Substituting, we get

$$\frac{V_o}{V_g} = (-\beta A)\frac{Z_i}{Z_i+R_o} = \frac{A}{1-5\alpha^2+\frac{R_o}{R}(3-\alpha^2)+j(\alpha^3-6\alpha-4\alpha\frac{R_o}{R})}$$

. Note that $\frac{V_o}{V_g}$ is real, if

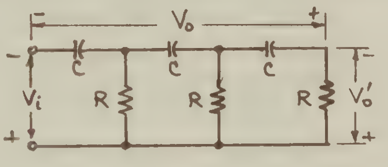
$\alpha^3-6\alpha-4\alpha\frac{R_o}{R} = 0$ or $\alpha^2 = 6+4\frac{R_o}{R} = \left(\frac{1}{2\pi fRC}\right)^2$ or $f = \frac{1}{2\pi RC}\cdot\frac{1}{\sqrt{6+4\frac{R_o}{R}}}$ QED

At this frequency, $\frac{V_o}{V_g}$ is negative and exceeds unity, if

$$A = -1+5\alpha^2-\frac{R_o}{R}(3-\alpha^2) = -1+5\left(6+4\frac{R_o}{R}\right)-\frac{R_o}{R}\left(3-6-4\frac{R_o}{R}\right) = 29+23\frac{R_o}{R}+4\left(\frac{R_o}{R}\right)^2$$

QED

(17-55) a) Consider the phase shift network redrawn as shown. Then, from Eq. (17-64) we see that at $\alpha^2 = 6 = \left(\frac{1}{2\pi fRC}\right)^2$, the phase shift of $\frac{V_o'}{V_i}$ is 180° or $V_o' = -\frac{1}{29}V_i$. Applying KVL around the outside loop, we have: $V_o = -V_o'+V_i = \frac{1}{29}V_i+V_i$

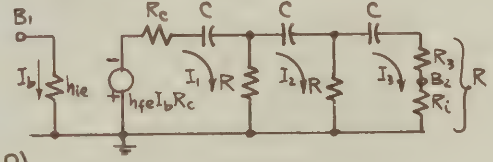


or $\frac{V_o}{V_i} = \frac{1}{29}+1 = \frac{30}{29} = -\beta$ ◁—

b) From part a, $f = \frac{1}{2\pi RC\sqrt{6}}$ ◁—

c) For oscillations to occur $-\beta A > 1$ or $A > \frac{29}{30} = 0.967$ ◁—

(17-56) Fig. 17-40b with Norton's current generator replaced by a Thévenin's voltage generator is indicated. Let $X = \frac{1}{wC}$. Then, the mesh eqs. are:



(1) $-h_{fe}I_bR_c = I_1(R_c+R-jX) - I_2R + I_3(0)$

(2) $0 = -I_1R + I_2(2R-jX) - I_3R$

(3) $0 = I_1(0) - I_2R + I_3(2R-jX)$; Let $\alpha = \frac{X}{R} = \frac{1}{wRC}$ and $k = \frac{R_c}{R}$

Then (3) becomes, $I_2 = I_3(2-j\alpha)$ and (2) becomes, $I_1 = I_3(3-\alpha^2-j4\alpha)$

Substituting the expressions of I_1 and I_2 in (1) and simplifying, we get:

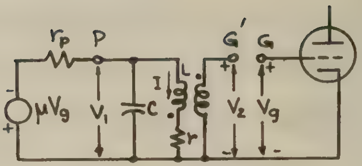
$-h_{fe}I_bk = I_3\{1+3k-(5+k)\alpha^2-j[(6+4k)\alpha-\alpha^3]\}$

The loop current gain is I_3/I_b and if this is to be real, then the coefficient of j must be zero or $\alpha^2 = 6+4k = \frac{1}{\omega^2R^2C^2}$

Thus, $f = \frac{1}{2\pi RC}\cdot\frac{1}{\sqrt{6+4k}}$ ...... Eq. (17-65)

At this frequency, $\frac{I_b}{I_3} = +\frac{1}{h_{fe}k}(4k^2+23k+29)$; For $\frac{I_3}{I_b} > 1$ then $h_{fe} > 4k+23+\frac{29}{k}$ QED

(17-57) If V_g is the voltage applied to the grid with the transformer secondary NOT connected (lead G′G open), then the loop gain $\equiv \frac{V_2}{V_g}$ where $V_2 = -j\omega MI$ $= \frac{-j\omega MV_1}{r+j\omega L}$. If Z is the impedance of C in parallel with $(r+j\omega L)$, then $V_1 = -\frac{\mu V_g Z}{Z+r_p} = \frac{-\mu V_g(r+j\omega L)}{r+j\omega L+j\omega r_pC[r+j(\omega L-\frac{1}{\omega C})]}$



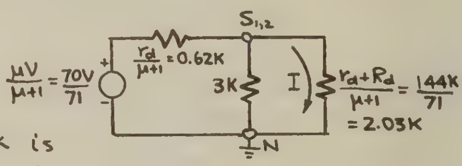
$$\therefore \frac{V_2}{V_g} = \frac{-j\omega MV_1}{(r+j\omega L)V_g} = \frac{j\mu\omega M}{r-\omega^2r_pLC+r_p+j(\omega L+\omega r_prC)} = \frac{\mu\omega M}{\omega(L+r_prC)+j(\omega^2r_pLC-r_p-r)}$$

In order for the loop gain to equal unity, $\omega^2r_pLC-r_p-r=0$ or $\omega^2=\frac{1}{LC}\left(1+\frac{r}{r_p}\right)$ and $\frac{\mu\omega M}{\omega(L+r_prC)}=1$ or $\mu M-L=r_prC=\frac{\mu rC}{g_m}$ or $g_m=\frac{\mu rC}{\mu M-L}$ QED

(17-58) a) If a signal causes the potential at G_1 to increase, then the source will follow this increase. Since the gate of Q2 is grounded, then a voltage increase at S_2 will cause D_2 to rise. Part of this increase in voltage is fed back to G_1. Hence, there will be zero phase shift around the loop provided that none is introduced by the reactive elements. At series resonance, the phase shift is zero and the oscillator frequency is given by

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi(25\times10^{-3}\times10^{-7})^{1/2}}=3180\text{ Hz}$$

b) We may neglect the loading of 1M on R<100K. If V is applied to G_1, then the equivalent circuit seen looking into each source is shown:



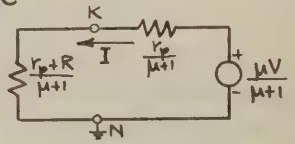
$S_{1,2}$
$\frac{r_d}{\mu+1}=0.62K$
$\frac{\mu V}{\mu+1}=\frac{70V}{71}$
3K
I
$\frac{r_d+R_d}{\mu+1}=\frac{144K}{71}$
$=2.03K$
N
+
-

The parallel combination of 3K and $\frac{r_d+R_d}{\mu+1}=2.03K$ is $\frac{3\times2.03}{5.03}K=1.21K$. Then, $I=\left(\frac{70V}{71}\right)\left(\frac{1.21K}{0.62K+1.21K}\right)\left(\frac{1}{2.03K}\right)=0.32V$

But loop gain $=\frac{IR}{V}=0.32R\geqslant1$ $\therefore R_{min}=3.12K$

(17-59) a) Using the same argument as in Prob. 17-58a, we conclude that the frequency at which the circuit will oscillate, if at all, is that for which Z becomes resistive. At parallel resonance, $Z=R$ and $f\approx\frac{1}{2\pi\sqrt{LC}}$

b) Under the given assumptions about R_g, R_K and C_b, the equivalent circuit seen looking at both cathodes is as shown. The voltage at G′ is $IZ=IR$ at resonance. The voltage at G_2 is V. Hence, the loop gain is $\frac{IR}{V}$ where $I=\frac{\mu V}{\mu+1}\frac{\mu+1}{2r_p+R}=\frac{\mu V}{2r_p+R}$.



Thus, loop gain $=\frac{\mu R}{2r_p+R}\geqslant1$ or $\mu R_{min}=2r_p+R_{min}$ $\therefore R_{min}=\frac{2r_p}{\mu-1}\approx\frac{2r_p}{\mu}=\frac{2}{g_m}$ if $\mu\gg1$

QED

(17-60) The phase shift around the circuit will be zero, if Z is real. But Z is given by:
$Z=\frac{(r+j\omega L)(1/j\omega C)}{r+j(\omega L-1/\omega C)}=\frac{(1/\omega C)(\omega L-jr)[r-j(\omega L-1/\omega C)]}{r^2+(\omega L-1/\omega C)^2}$. Setting the imaginary part of Z equal to zero, we have: $\omega L(\omega L-\frac{1}{\omega C})+r^2=0$ or $\omega^2L^2-\frac{L}{C}+r^2=0$

$$\therefore \omega^2=\frac{1}{LC}\left(1-\frac{r^2C}{L}\right) \qquad \text{QED}$$

Then, $Z=\frac{\omega Lr-r(\omega L-1/\omega C)}{r^2+(\omega L-1/\omega C)^2}=\frac{r/\omega^2C^2}{r^2+\left(\frac{\omega^2LC-1}{\omega C}\right)^2}=\frac{r}{r^2\omega^2C^2+(\omega^2LC-1)^2}$

but from above, $\omega^2LC-1=\frac{r^2C}{L}$. $\therefore Z=\frac{r}{\frac{r^2C^2}{LC}\left(1-\frac{r^2C}{L}\right)+\left(\frac{r^2C}{L}\right)^2}=\frac{r}{r^2C/L}=\frac{L}{rC}\equiv R$

The rest of the solution is given in Prob. 17-59b where we found $\mu R/(2r_p+R)\geqslant1$. Since $r_p=\mu/g_m$, then $\mu R=\frac{2\mu}{g_m}+R$ or $(g_m)_{min}=\frac{2\mu}{(\mu-1)R}=\frac{2\mu rC}{(\mu-1)L}$ QED

(17-61) Method I. If the load impedance is considered as $Z_2 \| (Z_1+Z_3)$, then $Z_L = \frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}$. Then, the gain without FB is $A = \frac{-\mu Z_L}{r_d+Z_L} = \frac{-\mu Z_2(Z_1+Z_3)}{r_d(Z_1+Z_2+Z_3)+Z_2(Z_1+Z_3)}$

and $-\beta = \frac{Z_1}{Z_1+Z_3}$ $\quad \therefore \; -\beta A = \frac{-\mu Z_1 Z_2}{r_d(Z_1+Z_2+Z_3)+Z_2(Z_1+Z_3)}$

Method II.

Apply a voltage V_{gs} between the gate and source and solve for $V_{g's}$ as outlined in the text. From Eq. (17-70b), $I_1 = I_2 \frac{Z_1+Z_2+Z_3}{Z_2}$. Then, Eq. (17-70a) becomes: $\mu V_{gs} = -I_1(r_d+Z_2) + I_2 Z_2 = I_2\left[-(r_d+Z_2)\left(\frac{Z_1+Z_2+Z_3}{Z_2}\right)+Z_2\right]$

$= I_2\left[-\left(\frac{r_d}{Z_2}\right)(Z_1+Z_2+Z_3)-Z_1-Z_3\right] \quad \therefore \; -\beta A = \frac{V_{g's}}{V_{gs}} = \frac{I_2 Z_1}{V_{gs}} = \frac{-\mu Z_1 Z_2}{r_d(Z_1+Z_2+Z_3)+Z_2(Z_1+Z_3)}$

(17-62) a) From Eq. (17-69) we obtain: $-\beta A = \frac{-\mu}{r_p\left(\frac{1}{Z_2}+\frac{1}{Z_1}+\frac{Z_3}{Z_1 Z_2}\right)+1+\frac{Z_3}{Z_1}}$. Now $\frac{1}{Z_1} = Y_1 = j\omega C_1$, $\frac{1}{Z_2} = Y_2 = j\omega C_2$ and $Z_3 = r + j\omega L_3$

$$\therefore -\beta A = \frac{-\mu}{r_p[j\omega(C_1+C_2)-(r+j\omega L_3)(\omega^2 C_1 C_2)]+1+(r+j\omega L_3)(j\omega C_1)} = \frac{-\mu}{j\omega[r_p(C_1+C_2)-\omega^2 L_3 r_p C_1 C_2 + r C_1] - r_p r \omega^2 C_1 C_2 + 1 - \omega^2 L_3 C_1}$$

In order for $-\beta A$ to be real, $\omega^2 L_3 r_p C_1 C_2 = r_p(C_1+C_2) + r C_1 \quad \therefore \; \omega^2 = \frac{1}{L_3}\left[\frac{1}{C_1}+\frac{1}{C_2}\left(1+\frac{r}{r_p}\right)\right]$ QED

b) $-\beta A = \frac{\mu}{-1+\omega^2(L_3 C_1 + r_p r C_1 C_2)} \geq 1$. Since $r_p = \frac{\mu}{g_m}$, the minimum g_m is given by $\mu + 1 = \omega^2\left(L_3 C_1 + \frac{\mu}{g_m} r C_1 C_2\right)$. Assuming that $\frac{r}{r_p}^{*} \ll 1$, so that $\omega^2 = \frac{1}{L_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$:

$\mu + 1 = \left(\frac{C_1+C_2}{L_3 C_1 C_2}\right)\left(L_3 C_1 + \frac{\mu}{g_m} r C_1 C_2\right) = \frac{C_1+C_2}{C_2} + \left(\frac{C_1+C_2}{L_3}\right)\left(\frac{\mu r}{g_m}\right) \quad \therefore \; g_m = \frac{r\mu C_2(C_1+C_2)}{L_3(\mu C_2 - C_1)}$ QED

(17-63) a) The output impedance between plate and cathode is $R_o = r_p + (\mu+1)R_K$. Hence, Eq. (17-69) must be modified by replacing r_p by R_o. Since $Z_1 = r_1 + j\omega L_1$, $Z_2 = r_2 + j\omega L_2$ and $Z_3 = -j/\omega C$. Then, Eq. (17-69) becomes:

$-\beta A = \frac{-\mu(r_1+j\omega L_1)(r_2+j\omega L_2)}{R_o(r_1+j\omega L_1+r_2+j\omega L_2-j/\omega C)+(r_2+j\omega L_2)(r_1+j\omega L_1-j/\omega C)}$. For unity loop gain,

$R_o(r_1+r_2) + j\left[R_o\omega(L_1+L_2) - \frac{R_o}{\omega C}\right] + (\mu+1)\left[r_1 r_2 - \omega^2 L_1 L_2 + j\omega(r_1 L_2 + r_2 L_1)\right] - j r_2/\omega C + L_2/C = 0$

The imaginary terms must add to zero, or $\omega\left[R_o(L_1+L_2)+(\mu+1)(r_1 L_2 + r_2 L_1)\right] = \frac{R_o + r_2}{\omega C}$

$$\therefore \; \omega^2 = \frac{1}{C(L_1+L_2)} \cdot \frac{1 + r_2/R_o}{1 + \frac{(r_1 L_2 + r_2 L_1)(\mu+1)}{R_o(L_1+L_2)}} \qquad \leftarrow$$

Note that if $r_1 = r_2 = 0$, $\omega^2 C(L_1+L_2) = 1$ which checks with Eq. (17-72)

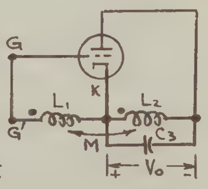
b) The loop gain equation is now: $R_o(r_1+r_2) + (\mu+1)(r_1 r_2 - \omega^2 L_1 L_2) + L_2/C = 0$

For small values of r_1 and r_2, we may neglect $r_1 r_2$ compared with $\omega^2 L_1 L_2$ and $R_o(r_1+r_2) \approx (\mu+1)\omega^2 L_1 L_2 - L_2/C \approx (\mu+1)\frac{L_1 L_2}{C(L_1+L_2)} - \frac{L_2}{C} = \frac{(\mu L_1 - L_2)L_2}{C(L_1+L_2)}$

Since $R_o = r_p + (\mu+1)R_K$, then

$$R_K = \frac{1}{\mu+1}\left[\frac{(\mu L_1 - L_2)L_2}{(r_1+r_2)C(L_1+L_2)} - r_p\right] \quad \text{for small } r_1 \text{ and } r_2 \qquad \leftarrow$$

(17-64) The circuit specified is as shown: Note that this is equivalent to the tuned-plate oscillator of Fig. 17-41a with $r = 0$. Hence, from Eq. (17-67), $\omega^2 = 1/L_2 C_3$.



A second approach is to apply a voltage V_{gk} between G & K with G-G' open. Then, $V_o = \frac{\mu Z_L}{r_p + Z_L} V_{gk}$ where $Z_L = (j\omega L_2) \| \left(\frac{1}{j\omega C_3}\right)$. But $V_{g'k} = \frac{M}{L_2} V_o \quad \therefore$ loop gain $= \frac{V_{g'k}}{V_{gk}} = \frac{M\mu}{L_2} \frac{1}{1 + r_p Y_L}$. For zero phase shift Y_L must be real. Hence, L_2 and C_3 must be in parallel resonance and $\omega^2 = 1/L_2 C_3$ ←

(17-65) a) $X = \frac{(\omega L - 1/\omega C)(-1/\omega C')}{\omega L - 1/\omega C - 1/\omega C'} = \frac{(\omega^2 - 1/LC)(-1/\omega C')}{\omega^2 - \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C'}\right)} = -\frac{1}{\omega C'}\,\frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2}$ where $\omega_s^2 = \frac{1}{LC}$ and

$\omega_p^2 = \frac{1}{L}\left(\frac{1}{C} + \frac{1}{C'}\right)$ QED

b) $\frac{\omega_p}{\omega_s} = \left[\frac{1}{L}\left(\frac{1}{C} + \frac{1}{C'}\right)/\left(\frac{1}{LC}\right)\right]^{1/2} = \left(1 + \frac{C}{C'}\right)^{1/2} \approx 1 + \frac{1}{2}\frac{C}{C'} + \cdots$

c) If $C = 0.04$ pF and $C' = 2$ pF, then $\frac{1}{2}\frac{C}{C'} \times 100\% = \frac{1}{2}\,\frac{0.04}{2.00} \times 100 = 1\%$ $\longleftarrow$

(17-66) a) $f_s = \frac{1}{2\pi}\left(\frac{1}{LC}\right)^{1/2} = \frac{1}{2\pi}\left(\frac{10^{12}}{0.33 \times 0.065}\right)^{1/2} = 1.09$ MHz $\longleftarrow$

b) $f_p/f_s = \left(1 + \frac{C}{C'}\right)^{1/2} = \left(1 + \frac{0.065}{1.0}\right)^{1/2} \approx 1.033$ $\therefore$ f_p exceeds f_s by 3.3% $\longleftarrow$

c) $Q = \frac{\omega_s L}{R} = \frac{1}{R}\sqrt{L/C} = \frac{1}{5.5 \times 10^3}\left(\frac{0.33}{0.065 \times 10^{-12}}\right)^{1/2} = 410$ $\longleftarrow$

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CHAPTER 18

(18-1) a) Eq. (18-6) with $x = X_1 \cos\omega_1 t + X_2 \cos\omega_2 t$ becomes: $i_y = G_1 x + G_2 x^2$
$= G_1 X_1 \cos\omega_1 t + G_1 X_2 \cos\omega_2 t + G_2 X_1^2 \cos^2\omega_1 t + G_2 X_2^2 \cos^2\omega_2 t + 2G_2 X_1 X_2 \cos\omega_1 t \cos\omega_2 t$
Note that since $2\cos^2\alpha = 1 + \cos 2\alpha$ and $2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$, then i_y contains terms whose frequencies are $\omega_1, \omega_2, 2\omega_1, 2\omega_2, \omega_1 - \omega_2$ & $\omega_1 + \omega_2$

b) Assume that i_y contains the term $G_3 x^3 = G_3(X_1^3 \cos^3\omega_1 t + 3X_1^2 X_2 \cos^2\omega_1 t \cos\omega_2 t + 3X_1 X_2^2 \cos\omega_1 t \cos^2\omega_2 t + X_2^3 \cos^3\omega_2 t)$. Now, since $4\cos^3\alpha = \cos 3\alpha + 3\cos\alpha$ and $4\cos^2\alpha\cos\beta = 2(1 + \cos 2\alpha)\cos\beta = 2\cos\beta + 2\cos 2\alpha\cos\beta = 2\cos\beta + \cos(2\alpha + \beta) + \cos(2\alpha - \beta)$, then, i_y contains (in addition to those frequencies in part a): $3\omega_1, 3\omega_2, 2\omega_1 \pm \omega_2$ and $2\omega_2 \pm \omega_1$.

If i_y also contains x^4, then it will contain terms of the form: $\cos^4\omega_1 t$ (giving rise to $4\omega_1$), $\cos^3\omega_1 t \cos\omega_2 t$ (giving rise to $3\omega_1 \pm \omega_2$), $\cos^2\omega_1 t \cos^2\omega_2 t$ (giving rise to $2\omega_1 \pm 2\omega_2$), etc.

(18-2) a) From a load line through $v_p = 300$V, $i_p = 0$ and $v_p = 0$, $i_p = \frac{300\text{V}}{2\text{K}} = 150$ mA we find: $I_{max} = 77$ mA, $I_Y = 43$ mA, $I_{min} = 15$ mA. Then using the 3-point schedule, we have: Eq. (18-12): $B_1 = (I_{max} - I_{min})/2 = (77 - 15)/2 = 31$ mA
Eq. (18-13): $B_2 = B_0 = (I_{max} + I_{min} - 2I_Y)/4 = (77 + 15 - 86)/4 = 1.5$ mA

Hence, the fundamental output current is $31/\sqrt{2} = 22$ mA rms $\longleftarrow$

b) $D_2 = (B_2/B_1) \times 100\% = (1.5/31) \times 100\% = 4.8\%$ $\longleftarrow$

c) $I_Y + B_0 = 43 + 1.5 = 44.5$ mA $\longleftarrow$

Note: If a 5-point schedule is used, $I_{\frac{1}{2}} = 61$ mA and $I_{-\frac{1}{2}} = 27$ mA, we find: $B_1 = 32$ mA, $B_2 = 1.5$ mA and $B_0 = 1.7$ mA.

(18-3) a) From Table 7-1, $\mu = 20$ and $r_p = 7.7$K. Thus, for a gain of $10 = |A| = \frac{\mu R_L}{r_p + R_L} = 20R_L/(7.7 + R_L)$ or $R_L = 7.7$K $\longleftarrow$

b) $V_G = -6$ V $\longleftarrow$

c) From a L.L. corresponding to $R_L = 7.7$K and $V_{PP} = 240$V on Fig. D-2, we

(18-3) (Cont'd) find $I_P = 6.5\ mA$

d) $I_{max} = 15.4\ mA$, $I_P = 6.5\ mA$, $I_{min} = 1\ mA$ $\quad \therefore\ B_1 = \frac{1}{2}(I_{max} - I_{min}) = \frac{1}{2}(15.4 - 1) = 7.2 mA$

$B_2 = \frac{1}{4}(I_{max} + I_{min} - 2I_P) = \frac{1}{4}(15.4 + 1 - 13) = 0.85\ mA$. Hence, $D_2 = \frac{0.85 \times 100\%}{7.2} = 11.8\%$

e) Fundamental voltage gain $= \frac{B_1 R_L}{V_{im}} = \frac{7.2 \times 7.7}{6} = 9.25$

If we assume that the grid draws no current, then the input power is zero and the power gain is thus infinite.

(18-4) $B_o = 34 - 31 = 3\ mA = B_2$; $P = \frac{B_1^2 R_L}{2}$ or $B_1^2 = \frac{2 \times 0.85}{4000} \times 10^6\ (mA)^2$ $\therefore B_1 = 20.6 mA$

Hence, $D_2 = \frac{B_2}{B_1}(100\%) = \frac{3 \times 100\%}{20.6} = 14.6\ \%$

(18-5) From Eq. (18-15), $i_y = G_1 x + G_2 x^2 + G_3 x^3 + G_4 x^4 + \ldots = G_1 X_m \sin\alpha + G_2 X_m^2 \sin^2\alpha + {}$ $+ G_3 X_m^3 \sin^3\alpha + G_4 X_m^4 \sin^4\alpha + \ldots$. Since $2\sin^2\alpha = 1 - \cos 2\alpha$, $4\sin^3\alpha = 3\sin\alpha - \sin 3\alpha$, $8\sin^4\alpha = \cos 4\alpha - 4\cos 2\alpha + 3$, etc. then, it follows that i_y contains odd sine terms and even cosine terms.

(18-6) From Eqs. (18-16) and (18-17), we have: $I_{max} = I_Y + B_o + B_1 + B_2 + B_3 + B_4$ (1)

$I_{\frac{1}{2}} = I_Y + B_o + \frac{1}{2}B_1 - \frac{1}{2}B_2 - B_3 - \frac{1}{2}B_4$ (2) $\qquad I_Y = I_Y + B_o - B_2 + B_4$ (3)

$I_{-\frac{1}{2}} = I_Y + B_o - \frac{1}{2}B_1 - \frac{1}{2}B_2 + B_3 - \frac{1}{2}B_4$ (4) $\qquad I_{min} = I_Y + B_o - B_1 + B_2 - B_3 + B_4$ (5)

From (1) and (5), we get: $I_{max} + I_{min} = 2I_Y + 2B_o + 2B_2 + 2B_4$ (6)

From (3), $B_o - B_2 + B_4 = 0$ (7)

Hence, from (6) and (7): $I_{max} + I_{min} - 2I_Y = 4B_2$ $\therefore B_2 = \frac{1}{4}(I_{max} + I_{min} - 2I_Y)$ $\longleftarrow$

From (2) and (4): $I_{\frac{1}{2}} + I_{-\frac{1}{2}} = 2I_Y + 2B_o - B_2 - B_4$ (8) . Then, from (6) and (8),

$I_{max} + I_{min} + 2(I_{\frac{1}{2}} + I_{-\frac{1}{2}}) = 6I_Y + 6B_o$ $\quad \therefore B_o = \frac{1}{6}(I_{max} + I_{min} + 2I_{\frac{1}{2}} + 2I_{\frac{1}{2}}) - I_Y$ $\longleftarrow$

From (7), $B_4 = B_2 - B_o = \frac{1}{12}(I_{max} - 4I_{\frac{1}{2}} + 6I_Y + 4I_{-\frac{1}{2}} + I_{min})$

From (1) and (5): $I_{max} - I_{min} = 2B_1 + 2B_3$ (9) . From (2) and (4): $I_{\frac{1}{2}} - I_{-\frac{1}{2}} = B_1 - 2B_3$ (10)

Adding (9) and (10), we get: $B_1 = \frac{1}{3}(I_{max} + I_{\frac{1}{2}} - I_{-\frac{1}{2}} - I_{min})$ $\longleftarrow$

From (9) and (10): $I_{max} - I_{min} - 2(I_{\frac{1}{2}} - I_{-\frac{1}{2}}) = 6B_3$ or $B_3 = \frac{1}{6}(I_{max} - 2I_{\frac{1}{2}} + 2I_{-\frac{1}{2}} - I_{min})$

(18-7)

i_y	I_{max}	$I_{0.707}$	I_Y	$I_{-0.707}$	I_{min}
α	0	45°	90°	135°	180°

Substituting these values into Eq. (18-16), we have:

$I_{max} = I_Y + B_o + B_1 + B_2 + B_3 + B_4$ (1) $\qquad I_{0.707} = I_Y + B_o + 0.707B_1 - 0.707B_3 - B_4$ (2)

$I_Y = I_Y + B_o - B_2 + B_4$ (3) $\qquad I_{-0.707} = I_Y + B_o - 0.707B_1 + 0.707B_3 - B_4$ (4)

$I_{min} = I_Y + B_o - B_1 + B_2 - B_3 + B_4$ (5)

From (1) and (5), we obtain: $I_{max} + I_{min} = 2I_Y + 2B_o + 2B_2 + 2B_4$ (6)

From (3): $B_o - B_2 + B_4 = 0$ (7) From (6) and (7): $I_{max} + I_{min} - 2I_Y = 4B_2$

$\therefore B_2 = \frac{1}{4}(I_{max} + I_{min} - 2I_Y)$ $\longleftarrow$

From (2) and (4): $I_{0.707} + I_{-0.707} = 2(I_Y + B_o - B_4)$ (8)

If we double (7) and add to (6): $I_{max} + I_{min} = 2I_Y + 4B_o + 4B_4$ (9)

If we double (8) and add to (9): $I_{max} + 2I_{0.707} + 2I_{-0.707} + I_{min} = 6I_Y + 8B_o$

$\therefore B_o = \frac{1}{8}(I_{max} + 2I_{0.707} - 6I_Y + 2I_{-0.707} + I_{min})$ $\longleftarrow$

Then, from (7), $B_4 = B_2 - B_o = \frac{1}{8}(I_{max} - 2I_{0.707} + 2I_Y - 2I_{-0.707} + I_{min})$ $\longleftarrow$

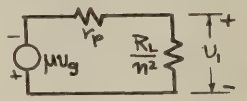
(18-7) (Cont'd) From (1) and (5) : $I_{max} - I_{min} = 2B_1 + 2B_3$ (10) From (2) and (4)

$I_{0.707} - I_{-0.707} = 1.414B_1 - 1.414B_3$ or $1.414 I_{0.707} - 1.414 I_{-0.707} = 2B_1 - 2B_3$ (11)

Adding (10) and (11), we get : $B_1 = \frac{1}{4}(I_{max} + 1.414 I_{0.707} - 1.414 I_{-0.707} - I_{min})$ ←

Subtracting (11) from (10) : $B_3 = \frac{1}{4}(I_{max} - 1.414 I_{0.707} + 1.414 I_{-0.707} - I_{min})$ ←

(18-8) If $n = \frac{N_2}{N_1}$ = turns ratio, then the impedance reflected into the primary is $\frac{R_L}{n^2}$ and the equivalent circuit is as shown. Then, the gain is $A = -\frac{v_2}{v_g} = -\frac{n v_1}{v_g} = \frac{n\mu R_L/n^2}{r_p + R_L/n^2}$



or $A = \frac{n\mu R_L/r_p}{n^2 + R_L/r_p} = \frac{n\mu\delta}{n^2+\delta}$ where $\delta = R_L/r_p$ QED

For a maximum wrt n, $\frac{dA}{dn} = 0 = \mu\delta\,\frac{n^2+\delta-2n^2}{(n^2+\delta)^2}$ or $n = \delta^{1/2}$

Hence, $A_{max} = \frac{\mu\delta^{3/2}}{2\delta} = \frac{1}{2}\mu\delta^{1/2} = \frac{1}{2}\mu n$

(18-9) Under the specified conditions, we obtain from Fig. 18-5 :

$I_{Bmax} = -30$ mA, $I_{B\frac{1}{2}} = -20.5$ mA, $I_{B0} = -15$ mA, $I_{B-\frac{1}{2}} = -10.5$ mA, $I_{Bmin} = -6.5$mA

$I_{Cmax} = -1.77$ A, $I_{C\frac{1}{2}} = -1.4$ A, $I_{C0} = -1.1$ A, $I_{C-\frac{1}{2}} = -0.8$ A, $I_{Cmin} = -0.53$A

Then, from Eqs. (18-18), we have:

a) $B_1 = -\frac{1}{3}(1.77 + 1.4 - 0.8 - 0.53) = -0.613$ A ←

b) $B_2 = -\frac{1}{4}(1.77 - 2.2 + 0.53) = -0.025$ A ; $B_3 = -\frac{1}{6}(1.77 - 2.8 + 1.6 - 0.53) = -6.67$mA

$B_4 = -\frac{1}{12}(1.77 - 5.6 + 6.6 - 3.2 + 0.53) = -8.32$ mA

$\therefore D_2 = \frac{B_2}{B_1} = \frac{25}{613}(100\%) = 4.07\%$; $D_3 = \frac{6.67}{613} = 1.09\%$ and $D_4 = \frac{8.32}{613} = 1.36\%$ ←

c) From Eq. (18-21), $P = (1+D^2)P_1$ where $P_1 = \frac{1}{2}B_1^2 R_L' = \frac{1}{2}(0.613)^2(10) = 1.88$ W

and $D^2 = D_2^2 + D_3^2 + D_4^2 = 16.5 + 1.19 + 1.85 = 19.54 \times 10^{-4} \approx 0.002$

Hence, $P = (1.002)(1.88) \simeq 1.89$ W ←

d) $B_0 = -\frac{1}{6}(1.77 + 2.8 + 1.6 + 0.53) + 1.1 = -15$ mA ←

(18-10) For $R_L' = 20\Omega$, we obtain from Fig. 18-5 :

$I_{Bmax} = -30$ mA, $I_{B\frac{1}{2}} = -20.5$ mA, $I_{B0} = -15$ mA, $I_{B-\frac{1}{2}} = -10.5$ mA, $I_{Bmin} = -6.5$mA

$I_{Cmax} = -1.45$ A, $I_{C\frac{1}{2}} = -1.35$ A, $I_{C0} = -1.1$ A, $I_{C-\frac{1}{2}} = -0.83$ A, $I_{Cmin} = -0.56$ A

Then, using Eqs. (18-18), we have:

$B_1 = -\frac{1}{3}(1.45 + 1.35 - 0.83 - 0.56) = -0.47$ A ; $B_2 = -\frac{1}{4}(1.45 - 2.2 + 0.56) = -47.5$ mA

$B_3 = -\frac{1}{6}(1.45 - 2.7 + 1.66 - 0.56) = -25$ mA ; $B_4 = -\frac{1}{12}(1.45 - 5.4 + 6.6 - 3.32 + 0.56) = +9.16$mA

Thus, $D_2 = \frac{47.5}{470}(100\%) = 10.1\%$; $D_3 = \frac{25}{470}(100\%) = 5.32\%$; $D_4 = \frac{9.16}{470}(100\%) = 1.95\%$

and $D = (10.1^2 + 5.32^2 + 1.95^2)^{1/2} = 11.5\%$; $P_1 = \frac{1}{2}(0.47)^2(20) = 2.21$ W ←

Hence, $P = (1+D^2)P_1 = (1 + 0.0134)(2.21) = 2.24$ W ←

(18-11) a) $R_L' = 5\Omega$. Then, $B_1 = \frac{1}{3}(I_{max} + I_{\frac{1}{2}} - I_{-\frac{1}{2}} - I_{min}) = -\frac{1}{3}(1.92 + 1.43 - 0.78 - 0.51) = -0.686$ A

Hence, $P_1 = \frac{1}{2}B_1^2 R_L' = \frac{1}{2}(0.686)^2(5) = 2.5 \times 0.47 = 1.18$ W ←

b) $R_L' = 30\Omega$. Then, $B_1 = -\frac{1}{3}(1.33 + 1.31 - 0.86 - 0.59) = -0.396$ A

and $P_1 = \frac{1}{2}(0.396)^2(30) = (15)(0.157) = 2.35$ W ←

(18-12) $R_L' = 10\Omega$: $I_{Bmax} = -30mA$, $I_{B\frac{1}{2}} = -22.5mA$, $I_{Bo} = -15mA$, $I_{B-\frac{1}{2}} = -7.5mA$, $I_{Bmin} = 0$

$I_{Cmax} = -1.77A$, $I_{C\frac{1}{2}} = -1.49A$, $I_{Co} = -1.1A$, $I_{C-\frac{1}{2}} = -0.6A$, $I_{Cmin} = -0.08A$

a) $B_1 = -\frac{1}{3}(1.77 + 1.49 - 0.6 - 0.08) = -0.86A$ ⟵

b) $B_2 = -\frac{1}{4}(1.77 - 2.2 + 0.08) = +87.5\ mA$; $B_3 = -\frac{1}{6}(1.77 - 2.98 + 1.2 - 0.08) = +15mA$

$B_4 = -\frac{1}{12}(1.77 - 5.96 + 6.6 - 2.4 + 0.08) = -7.5\ mA$ $\therefore D_2 = \frac{87.5}{860}(100\%) = 10.2\%$

$D_3 = \frac{15}{860}(100\%) = 1.75\%$, $D_4 = \frac{7.5}{860}(100\%) = 0.87\%$ ⟵

c) Then, $D = (103.5 + 3.05 + 0.75)^{1/2} = (107.3)^{1/2} = 10.35\%$; $P_1 = \frac{1}{2}(0.86)^2(10) = 3.7W$

Hence, $P = (1.01)(3.7) = 3.74W$ ⟵

d) $B_o = -\frac{1}{6}(1.77 + 2.98 + 1.2 + 0.08) + 1.1 = +95\ mA$ ⟵

(18-13) a) We have, $P = \frac{1}{2}\frac{V_m^2}{R_L'} = \frac{1}{2}\frac{V_{cc}^2}{R_L'} = 5W$ $\therefore R_L' = \frac{V_{cc}^2}{10} = \frac{400}{10} = 40\Omega = \frac{R_L}{n^2}$

Hence, $n^2 = \frac{4}{40} = \frac{1}{10}$ and $n = \frac{1}{\sqrt{10}}$ ⟵

b) $P = \frac{1}{2}I_m^2 R_L'$ or $I_m^2 = \frac{2P}{R_L'} = \frac{2\times5}{40} = \frac{1}{4}$ $\therefore I_m = 0.5A$ ⟵

c) $I_C = I_m = 0.5A$ and $V_{CE} = V_{cc} = 20V$ ⟵

d) $\eta = \frac{5W}{20\times0.5} = \frac{5}{10} = 50\%$ ⟵

(18-14) $I_P = \frac{P_D}{V_P} = \frac{10W}{250V} = 40\ mA$. For this current and $V_P = 250V$, we find $V_G = -20V$ Hence, the grid swings from 0 to $-20V$ to $-40V$. From load lines drawn through the Q-point we find:

a) $R_L' = 3K$: $I_{max} = 75\ mA$, $I_{min} = 14\ mA$ $\therefore B_1 = \frac{1}{2}(I_{max} - I_{min}) = \frac{1}{2}(75 - 14) = 30.5\ mA$

$B_2 = \frac{1}{4}(I_{max} + I_{min} - 2I_Y) = \frac{1}{4}(75 + 14 - 80) = 2.25\ mA$. Then, $D_2 = \frac{2.25}{30.5} = 7.4\%$ ⟵

and $P = \frac{1}{2}B_1^2 R_L' = \frac{1}{2}(30.5\times10^{-3})^2(3000) = 1.40\ W$ ⟵

b) $R_L' = 7K$: $I_{max} = 57\ mA$, $I_{min} = 28\ mA$ $\therefore B_1 = \frac{1}{2}(57 - 28) = 14.5\ mA$

$B_2 = \frac{1}{4}(57 + 28 - 80) = 1.25\ mA$. Then, $D_2 = \frac{1.25}{14.5} = 8.6\%$ and $P = \frac{1}{2}(14.5\times10^{-3})(7\times10^{3}) = 0.74W$

(18-15) a) From Fig. D-7 and a load line passing through $V_P = 300V$, $I_P = 0$ and $V_P = 0$, $I_P = 75\ mA$, we find : $V_{max} = 295V$, $V_Y = 225V$, $V_{min} = 100V$; $I_{max} = 50$, $I_Y = 18$, $I_{min} = 1mA$

Then, $B_1 = \frac{1}{2}(I_{max} - I_{min}) = \frac{1}{2}(50 - 1) = 24.5\ mA$; $B_2 = B_o = \frac{1}{4}(50 + 1 - 36) = 3.75\ mA$

Hence, $P_1 = \frac{1}{2}(B_1^2)R_L' = \frac{1}{2}(24.5\times10^{-3})^2(4000) = 1.2W$; $D_2 = \frac{3.75}{24.5}(100\%) = 15\%$ ⟵

Since $D \approx D_2$, then $P = (1 + D^2)P_1 \simeq (1.0225)(1.2) = 1.23W$ ⟵

From Eq. (18-28), $\eta = \frac{P_1}{V_{PP}(I_P + B_o)}(100\%) = \frac{120}{300\times21.75\times10^{-3}} = 18\%$ ⟵

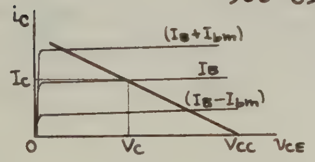
b) If we draw a 4K L.L. through the quiescent point : $V_P = 300V$, $V_G = -22.5V$, we find: $I_{max} = 96\ mA$, $I_Y = 62\ mA$ and $I_{min} = 32\ mA$.

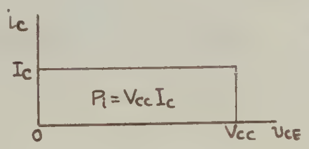
Thus, $B_1 = \frac{1}{2}(96 - 32) = 32\ mA$, $B_2 = B_o = \frac{1}{4}(96 + 32 - 124) = 1\ mA$

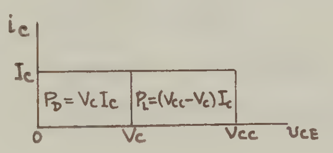
$D_2 = \frac{1}{32}(100\%) = 3.1\%$; $P_1 = \frac{1}{2}(32\times10^{-3})^2(4000) = 2.05W \approx P$ since $D^2 \ll 1$.

Then, $\eta = \frac{2.05}{300\times63\times10^{-3}} = 10.8\%$ ⟵

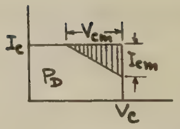
(18-16) a)



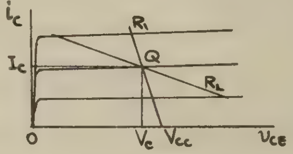




(18-16) (Cont'd) b) $P_i = V_{cc} I_c$ as in $\underline{a}$. ; $P_L = (V_{cc} - V_c) I_c$ as in $\underline{a}$. ;
$P = V_e I_e = \frac{1}{2} V_{cm} I_{cm}$ = shaded triangle shown, since V_{cm} and I_{cm} are the voltage and current swings, respectively, in going from I_B to $I_B + I_{bm}$ or from I_B to $I_B - I_{bm}$; $P_D = V_c I_c - P$



c) $\eta = \frac{P}{P_i} = \frac{\frac{1}{2} V_{cm} I_{cm}}{V_{cc} I_c} = \frac{\text{shaded triangle in } \underline{b}}{\text{largest rectangle in } \underline{a}}$



d) With or without signal, $P_i = V_{cc} I_c$. The power loss in the dc resistance, R_1, is $I_c^2 R_1 = (V_{cc} - V_c) I_c$. With no signal $P_D = V_c I_c$ as in part $\underline{a}$. With signal, $P = \frac{V_{cm} I_{cm}}{2}$ = shaded triangle as in part $\underline{b}$. $P_D = V_c I_c - P$ = rectangle − triangle as in $\underline{b}$.
Note that since V_{cc} for the shunt-fed-load case is much smaller than for the series-fed-load case, then the rectangle $V_{cc} I_c = P_i$ is smaller in the former case and η is thus larger.

(18-17) a) For zero second harmonic distortion, $I_{max} = 2I_Y - I_{min} = 2(72) - 15 = 129$ mA. Corresponding to this I_{max} we find $V_{min} = 35$ V for $V_G = 0$. Hence,
$R_L = \frac{V_P - V_{min}}{I_{max} - I_Y} = \frac{250-35}{129-72} = \frac{215}{57} = 3.8K$ ⟵
b) A 2.5K load line is drawn through the Q-point: $V_P = 250V$, $V_{G1} = -14V$, $I_Y = 72$ mA in Fig. 7-15. We then obtain: $I_{max} = 157$ mA, $I_{\frac{1}{2}} = 117$, $I_Y = 72$, $I_{-\frac{1}{2}} = 40$ and $I_{min} = 15$
Using Eqs. (18-18) we get: $B_1 = 72$ mA and $B_3 = -2$ mA $\therefore D_3 = \frac{2}{72}(100\%) = 2.8\%$ ⟵

(18-18) a) For zero second harmonic distortion, $I_{max} = 2I_Y - I_{min} = 2(65) - 10 = 120$ mA
Corresponding to $I_{max} = 120$ mA and $V_G = 0$, we find $V_{min} = 36V$
Hence, $R_L = \frac{200-36}{120-65} = 3K$ ⟵
b) For a 3K LL and $V_G = -30V$, we find $V_{max} = 370V$. Then from Eq. (18-5),
$P = \frac{1}{8}(V_{max} - V_{min})(I_{max} - I_{min}) = \frac{1}{8}(370-36)(120-10) \times 10^{-3} = 4.6W$ ⟵
c) From Eq. (18-26), $P_D = V_Y I_Y - V_Y I_Y = 200 \times 0.065 - 4.6 = 8.4W$ ⟵
d) From Eq. (18-29), $\eta = \frac{4.6}{200 \times 0.065} = 35.4\%$ ⟵

(18-19) A load line is drawn through the Q-point: $V_P = 250V$, $I_P = 72$ mA and $V_G = -14V$ on Fig. 7-15. We read: I_{max} for $V_G = 0$, $I_{\frac{1}{2}}$ for $V_G = -7V$, $I_Y = I_P$ for $V_G = -14V$, $I_{-\frac{1}{2}}$ for $V_G = -21V$ and I_{min} for $V_G = -28V$. Then we use Eqs. (18-18), (18-19), (18-20) & (18-21).

	R_L'	I_{max}	$I_{\frac{1}{2}}$	I_Y	$I_{-\frac{1}{2}}$	I_{min}	B_1	B_2	B_3	B_4	D_2	D_3	D_4	P
a)	1K	177.5	120	72	38	15	81.5	12.1	-0.25	-0.62	14.7	0.3	0.75	3.3W
b)	4K	125	110	72	39	15	60	-1	-5.3	-2	1.7	8.9	3.3	7.2

(18-20) a) With $x_1 = X_m \cos\omega t$ and $x_2 = -X_m \cos\omega t$ substituted in $i_c = I_c + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$ we obtain: $i = K(i_{c1} - i_{c2}) = K[I_c + a_1 X_m \cos\omega t + a_2 X_m^2 \cos^2\omega t + a_3 X_m^3 \cos^3\omega t + \ldots$
$- (I_c - a_1 X_m \cos\omega t + a_2 X_m^2 \cos^2\omega t - a_3 X_m^3 \cos^3\omega t + \ldots)] = 2K(a_1 X_m \cos\omega t + a_3 X_m^3 \cos^3\omega t + \ldots)$
This series contains only odd powers of $\cos\omega t$. Since $(\cos\omega t)^n$ with n odd may be expressed as a sum of terms containing $\cos p\omega t$ where p is odd, then i contains only odd cosine terms.
b) The collector supply current is $(i_{c1} + i_{c2}) = 2(I_c + a_2 X_m^2 \cos^2\omega t + a_4 X_m^4 \cos^4\omega t + \ldots)$
Since $(\cos\omega t)^n$ with n even may be expressed as a sum of terms containing $\cos p\omega t$ with p even, then, $(i_{c1} + i_{c2})$ contains I_c and even harmonics.

(18-21) We have: $i = k(i_1 - i_2)$. From Eq. (18-34), $i_2(\omega t) = i_1(\omega t + \pi)$ $\therefore$ $i(\omega t) = k[i_1(\omega t) - i_1(\omega t + \pi)]$
Then, $i(\omega t+\pi) = k[i_1(\omega t+\pi) - i_1(\omega t + 2\pi)] = k[i_1(\omega t+\pi) - i_1(\omega t)] = -k[i_1(\omega t) - i_1(\omega t+\pi)]$
Thus, we see that $i(\omega t) = -i(\omega t+\pi)$, thereby verifying Eq. (18-37)

(18-22) For a half wave sine, $I_{dc} = \frac{I_m}{\pi}$ and $I_{rms} = \frac{I_m}{2}$. Hence, $P = I_{rms}^2 R_L = \frac{I_m^2 R_L}{4}$
$= \frac{\pi^2}{4} I_{dc}^2 R_L = \frac{\pi^2}{4} \times (10 \times 10^{-3})^2 (10^3) = 0.246\ \mathrm{W}$ ←

(18-23) From Eq. (18-41) we have: $P_c = \frac{2}{\pi} \frac{V_{cc} V_m}{R_L'} - \frac{V_m^2}{2R_L'}$. Note that at $V_m = 0$, $P_c = 0$, and at $V_m = \frac{4V_{cc}}{\pi}$, $P_c = 0$. Since P_c cannot be negative, there exists a maximum for $\frac{dP_c}{dV_m} = 0 = \frac{2V_{cc}}{\pi R_L'} - \frac{2V_m}{2R_L'}$ or $V_m = \frac{2V_{cc}}{\pi}$. At this value, $(P_c)_{max} = \frac{2}{\pi} \frac{V_{cc}}{R_L'} \frac{2V_{cc}}{\pi} - \frac{4V_{cc}^2}{2\pi^2 R_L'} = \frac{2V_{cc}}{\pi^2 R_L'}$ ... Eq. (18-42).

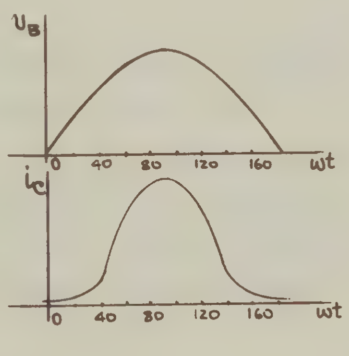
(18-24) a) $P = \frac{1}{2} V_m I_m$ where $I_m \approx \frac{V_{cc}}{R_L'}$, $R_L' = \left(\frac{N_1}{N_2}\right)^2 R_L = \frac{1}{4} \times 20 = 5\ \Omega$
Hence, $P = \frac{1}{2} \frac{V_{cc}^2}{R_L'} = \frac{1}{2} \frac{400}{5} = 40\ \mathrm{W}$ ←
b) $P_i = \frac{2I_m}{\pi} V_{cc} \approx 2 \frac{V_{cc}}{R_L'} \frac{V_{cc}}{\pi} = \frac{2V_{cc}^2}{\pi R_L'} = \frac{2 \times 400}{\pi \times 5} = 50.8\ \mathrm{W}$
Collector dissipation in each transistor is $\frac{1}{2}(P_i - P) = \frac{1}{2}(50.8 - 40) = 5.4\ \mathrm{W}$

(18-25) a) From Fig. 18-5, we obtain: $I_{Bmax} = -20\ \mathrm{mA}$, $I_{B\frac{1}{2}} = -10\ \mathrm{mA}$
$I_{Cmax} = -1.26\ \mathrm{A}$, $I_{C\frac{1}{2}} = -0.75\ \mathrm{A}$
Then, $B_3 = \frac{1}{3}(I_{max} - 2I_{C\frac{1}{2}}) = -\frac{1}{3}(1.26 - 1.5) = +80\ \mathrm{mA}$
$B_1 = \frac{2}{3}(I_{max} + I_{C\frac{1}{2}}) = -\frac{2}{3}(1.26 + 0.75) = -1.34\ \mathrm{A}$ $\therefore$ $D_3 = \left|\frac{B_3}{B_1}\right| = \frac{80}{1340} = 6\ \%$ ←
b) $P = (1 + D^2) \frac{B_1^2 R_L'}{2} \approx (1 + 0.0036)(1.34)^2 \frac{15}{2} = 13.5\ \mathrm{W}$ ←
c) From Fig. 18-5, $V_{min} = -1\ \mathrm{V}$; Hence, $\eta = 78.5\left(1 - \frac{V_{min}}{V_{cc}}\right) = 78.5\left(1 - \frac{1}{20}\right) = 74.5\ \%$ ←

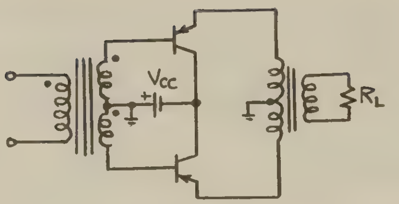
(18-26) a) From Fig. 18-5 : $I_{Bmax} = -30\ \mathrm{mA}$, $I_{B\frac{1}{2}} = -15$
$I_{Cmax} = -1.84\ \mathrm{A}$, $I_{C\frac{1}{2}} = -1.1\ \mathrm{A}$
Then, $B_3 = -\frac{1}{3}(1.84 - 2.2) = 0.12\ \mathrm{A}$; $B_1 = -\frac{2}{3}(1.84 + 1.1) = -1.96\ \mathrm{A}$
$\therefore$ $D_3 = \frac{0.12}{1.96}(100\%) = 6.1\ \%$ ←
b) $P = (1 + 0.0036)(1.96)^2 \frac{7.5}{2} = 14.4\ \mathrm{W}$ ←
c) $V_{min} = -1.3\ \mathrm{V}$. $\therefore$ $\eta = 78.5\left(1 - \frac{1.3}{15}\right) = 78.5\left(\frac{13.7}{15}\right) = 71.2\ \%$ ←

(18-27) $v_B = 0.4 \sin \omega t$
Using Fig. 18-5 we get:

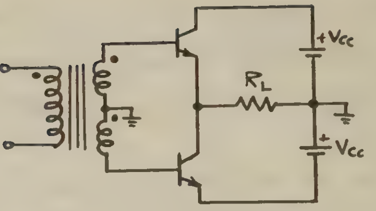
ωt	0°	20°	30°	60°	90°	120°	150°	160°	180°
v_B	0	0.137V	0.20	0.35	0.40	0.35	0.20	0.137	0
i_C	75mA	75mA	0.2A	1.05A	1.3A	1.05A	0.2A	75mA	75mA



(18-28) a)



b)



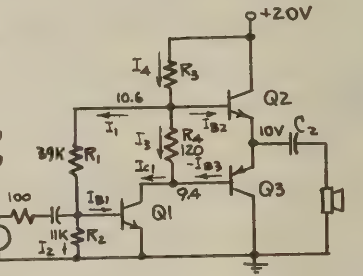
(18-29) In Fig. 18-14 the bases of the two transistors are connected together so that when v_s increases, the base current of the n-p-n transistor increases while that of the p-n-p transistor decreases in magnitude. The reverse occurs when v_s decreases. Hence, the collector currents are out of phase by 180° or $(i_c)_{npn} = i_{c_1} = -(i_c)_{pnp} = -i_{c_2}$ or $i_{c_1}(\omega t) = i_{c_2}(\omega t + \pi)$. Then, the load current is $i_L = i_{c_1}(\omega t) - i_{c_2}(\omega t)$

Using Eq. (18-16), we have: $i_{c_1}(\omega t) = I_Y + B_0 + B_1 \cos\omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \dots$

Then, $i_{c_2}(\omega t) = I_Y + B_0 + B_1 \cos(\omega t + \pi) + B_2 \cos(2\omega t + 2\pi) + B_3 \cos(3\omega t + 3\pi) + \dots$

$= I_Y + B_0 - B_1 \cos\omega t + B_2 \cos 2\omega t - B_3 \cos 3\omega t + \dots$

Hence, $i_L = 2(B_1 \cos\omega t + B_3 \cos 3\omega t + \dots)$ ∴ no even harmonics are present in the load.



(18-30) a) With $V_s = 0$, Q1 is biased to operate Class A while Q2 and Q3 are biased to operate Class B. Actually, Q2 and Q3 are slightly forward-biased by the drop across R_4 in order to avoid crossover distortion. Assume that V_s is sinusoidal. Then, the positive cycle of V_s is amplified and inverted by Q1, thus turning Q2 off. Q3 operates as an emitter follower and if X_{C_2} is negligible at the signal frequency, then the voltage across the speaker consists of a power-amplified negative cycle of the signal.
During the negative cycle of V_s, Q3 is turned off and Q2 operates as an emitter follower and the cycle of the voltage across the speaker is completed. R_1 provides negative FB for dc stability and distortion reduction. C_2 blocks dc from the speaker.

b) Assume that $V_{BE} = 0.6V$ for all transistors, when $V_s = 0$. Then, $I_1 = \frac{10.6 - 0.6}{39K} = 256 \mu A$

$I_2 = \frac{0.6}{11K} = 54 \mu A$, $I_{B1} = I_1 - I_2 = 202 \mu A$, $I_{C1} = h_{FE1} I_{B1} = 50 \times 202 \mu A = 10.1 \text{ mA}$

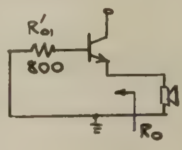
$I_3 = \frac{10.6 - 9.4}{120\Omega} = 10 \text{ mA}$ ∴ $I_{B2} = -I_{B3} = 10.1 - 10 = 0.1 \text{ mA}$

Hence, $I_4 = I_1 + I_{B2} + I_3 = 0.256 + 0.1 + 10 = 10.356 \text{ mA}$ ∴ $R_3 = \frac{20 - 10.6}{10.356} K \approx 910\Omega$ ◁—

c) If $R_4 = 0$, then the impedance seen looking to the left of the bases of Q2 and Q3 is $R'_{o1} = R_{o1} \| R_3$ where R_{o1} is given by Eq. (17-53), namely,

$Y_{o1} = \frac{1}{R_{o1}} = h_{oe} + \frac{1}{R_1 + R'_s \| h_{ie}} \left(1 + h_{fe} \frac{R'_s}{R'_s + h_{ie}}\right)$; $R_1 = 39K$, $R'_s = R_s \| R_2 = 0.1K \| 11K \approx 0.1K$

$= 2.5 \times 10^{-5} + \frac{10^{-3}}{39 + \frac{100 \times 1.1}{1.2}} \left(1 + 50 \frac{100}{1200}\right) = 15.7 \times 10^{-5} \mho$ ∴ $R_{o1} = 6.4K$



and $R'_{o1} = 6.4K \| 0.91K \doteq 800\Omega$ ∴ $R_o = \frac{h_{ie2} + R'_{o1}}{1 + h_{fe2}} = \frac{50 + 800}{101} = 8.4\Omega$ ◁—

d) With $V_{CE}(sat) \approx 0$, $(P)_{max} = I^2_{rms} R_{speaker} = \left(\frac{10}{\sqrt{2}} \frac{1}{8.4 + 8}\right)^2 (8) = (0.431)^2 8$

Hence, $(P)_{max} = 1.48$ W ◁—

CHAPTER 19

(19-1) a) Energy corresponding to 5,893 Å is $E = \frac{12,400}{5,893} = 2.11$ eV from Eq. (19-5).

Maximum energy of electrons leaving cesium is $2.11 - 1.8 = 0.31$ eV

Hence, $v_{max} = 5.93\times10^5 (0.31)^{1/2} = 3.3\times10^5$ m/sec ⟵

b) No emission from platinum, because $2.11 < 6.0$ eV ⟵

c) Energy corresponding to 743 Å is $E_W = \frac{12,400}{743} = 16.7$ eV

(i) cesium : $16.7 - 1.8 = 14.9$ eV ∴ $v_{max} = 5.93\times10^5 (14.9)^{1/2} = 2.29\times10^6$ m/sec ⟵

(ii) platinum: $16.7 - 6.0 = 10.7$ eV ∴ $v_{max} = 5.93\times10^5 (10.7)^{1/2} = 1.94\times10^6$ m/sec ⟵

(19-2) From Eq. (19-5), $E = \frac{12,400}{5,500} = 2.25$ eV or $2.25\times1.6\times10^{-19} = 3.6\times10^{-19}$ J ⟵

(19-3) The energy of radiation is $\frac{12,400}{1065} = 11.6$ eV/ph. Hence, the applied retarding potential to reduce the current to zero is $11.6 - 1.8 - 0.5 = 9.3$ V ⟵

(19-4) a) The energy of radiation is $\frac{12,400}{2,537} = 4.88$ eV/ph. The energy of the fastest electrons leaving the surface is $4.88 - E_W$. These electrons reach the anode with an energy of $4.88 - E_W - 1.00$ eV. Hence, an applied retarding potential of this amount will reduce the current to zero.

Thus, $4.88 - E_W - 1.00 + 0.54 = 0$ or $E_W = 4.42$ eV ⟵

b) $\lambda_c = \frac{12,400}{E_W} = \frac{12,400}{4.42} = 2,800$ Å. ⟵

(19-5) Total energy $= (10^4 \text{ photons})\times(\frac{12,400}{2,537} \text{ eV/ph})\times(1.6\times10^{-19} \text{ J/eV}) = 7.75\times10^{-15} \text{ J} = 7.75\times10^{-15}$ W-sec

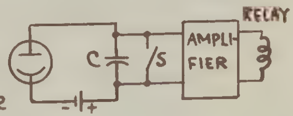
Sensitivity $= (6\times10^{-3} \frac{\text{A}}{\text{W}})(\frac{\text{Coulomb}}{\text{A-sec}})(\frac{\text{electrons}}{1.6\times10^{-19} \text{ coulomb}}) = 3.75\times10^{16}$ electrons/W-sec

∴ number of emitted electrons $= 7.75\times10^{-15} \text{ W-sec} \times 3.75\times10^{16} \frac{\text{electrons}}{\text{W-sec}} = 291$ electrons ⟵

(19-6) Total flux $= 120\times4\pi$ lumens. Since the area of a sphere is $4\pi R^2$, then the flux density at a distance of 3 ft from the source is $\frac{120\times4\pi}{4\pi\times3^2\times144} = 0.0927$ lumens/in². The flux intercepted by the photocell is:

$(0.9 \text{ in}^2)\times(0.0927 \text{ lumens/in}^2) = 0.0834$ lumens. ∴ $I = 14\ \mu\text{A/lumen} \times 0.0834 = 1.17\ \mu$A ⟵

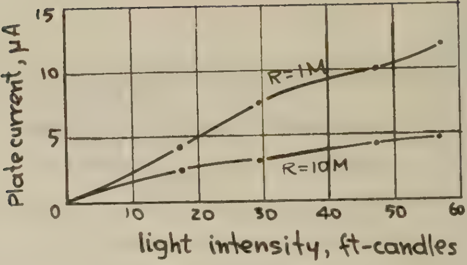
(19-7) Multiply the energy distribution curve of the light source (at a given wavelength) by the spectral response of the photocell (at the same wavelength). Find the area under the resultant curve. This area is proportional to the total photocurrent. Hence, the tube having the largest area under this product curve will yield the maximum current.



(19-8) One possible solution is indicated in the sketch. A capacitor C, shunted by a normally closed switch S, is placed in series with a photocell and a battery. The switch is opened at the start of the exposure. The voltage across the capacitor is proportional to the product of the photocurrent and time (more precisely, to $\int I dt$). Since the photocurrent is proportional to the light intensity then, the capacitor voltage is proportional to the blackening of the emulsion. The capacitor voltage is applied to an amplifier which drives a relay. The relay trips at a given current which is proportional to the input voltage. When the relay closes, the light source is shut off, S is closed and the apparatus is ready for the next print.

(19-9)

j	17.5	29.5	47.5	57.5
i 1M	4	7.5	10	12
10M	2.5	3	4	4.5



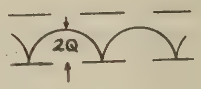
(19-10) a) For 20V across R = 2M, I = 10 μA. From Fig. 19-6a, 10 μA with a plate voltage of 60V is obtained at $j \approx 108$ ft-c. ⟵

b) The minimum tube voltage is 80−50 = 30V. From Fig. 19-6a, we find I = 18.5 μA at j = 208 ft-c. $\therefore R_{max} = 50/(18.5\times10^{-6}) = 2.7$ M ⟵

(19-11) From Eq. (19-7), $10^6 = \delta^n = 3.5^n$ or $6 = n\log_{10}3.5 = 0.544n$ $\therefore n = 11$ stages ⟵

(19-12) From Eq. (19-7), $\delta^n = \frac{I}{I_o}$ or $\delta^9 = \frac{0.1}{10^{-8}} = 10^{+7}$. Then, $9\log_{10}\delta = 7$ $\therefore \delta = 5.95$ ⟵

(19-13) a) The maximum height 2Q of the cycloidal path cannot exceed 1 cm or the electron will strike the top field plates. From Eqs. (1-58) and (1-53)



$$Q = \frac{u}{\omega} = \frac{\mathcal{E}}{B}\frac{m}{eB} = \frac{\mathcal{E}}{(\frac{e}{m})B^2} = 0.5\text{ cm} = 5\times10^{-3}\text{ m}$$

$$\therefore B^2 = \frac{10^4}{1.76\times10^{11}\times5\times10^{-3}} = 11.35\times10^{-6} \text{ or } B = 3.36\text{ mWb/m}^2 = 33.6\text{ G}$$ ⟵

b) The distance between targets is $2\pi Q = \frac{2\pi\mathcal{E}}{(\frac{e}{m})B^2} = \frac{2\pi\times10^4}{1.76\times10^{11}\times25\times10^{-6}} = 1.43$ cm ⟵

(19-14) a) From Fig. 19-8c, $v = \frac{R_L}{R_L+R_\lambda}V_{PP} = \frac{4k}{4k+R_\lambda}(40V) = 20V$ $\therefore R_\lambda = 4k$

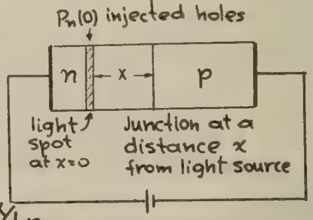
For $R_\lambda = 4k$, we find from Fig. 19-16 that illumination ≈ 9 ft-c. ⟵

b) $P_\lambda = \frac{V_o^2}{R_\lambda} = \frac{(20)^2}{4\times10^3} = 100$ mW ⟵

c) At 100 ft-c., $R_\lambda = 400\,\Omega$. Thus, $I^2R_\lambda = P_T = 300$ mW or $I^2 = \frac{300\times10^{-3}}{400} = 750\times10^{-6}$ or $I = 27.4\times10^{-3}$ A. Then $V_{R_L} = V_{PP} - IR_\lambda = 40 - 27.4\times0.4 = 29\text{ V} = IR_L$

Hence, $R_L = \frac{29}{27.4\times10^{-3}} = 1070\ \Omega$ ⟵

(19-15) From Eq. (5-52), $P_n(x) = p_n - p_{no} = P_n(0)\,e^{-x/L_p}$. This gives the concentration of injected holes in the n-material as a function of distance x from the light spot. L_p is the diffusion length of holes in the n material.

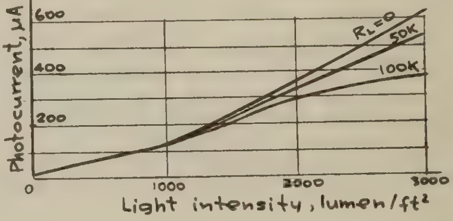


The diffusion current is: $I = -AeD_p\frac{dP_n(x)}{dx} = +\frac{AeD_pP_n(0)}{L_p}e^{-x/L_p}$

Thus, $\ln I = \ln\left[\frac{AeD_pP_n(0)}{L_p}\right] - \frac{x}{L_p}$ $\therefore$ slope $= -\frac{1}{L_p}$. Similarly for the p-side.

(19-16)

light intensity		0	335	1300	2000	3000
i (μA)	100K	7	60	150	295	375
	50K	7	60	155	315	530
	0	7	60	160	340	620



(19-17) a) At switching, $V_R = V_{BE} + 1.2\times 8 = 0.6 + 9.6 = 10.2\,V$ ∴ $V_D = 20 - 10.2 = 9.8\,V$ ⟵

b) Current through diode $= \frac{V_R}{50K} + I_B = \left(\frac{10.2}{50} + \frac{1.2}{101}\right) mA = 216\ \mu A$

From Fig. 19-23 at $V_D = 9.8V$ and $I_D = 216\ \mu A$, illumination ≈ 400 ft-c. ⟵

c) From the intersection of the L.L. and the 300 ft-c. curve in Fig. 19-23, we obtain: $I_D = 190\ \mu A$, $V_D = 12V$ ∴ $V_R = 8\,V = 0.6 + 8I$

or $I = \frac{7.4}{8} = 0.925\ mA$ ⟵

Since the base current is $\frac{925}{100} = 9.25\ \mu A$ and $I_D = 190\ \mu A$, the assumption that the transistor does not load the 50K resistor is justified.

(19-18) a) From Fig. 19-25b, $\Delta I = (0.6 - 0.5)\,mA = 0.1\ mA$.

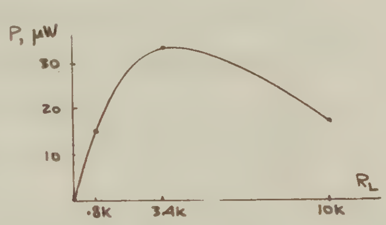
Since the current flows through $R = 0.75K \| 18K \| 82K \| 1K \approx 430\,\Omega$, then

$\Delta V_i = v_i = 430 \times 0.1 \times 10^{-3} = 43\ mV$ ⟵

b) $A_V = A_I \frac{R_c}{R_i} = -25 \frac{4K}{1K} = -100$ ∴ $\Delta V_o = 100 \times 43 \times 10^{-3} = 4.3\,V$ ⟵

(19-19) a)

R_L	I (mA)	V (V)	P (μW)
0	0.145	0	0
800Ω	0.14	0.11	15.4
3.4K	0.10	0.34	34
10K	0.04	0.41	16.4



b) The optimum value of $R_L = 3.4K$ ⟵

This may be verified by plotting on Fig. 19-24 the hyperbola: $IV = 34\ \mu W$. This hyperbola intersects the LS223 characteristic at one point. ∴ maximum power is generated at one point only.

CHAPTER 20

(20-1) a) $I_m = \frac{V_m}{R_f + R_L} = \frac{110\sqrt{2}}{1020} = 152.5\ mA$ ⟵

b) $I_{dc} = \frac{I_m}{\pi} = \frac{152.5}{\pi} = 48.5\ mA$ ⟵

c) $I_{rms} = \frac{1}{2} I_m = \frac{1}{2}(152.5) = 76.2\ mA$ ⟵

d) $V_{dc} = -\frac{I_m R_L}{\pi} = -48.5 \times 1 = -48.5\ V$ ⟵

e) $P_i = I_{rms}^2 (R_f + R_L) = (76.2\times 10^{-3})^2 (1020) = 5800\times 10^{-6} \times 1020 = 5.92\ W$ ⟵

f) $\%\ \text{Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}}(100\%) = \frac{V_m/\pi - I_{dc} R_L}{I_{dc} R_L}(100\%) = \frac{49.5 - 48.5}{48.5} = 2.06\%$

(20-2) We have: $P_{dc} = I_{dc}^2 R_L = \frac{V_m^2 R_L}{\pi^2 (R_f + R_L)^2}$; For max P_{dc} set $\frac{dP_{dc}}{dR_L} = 0$, or

$\frac{V_m^2}{\pi^2}\left[\frac{(R_f+R_L)^2 - 2(R_f+R_L)R_L}{(R_f+R_L)^4}\right] = 0$. Then $R_f + R_L = 2R_L$ ∴ $R_L = R_f$ QED

(20-3) a) From Eqs. (20-6), (20-10) & (20-12), we obtain: $\eta_r = \frac{P_{dc}}{P_i} \times 100\% = \frac{I_{dc}^2 R_L \times 100\%}{I_{rms}^2 (R_f + R_L)}$

$= \left(\frac{I_{dc}}{I_{rms}}\right)^2 \frac{100\%}{1 + R_f/R_L} = \left(\frac{I_m/\pi}{I_m/2}\right)^2 \frac{100\%}{1 + R_f/R_L} = \frac{40.6}{1 + R_f/R_L}\ \%$

b) From Eqs. (20-17) & (20-19), $\eta_r = \left(\frac{I_{dc}}{I_{rms}}\right)^2 \frac{100\%}{1 + R_f/R_L} = \left(\frac{2I_m/\pi}{I_m/\sqrt{2}}\right)^2 \frac{100\%}{1 + R_f/R_L} = \frac{81.2}{1 + R_f/R_L}\ \%$ QED

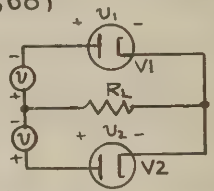
(20-4) Consider a half-wave rectifier: $V_{NL} = \frac{V_m}{\pi}$ and $V_{FL} = \frac{V_m}{\pi}\,\frac{R_L}{R_L + R_f}$

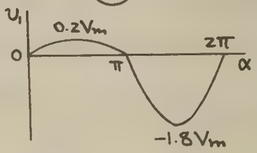
$\therefore$ % Regulation $= \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\% = \frac{(V_m/\pi)\left(1 - \frac{R_L}{R_L+R_f}\right)}{(V_m/\pi)\,\frac{R_L}{R_L+R_f}} \times 100\% = \frac{R_f}{R_L} \times 100\%$

For a full-wave rectifier, V_{NL} and V_{FL} are doubled. Hence, the regulation remains the same as above.

(20-5) a) From Eqs. (20-17) & (20-18), $I_{dc} = 2I_m/\pi = \frac{2V_m}{\pi(R_L+R_f)} = \frac{2 \times 280\sqrt{2}}{\pi(2500)} = 101$ mA ←

b) $(I_{dc})_{tube} = \frac{1}{2} I_{dc} = 50.5$ mA ←





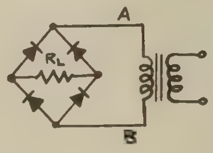
c) The voltage to center tap is impressed across R_f in series with R_L. Hence, the voltage across the conducting tube is sinusoidal with a peak value $V_m \frac{500}{2500} = 0.2V_m$. During non-conduction of V1 we see by traversing the outside path of the circuit sketched that $v_1 = -2v - v_2$. Since V2 is now conducting, its peak value is $0.2V_m$ and that of v_1 is $-2V_m + 0.2V_m = -1.8V_m$. Thus, the voltage v_1 across V1 is as shown, and its ac value is:

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{\pi} (0.2V_m)^2 \sin^2\alpha \, d\alpha + \frac{1}{2\pi}\int_{\pi}^{2\pi} (-1.8V_m)^2 \sin^2\alpha \, d\alpha = \frac{V_m^2}{4}\left[(0.2)^2 + (1.8)^2\right]$$

Hence, $V_{rms} = 0.905\,V_m = 0.905 \times 280\sqrt{2} = 358$ V ←

d) $P_{dc} = I_{dc}^2 R_L = (0.101)^2(2000) = 20.4$ W ←

e) From Prob. 20-4, % regulation $= \frac{R_f}{R_L} \times 100\% = \frac{500}{2000} \times 100\% = 25\%$ ←



(20-6) The load and the transformer cannot be interchanged. If they were, the circuit shown would result. Note that if A is positive with respect to B, all diodes will be reverse-biased. If B is positive with respect to A, then all four diodes conduct, thus short-circuiting the transformer.

(20-7) From Eqs. (20-17) & (20-18), $I_{dc} = \frac{2I_m}{\pi} = \frac{2}{\pi}\,\frac{V_m}{R_L} = \frac{2\sqrt{2}\,V_{rms}}{\pi(5010)} = 1 \times 10^{-3}$

Hence, $V_{rms} = \frac{5.01\pi}{2\sqrt{2}} = 5.56$ V ←

(20-8) Assume negligible voltage drop across each conducting diode. Then, C_1 charges to V_m, the peak transformer voltage, through D1. On the inverse cycle, the voltage across D1 is $V_m + V_m = 2V_m$ with the anode negative. This is the peak inverse voltage of D1. During the inverse cycle, D2 conducts and C_2 is charged to the peak value of $2V_m$, which shows that this circuit is a doubler at no load.

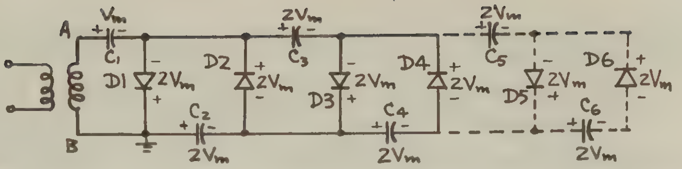
a) From the above: $C_1 \rightarrow V_m$, $C_2 \rightarrow 2V_m$ ←

b) D1 → $2V_m$. When D1 is conducting its drop is zero. Hence, the PIV across D2 is the peak voltage of C_2 or $2V_m$. ←

In the bridge doubler of Fig. 20-6, the peak voltage of each capacitor is $2V_m$. The PIV across each diode is also $2V_m$. The regulation of the circuit of Fig. 20-6 should be better, because the load is across the two capacitors in series.

If the cathode and anode leads of each diode are interchanged then, the direction of current flow is reversed. Hence, the capacitors charge with the opposite polarities and the output becomes positive with respect to ground.

(20-9) a) If A is positive with respect to B, D1 conducts and C_1 charges with the polarity shown to the transformer peak value V_m. When A is negative with respect to B, then D1 is back-biased by a PIV $2V_m$. This polarity is such as to forward-bias D2, charging C_2 to $2V_m$ as shown.



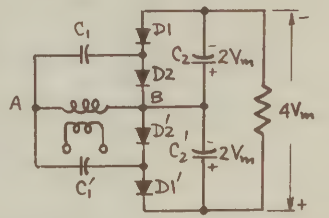
When A is again positive wrt B, D1 conducts and C_2 is put in parallel with D2. Hence, the PIV across D2 is the voltage across C_2 or $2V_m$. This polarity is such as to cause D3 to conduct, charging C_3 to $2V_m$ as shown. When A is negative wrt B, D2 conducts and C_3 is put in parallel with D3. Hence, D3 is back-biased by $2V_m$.

b) From the analysis above we conclude that the peak voltage across C_1 is V_m while the peak voltage across all other capacitors is $2V_m$. The PIV across each diode is $2V_m$.

c) Add diodes and capacitors as indicated by the dashed circuit above. The outputs from the anodes of D2, D4, D6 etc and ground are $2V_m$, $4V_m$, $6V_m$ etc or n-fold multiplication with n even.

d) If point A is grounded (instead of B), then the outputs from the anodes of D1, D3, D5 etc are V_m, $3V_m$, $5V_m$ etc or n-fold multiplication with n odd.

(20-10) From the circuit shown, we see that when A is positive wrt B, D2 conducts, whereas, when B is positive wrt A, D2′ conducts. Hence, whatever happens in the voltage doubler consisting of D1 and D2, happens one-half a cycle later in the voltage doubler consisting of D′ & D′₂. Thus, C_2 receives its charge during one half cycle, and C_2' during the next half cycle. Hence, this is a full-wave circuit and the output across C_2 and C_2' is series is $4V_m$.



(20-11) a) $L\frac{di}{dt} + R_L i = V_m \sin\omega t$: The steady state solution is $\frac{V_m}{Z}\sin(\omega t - \psi)$ where $Z = (R^2+\omega^2L^2)^{1/2}$ and $\tan\psi = \omega L/R_L$. The transient solution is $Ae^{-R_Lt/L}$. Thus, the total solution is $i = \frac{V_m}{Z}\sin(\omega t-\psi) + Ae^{-R_Lt/L}$. The initial condition is @ $t=0$, $i=0$. $\therefore A = \frac{V_m}{Z}\sin\psi$

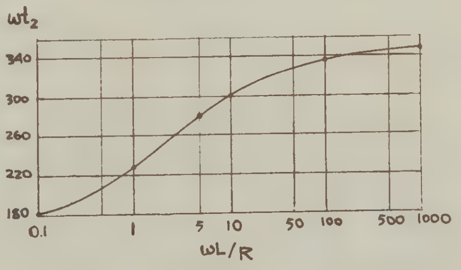
Hence, $i = \frac{V}{\sqrt{R_L^2+\omega^2L^2}}\left[\sin(\omega t-\psi) + e^{-R_Lt/L}\sin\psi\right]$

b) At $t=t_2$, $i=0$ $\quad\therefore \sin(\omega t_2-\psi) + e^{-(R_L/\omega L)\omega t_2}\sin\psi = 0 \qquad (1)$

Let $\omega t_2 - \psi \equiv \alpha$ and $\omega L/R \equiv x$. Then, $(R_L/\omega L)\omega t_2 = \frac{1}{x}(\alpha+\psi)$ and eq'n (1) becomes: $-\sin\alpha = e^{-\alpha/x}e^{-\psi/x}\sin\psi$. This transcendental eq'n. is solved graphically by finding the intersection of the sine curve and of the exponential.

e.g. for $x = \frac{\omega L}{R} = 5 = \tan\psi$, $\psi = 78.7° = 1.37\text{rad}$.

$\sin\psi = 0.921$ and $-\sin\alpha = e^{-0.2\alpha}e^{-\frac{1.37}{5}}(0.981)$
$= 0.746\,e^{-0.2\alpha}$. We find $\alpha = 201.6°$ and
$\omega t_2 = \alpha + \psi = 201.6° + 78.7° \approx 280°$ in agreement with Fig. 20-8.

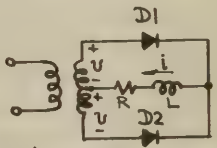


c) For $\omega L/R_L = 5$, $iR_L/V_m = 0.196[\sin(\omega t - 78.7°) + e^{-0.2\omega t}(0.981)]$

(20-11) (Cont'd) Let us check the point $\omega t = \pi$ rad $= 180°$.

$iR_L/V_m = 0.196[\sin(101.3°) + e^{-0.628}(0.981)] = 0.196(0.981+0.523) = 0.295$

The value in Fig. 20-8 at $\omega t = \pi$ is 0.32 or about 10% higher (indicating that the plot was not drawn very accurately).



(20-12) a) With the voltage polarity indicated D1 conducts. The voltage across D2 is $2v$ in the reverse-biased direction. Hence, only D1 conducts. When the polarity reverses, only D2 conducts. We will show now that it is impossible to have cutout (as with a capacitor filter); that is, we will show that the current cannot remain zero for a finite length of time. If $i=0$ and $\frac{di}{dt}=0$, then the voltage across R and L, $iR + L\frac{di}{dt} = 0$. Hence, v appears across D1 in such a direction that D1 must conduct. (It is possible for $i=0$ but $\frac{di}{dt} \neq 0$.)

b) $v = L\frac{di}{dt} + iR = V_m \sin \omega t$. The steady state solution is $\frac{V_m}{Z}\sin(\omega t - \varphi)$ where $Z = (R^2+\omega^2L^2)^{1/2}$ and $\tan\varphi = \omega L/R$. The transient solution is $Ae^{-Rt/L}$. The total solution is : $i = \frac{V_m}{Z}\sin(\omega t-\varphi) + Ae^{-Rt/L} = \frac{V_m}{Z}\sin(\alpha-\varphi) + Ae^{-\alpha R/\omega L}$ where $\alpha = \omega t$

Since $i(0) = i(\pi)$, $-\frac{V_m}{Z}\sin\varphi + A = \frac{V_m}{Z}\sin(\pi-\varphi) + Ae^{-\pi R/\omega L}$ $\therefore A = \frac{2V_m}{Z}\frac{\sin\varphi}{(1-e^{-\pi R/\omega L})}$

Hence, $i = \frac{V_m}{Z}\left[\sin(\omega t-\varphi) + \frac{2\sin\varphi\, e^{-Rt/L}}{(1-e^{-\pi R/\omega L})}\right]$

c) $I_{dc} = \frac{1}{\pi}\int_0^\pi i\,d\alpha = \frac{1}{\pi}\left[\int_0^\pi \frac{V_m}{Z}\sin(\alpha-\varphi)\,d\alpha + \int_0^\pi Ae^{-R\alpha/\omega L}\,d\alpha\right]$

$= \frac{V_m}{\pi Z}[\cos\varphi - \cos(\pi-\varphi)] + \frac{A}{\pi}\frac{\omega L}{R}(1-e^{-R\pi/\omega L})$

$= \frac{2V_m}{\pi Z}\cos\varphi + \left(\frac{2V_m}{Z}\frac{\sin\varphi}{1-e^{-R\pi/\omega L}}\right)\left(\frac{\omega L}{\pi R}\right)(1-e^{-R\pi/\omega L}) = \frac{2V_m}{\pi Z}(\cos\varphi + \sin\varphi\tan\varphi)$

$= \frac{2V_m}{\pi Z}\left(\cos\varphi + \frac{\sin^2\varphi}{\cos\varphi}\right) = \frac{2V_m}{\pi Z}\frac{\cos^2\varphi+\sin^2\varphi}{\cos\varphi} = \frac{2V_m}{\pi R}$ since $Z\cos\varphi = R$ ◁—

d) Fourier series may be written in the form: $i = B_0 + \sum_{k=1}^{\infty} B_k \cos k\alpha + \sum_{k=1}^{\infty} A_k \sin k\alpha$

Since the current repeats itself every $\alpha = \pi$, then $B_2 = \frac{2}{\pi}\int_0^\pi i\cos 2\alpha\,d\alpha$ and $A_2 = \frac{2}{\pi}\int_0^\pi i\sin 2\alpha\,d\alpha$. Let $D_2 \equiv \frac{1}{2}(B_2 - jA_2) = \frac{1}{\pi}\int_0^\pi i(\cos 2\alpha - j\sin 2\alpha)\,d\alpha$

or $D_2 = \frac{1}{\pi}\int_0^\pi i e^{-j2\alpha}\,d\alpha = \frac{1}{\pi}\int_0^\pi \left[\frac{V_m}{Z}\sin(\alpha-\varphi) + Ae^{-R\alpha/\omega L}\right]e^{-j2\alpha}\,d\alpha$

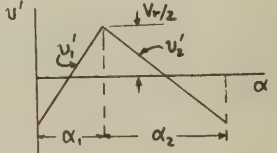
$= \frac{1}{\pi}\int_0^\pi \frac{V_m}{Z}\left(\frac{e^{j(\alpha-\varphi)} - e^{-j(\alpha-\varphi)}}{2j}\right)e^{-j2\alpha}\,d\alpha + \frac{1}{\pi}\int_0^\pi Ae^{-\left(\frac{R\alpha}{\omega L}+j2\alpha\right)}\,d\alpha$. After some manipulation, this integrates to: $D_2 = -\frac{2}{3}\frac{V_m}{\pi}\frac{e^{-j\psi}}{\sqrt{R^2+(2\omega L)^2}}$ where $\tan\psi = \frac{2\omega L}{R}$

Thus, $B_2 = -\frac{4}{3}\frac{V_m}{\pi}\frac{\cos\psi}{\sqrt{R^2+(2\omega L)^2}}$ and $A_2 = -\frac{4}{3}\frac{V_m}{\pi}\frac{\sin\psi}{\sqrt{R^2+(2\omega L)^2}}$

Hence, the first term in the Fourier series is

$B_2\cos 2\alpha + A_2 \sin 2\alpha = -\frac{4}{3}\frac{V_m}{\pi}\frac{1}{\sqrt{R^2+4\omega^2L^2}}(\cos 2\alpha\cos\psi + \sin 2\alpha \sin\psi)$

$= -\frac{4V_m}{3\pi}\frac{\cos(2\omega t-\psi)}{\sqrt{R^2+4\omega^2 L}}$. This checks with Eq. (20-25)



(20-13) $(V'_{rms})^2 = \frac{1}{\alpha_1+\alpha_2}\left[\int_0^{\alpha_1}(v_1')^2\,d\alpha + \int_{\alpha_1}^{\alpha_1+\alpha_2}(v_2')^2\,d\alpha\right]$ where $v_1' = \frac{V_r}{2}\left(\frac{2\alpha}{\alpha_1}-1\right)$

Consider the first integral: $\int_0^{\alpha_1}(v_1')^2\,d\alpha = \frac{V_r^2}{4}\int_0^{\alpha_1}\left(\frac{4\alpha^2}{\alpha_1^2} - \frac{4\alpha}{\alpha_1}+1\right)d\alpha$

$= \frac{V_r^2}{4}\left(\frac{4\alpha^3}{3\alpha_1^2} - \frac{4\alpha^2}{2\alpha_1} + \alpha\right)\Big|_0^{\alpha_1} = \frac{V_r^2\alpha_1}{4}\left(\frac{4}{3}-2+1\right) = \frac{V_r^2}{4}\frac{\alpha_1}{3}$. Because of similarity, the second integral is $\frac{V_r^2}{4}\frac{\alpha_2}{3}$.

Hence, $(V'_{rms})^2 = \frac{1}{\alpha_1+\alpha_2}\left(\frac{V_r^2}{4}\right)\left(\frac{\alpha_1+\alpha_2}{3}\right) = \left(\frac{V_r^2}{4}\right)\left(\frac{1}{3}\right)$ and $V'_{rms} = \frac{V_r}{2\sqrt{3}}$ … Eq.(20-41)

(20-14) a) From Eq. (20-35), $\omega t_1 = \alpha_1 = \pi - \psi = 180° - \tan^{-1}(377 \times 250 \times 40 \times 10^{-6}) = 180° - 75° = 105°$

b) From Eq. (20-33), $i = V_m\sqrt{\omega^2C^2 + 1/R_L^2}\ \sin(\omega t - \psi) = 35\sqrt{2}\sqrt{(377\times 40 \times 10^{-6})^2 + (\frac{1}{250})^2}\ \sin(\alpha + 75°)$

or $i = 772 \sin(\alpha + 75°)$ mA ; Diode conducting: $v_o = v = 35\sqrt{2}\sin\alpha = 49.5 \sin\alpha$

Diode non-conducting: Eq. (20-38), $v_o = 49.5 \sin 105°\, e^{-\frac{\alpha - \alpha_1}{\omega R_L C}} = 47.8\, e^{-(\frac{\alpha - 105°}{216})}$

From the plot of these eq'ns. we find cut-in at 36° ←

The peak current is $772 \sin(36° + 75°) = 720$ mA ←

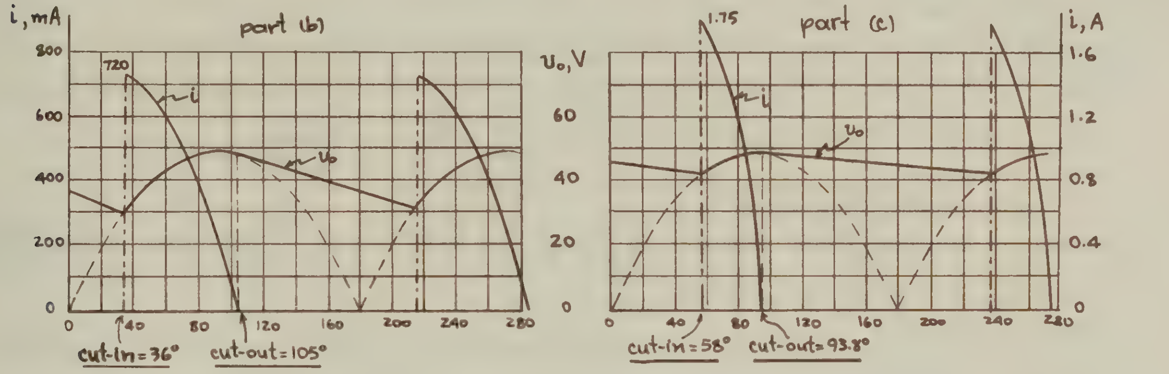
c) $\tan\psi = \omega C R_L = 377 \times \overset{*}{160} \times 10^{-6} \times 250 = 15.1$ or $\psi = 86.2°$ $\therefore$ cut-in angle $= \pi - \psi = 93.8°$

$i = \frac{49.5}{250}\left[1 + (15.1)^2\right]^{1/2} \sin(\alpha + 86.2°) = 2.98 \sin(\alpha + 86.2°)$

Diode conducting: $v_o = 49.5 \sin\alpha$; Diode nonconducting: $v_o = 49.5\, e^{-(\frac{\alpha - 93.8°}{866})}$

From a plot of these eq'ns. we find cut-in at 238° or 238° − 180° = 58° ←

Then, the peak current is $2.98 \sin(58° + 86.2°) = 1.75$ A ←



(20-15) If the input to the circuit of Fig. 20-15 is V, then, the output V' is equal to the short-circuit current V/jX_L divided by the admittance of the output, or

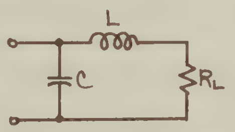
$V' = \frac{V}{jX_L}\frac{1}{1/jX_L + j/X_C + 1/R_L} = \frac{V}{1 - X_L/X_C + jX_L/R_L}$; $V'_{rms} = \frac{V_{rms}}{[(X_L/R_L)^2 + (X_L/X_C - 1)^2]^{1/2}}$,

From Eq. (20-45), $V_{rms} = 2\sqrt{2}V_m/3\pi$. Since $V_{dc} = 2V_m/\pi$, then $r \equiv \frac{V'_{rms}}{V_{dc}}$

or $r = \frac{2\sqrt{2}V_m}{3\pi(\frac{2V_m}{\pi})}\frac{1}{\sqrt{(X_L/R_L)^2 + (X_L/X_C - 1)^2}} = \frac{\sqrt{2}/3}{\sqrt{(X_L/R_L)^2 + (X_L/X_C - 1)^2}}$ QED

If $X_L \gg X_C$ and $R_L \gg X_C$, then $r \approx \frac{\sqrt{2}/3}{X_L/X_C} = \frac{\sqrt{2}}{3}(\frac{1}{2\omega L})(\frac{1}{2\omega C})$ ... Eq. (20-49)

(20-16) a) This is a capacitor input filter followed by an R-L load. The dc output voltage is, thus, given by Eq. (20-44) less the IR drop across the inductor.



or $V_{dc} = V_m - \frac{I_{dc}}{4fC} - I_{dc}R = V_m - I_{dc}(R + \frac{1}{4fC})$ where R is the resistance of the inductor.

b) By Eq. (20-58), the ripple voltage across C is $V'_2 = \sqrt{2}I_{dc}X_C$. This is reduced by the ratio R_L/X_L (assuming that $X_L \gg R$) at the output. Hence, $r = \frac{\sqrt{2}I_{dc}X_C}{V_{dc}}\frac{R_L}{X_L} = \sqrt{2}\frac{X_C}{R_L}\frac{R_L}{X_L} = \sqrt{2}\frac{X_C}{X_L}$. Thus, the ripple now is 3 times as great as for an L-section filter; see Eq. (20-49). The output voltage is higher, but the regulation is worse than that of an L-section filter.

(20-17) a) From Eq. (20-56), $LC = 1.76\left(\frac{0.471}{r}\right)^{1/2}$ $\therefore L = \frac{1.76}{40\times10^{-6}}(4710)^{1/2} = 3.02\text{ H}$ $\longleftarrow$

b) Taking the 50 Ω to be the transformer and inductor resistance, we have: $V_{dc} = \frac{2V_m}{\pi} - I_{dc}R = \frac{(2\sqrt{2})(40)}{\pi} - 0.1\times50 = 36-5 = 31\text{ V}$ $\longleftarrow$

(20-18) Single-section ripple voltage is $V_1' = V_{rms}\left(\frac{X_C}{X_L}\right) = V_{rms}\frac{1}{(2\omega C)(2\omega L)} = \frac{V_{rms}}{4\omega^2 LC}$

If each choke has an inductance L_o and each capacitor a capacitance C_o, then $V_1' = \frac{V_{rms}}{16\omega^2 L_o C_o}$

Two-section ripple voltage is $V_2' = V_{rms}\left(\frac{X_C}{X_L}\right)^2 = V_{rms}\left(\frac{1}{2\omega C_o}\right)^2\left(\frac{1}{2\omega L_o}\right)^2 = \frac{V_{rms}}{16\omega^4 L_o^2 C_o^2}$

or $V_2' = \frac{V_1'}{\omega^2 L_o C_o}$. Hence, if $\omega^2 L_o C_o > 1$, use a two-section filter.

(20-19) a) We must show first that cutout does not take place. $V_{dc} = \frac{2V_m}{\pi} = \frac{2\sqrt{2}\times40}{\pi} = 36\text{V}$ but $V_{dc} = I_{dc}R_L = 0.2R_L = 36$ $\therefore R_L = 180\ \Omega$. From Eq. (20-53), $L_c = \frac{R_L}{1130} = \frac{180}{1130}$ $= 0.16\text{ H}$. Since $L = 2\text{H} > 0.16\text{ H}$, then there is no cutout.

The input voltage to the filter is: $v = \frac{2V_m}{\pi}\left(1 - \frac{2}{3}\cos\omega t - \frac{2}{15}\cos 4\omega t + \dots\right)$

The 120-Hz ripple voltage is $V_{120}' = \left(\frac{4V_m}{3\pi\sqrt{2}}\right)\left(\frac{X_C}{X_L}\right) = \frac{4V_{rms}}{3\pi}\frac{1}{4\pi^2 f^2 LC}$

$= \frac{40}{3\pi^3(120)^2\times4\times80\times10^{-6}} = 0.0935\text{ V} = 93.5\text{ mV}$ $\longleftarrow$

b) For a two-section filter, $V_{120}' = \frac{4V_m}{3\pi\sqrt{2}}\left(\frac{X_C}{X_L}\right)^2 = \frac{4\ V_{rms}}{3\pi(4\pi^2f^2LC)^2} = \frac{4\times40\times10^{12}}{3\pi^5(120)^4(16)(4)(1600)}$

or $V_{120}' = 0.00825\text{ V} = 8.25\text{ mV}$ $\longleftarrow$

c) For $f = 240\text{Hz}$, single section using both inductors and capacitors:

$V_{240}' = \frac{4V_m}{15\pi\sqrt{2}}\left(\frac{X_C}{X_L}\right) = \frac{4\times40}{15\pi}\frac{1}{(2\pi fC)(2\pi fL)}$. This voltage is $\left(\frac{1}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{20}$ of that in part (a), or $V_{240}' = \frac{93.5}{20} = 4.67\text{ mV}$ $\longleftarrow$

(20-20) a) $V_{dc} = \frac{2V_m}{\pi} - I_{dc}R$ where $R = 200+100+400 = 700\,\Omega$ and $I_{dc} = \frac{V_{dc}}{R_L}$

or $V_{dc} = \frac{2\sqrt{2}}{\pi}V_{rms} - V_{dc}\left(\frac{R}{R_L}\right)$ $\therefore V_{dc} = \frac{2\sqrt{2}(375)}{\pi}/(1.35) = 250\text{ V}$ $\longleftarrow$

Since $\frac{4\omega^2L^2}{R_L^2} = \frac{4\times(377)^2(40)^2}{(2000)^2} = 225 \gg 1$, we may use Eq. (20-28).

$r = \frac{1}{3\sqrt{2}}\frac{R_L}{\omega L} = \frac{1}{3\sqrt{2}}\left(\frac{2000}{377\times40}\right) = 0.031$ $\longleftarrow$

b) From Eq. (20-44), $V_{dc} = V_m - \frac{I_{dc}}{4fC} = V_m - \frac{V_{dc}}{4fR_LC}$ or $V_{dc} = \frac{V_m}{1 + 1/4fR_LC}$

or $V_{dc} = \frac{\sqrt{2}\times375}{1+10^6/(4\times60\times2000\times32)} = \frac{531}{1.065} = 500\text{V}$ $\longleftarrow$

From Eq. (20-43), $r = \frac{1}{4\sqrt{3}\ fR_LC} = \frac{10^6}{(4\sqrt{3})(60)(2\times10^3)(32)} = 0.0376$ $\longleftarrow$

c) As in part a, $V_{dc} = 250\text{V}$ $\longleftarrow$

From Eq. (20-50), $r = \frac{0.83}{LC} = \frac{0.83}{40\times32} = 6.5\times10^{-4}$ $\longleftarrow$

d) $V_{dc} = 250\text{ V}$ as above. $\longleftarrow$

From Eq. (20-55), $r = \frac{\sqrt{2}}{3}\frac{10^{12}}{[16\pi^2\times3600\times20\times16]^2} = 1.42\times10^{-5}$ $\longleftarrow$

e) From Table 20-1, $V_{dc} = V_m - \frac{4170}{C}\frac{V_{dc}}{R_L} - 700\frac{V_{dc}}{R_L}$ or $V_{dc} = \frac{\sqrt{2}\times375}{1 + \frac{4170}{2000\times16} + \frac{700}{2000}}$

or $V_{dc} = 360\text{ V}$

From Eq. (20-60), $r = \frac{3300}{16\times16\times40\times2000} = 1.6\times10^{-4}$ $\longleftarrow$

(20-21) Using Arguimbau's mothod: The fundamental current is $\sqrt{2}I' = \frac{1}{\pi}\int_0^{2\pi} i\cos\alpha\, d\alpha$

The current is zero almost everywhere except near $\alpha = 0$ where $\cos\alpha \simeq 1$.

$\therefore \sqrt{2}I' \leq \frac{1}{\pi}\int_0^{2\pi} i\, d\alpha = 2I_{dc}$ or $I' \leq \sqrt{2}\, I_{dc}$

The upper limit to the fundamental ripple voltage is $V' = I'X_c = \sqrt{2}\, I_{dc}X_c$

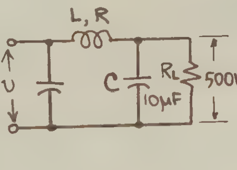
This is the voltage across the first capacitor.

$\therefore r = \frac{V'_{rms}}{V_{dc}} = \frac{V'}{V_{dc}}\frac{X_{c_1}}{X_{L_1}} = \sqrt{2}\,\frac{X_{c_1}}{X_{L_1}}\,\frac{I_{dc}X_c}{V_{dc}} = \sqrt{2}\left(\frac{X_c}{R_L}\right)\left(\frac{X_{c_1}}{X_{L_1}}\right)$ where all reactances are calculated at the fundamental.

(20-22) Using Table 20-1, $V_{dc} = V_m - \frac{4170}{C}I_{dc} - I_{dc}R = 300\sqrt{2} - \frac{4170}{4}(0.05) - 0.05\times 200$

or $V_{dc} = 415 - 52 - 10 = 353$ V

$r = \frac{3300}{CC_1L_1R_L} = \frac{3300\times 0.05}{4\times 4\times 20\times 353} = 1.46\times 10^{-3}$ $\therefore V'_{rms} = r\,V_{dc} = 0.515$ V ←



(20-23) $v = 525 - 40\sin 754t$; $I_{dc}R = 0.1R = 25$V $\therefore R = 250\ \Omega$ ←

The ripple voltage is $rV_{dc} = \frac{40}{\sqrt{2}}\,\frac{X_c}{X_L} = \frac{40}{\sqrt{2}}\,\frac{1}{(754)^2LC} = 0.001\times 500$

$\therefore L = \frac{40}{\sqrt{2}}\,\frac{1}{(754)^2\times 10^{-5}\times 0.5} = 10$ H ←

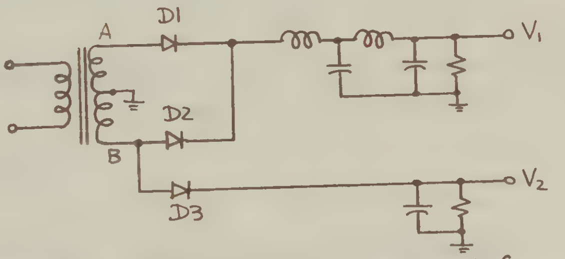
(20-24) Refer to Table 20-1. $V_m = \sqrt{2}\times 300 = 424$ V

a1 : 270, a2 : 424, a3 : 270, a4 : 424 V ←

b1 : 270, b2 : 382, b3 : 270, b4 : 270 V ←

c1 : const., c2 : Incr., c3 : Decr., c4 : From 0 current to the critical value of current r increases. Beyond this value r is constant.

d) $2V_m = 848$V for all cases.



(20-25) If the transformer, diode and coil resistancos are negligible, then

$V_{1dc} = \frac{2}{\pi}V_m = \frac{2\sqrt{2}}{\pi}(60) = 54$V ←

$V_{2dc} \approx V_m = \sqrt{2}\times 60 = 85$V ←

(20-26) Diodes D1 and D2 (next to the transformer) form a full-wave rectifier feeding the load connected at B with $V_{dc} = +\frac{2\sqrt{2}}{\pi}(40) = +36$V (neglecting any series resistances), under load. ←

All four diodes form a full-wave bridge feeding the load at A through the entire secondary winding ($2V_m$). Hence, the voltage at A is twice that found above i.e. +72V under load. ←

(20-27) The top two tubes constitute a full-wave rectifier feeding an L-section filter. Thus, the dc voltage at A under load is $+\frac{2\sqrt{2}\times 350}{\pi} = +316$V ←

The lower two tubes constitute a voltage doubler of the type analyzed in Prob. 20-8. Hence the voltage output at B is $-2V_m = -2\sqrt{2}\times 350 = -990$V provided that the load connected there is light. ←

(20-28) a) At A : at no load , $V_{dc} = -V_m = -30\sqrt{2} = -42.4$ V ←

At B : " " " , $V_{dc} = +V_m = +42.4$ V ←

b) For all diodes , PIV $= 2V_m = 84.8$ V ←

c) $V_{dc} = -\dfrac{2V_m}{\pi} = -\dfrac{84.8}{\pi} = -27$ V ←

d) Note at B we have a capacitor filter fed by a half-wave rectifier. Refer to Fig. 20-14 and Sec. 20-8. For the case of a HW rectifier, the non-conduction time is, $T_2 = \frac{T}{2} + \frac{T}{2}\frac{V_{dc}}{V_m}$. From Eq. (20-42a) , we have:

$V_r = \dfrac{I_{dc} T_2}{C} = \dfrac{I_{dc}}{C}\dfrac{T}{2}\left(1+\dfrac{V_{dc}}{V_m}\right)$ where $T = \frac{1}{f}$. Then, from Eq. (20-40) ,

$V_{dc} = V_m - \dfrac{V_r}{2} = V_m - \dfrac{I_{dc}}{4fC}\left(1+\dfrac{V_{dc}}{V_m}\right)$ or $V_{dc} = \dfrac{V_m - I_{dc}/4fC}{1 + I_{dc}/4fCV_m}$; $I_{dc} = 20$ mA

Thus, $I_{dc}/4fC = \dfrac{0.02\times 10^6}{4\times 60\times 8} = 10.4$; $\therefore V_{dc} = \dfrac{42.4 - 10.4}{1 + 10.4/42.4} = 25.7$V ←

(20-29) Refer to Fig. 20-20 . Since $R \gg r_o$ and $R \gg R_z$, then the circuit behaves approximately, as if $r_o = 0$ and $R \| R_z \approx R_z$. By Eq. (12-35), $R_o = \dfrac{R_z + h_{ie}}{1+h_{fe}}$

With $R \gg r_o$, $\Delta V_R \approx \dfrac{R_z}{R_z+R}\Delta V_i$. Since the circuit is an emitter follower, then, $\Delta V_o \approx \Delta V_R$ $\therefore$ $S_v = \dfrac{\Delta V_o}{\Delta V_i} \approx \dfrac{R_z}{R_z+R}$ . . . Eq. (20-69) .

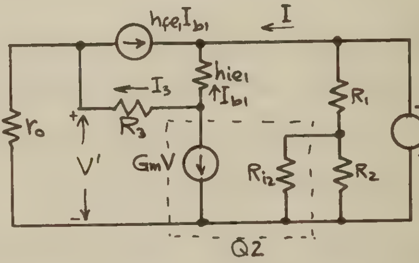
(20-30) From Fig. 20-22 , $i_{b2} = \dfrac{R_2}{R_1+R_2} v_o \dfrac{1}{R_1\|R_2 + R_i}$ where $R_i = h_{ie_2} + (1+h_{fe_2})R_z$

and $i_{c2} \approx h_{fe} i_{b2} = h_{fe_2}\dfrac{R_2}{R_1+R_2}\dfrac{v_o}{(R_1\|R_2)+h_{ie_2}+(1+h_{fe_2})R_z} \equiv G_m v_o$ . . . Eq. (20-73)

From Eq. (20-71) , $v_i = R_3 i_{c2} = R_3 h_{fe_2}\dfrac{R_2}{R_1+R_2}\dfrac{v_o}{(R_1\|R_2)+h_{ie_2}+(1+h_{fe_2})R_z}$

Hence, $S_v = \dfrac{v_o}{v_i} = \dfrac{R_1+R_2}{R_2}\dfrac{(R_1\|R_2)+h_{ie2}+(1+h_{fe2})R_z}{h_{fe2}R_3}$ . . . Eq. (20-74)

(20-31) Fron tho approximate small-signal equivalent circuit shown, we have:



$V = V' + I_3R_3 - I_{b1}h_{ie1}$ (1) ; $V' = r_o(I_3 - h_{fe1}I_{b1})$ (2)

$I_3 = -I_{b1} - G_mV$ (3) ; $I_{b1} = -\dfrac{I}{1+h_{fe1}}$ (4)

Substituting (4) into (3) : $I_3 = \dfrac{I}{1+h_{fe1}} - G_mV$ (5)

Subst. (5) & (4) into (2) : $V' = r_o\left(\dfrac{I}{1+h_{fe1}} - G_mV\right) + h_{fe1}r_o\dfrac{I}{1+h_{fe1}}$ (6)

Subst. (6) & (5) into (1) : $V = r_o\left(\dfrac{I}{1+h_{fe1}} - G_mV + h_{fe1}\dfrac{I}{1+h_{fe1}}\right) + R_3\left(\dfrac{I}{1+h_{fe1}} - G_mV\right) + \dfrac{I}{1+h_{fe1}}h_{ie1}$

or $V[1 + G_m(R_3+r_o)] = \dfrac{I}{1+h_{fe1}}[R_3 + h_{ie1} + (1+h_{fe1})r_o] = \dfrac{I}{1+h_{fe1}}(R_3+h_{ie1}) + Ir_o$

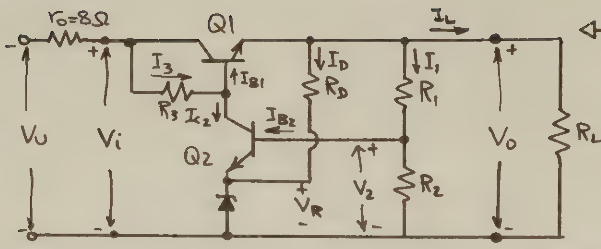
Let $R = \dfrac{V}{I} = \dfrac{r_o + \frac{R_3+h_{ie1}}{1+h_{fe1}}}{1 + G_m(R_3+r_o)}$. Hence, $R_o = R\|(R_1 + R_2\|R_{i2}) \approx R$ . . . Eq. (20-75)

(20-32) a) Let $I_{c2} \approx I_{E2} = 10$ mA and $I_D = 10$ mA so that $I_z = 20$ mA . Then, we have:

$R_D = \dfrac{V_o - V_R}{I_D} = \dfrac{12-6}{10\times 10^{-3}} = 600\,\Omega$; $I_{B2} = \dfrac{I_{c2}}{h_{FE2}} = \dfrac{10\times 10^{-3}}{200} = 50\,\mu$A . Let $I_{B2} \ll I_1 = 10$mA

Then, $V_2 = V_{BE2} + V_R = 0.6 + 6 = 6.6$V , and $R_1 = \dfrac{V_o - V_2}{I_1} = \dfrac{12-6.6}{10\times 10^{-3}} = 540\,\Omega$ ←

(20-32) (Cont'd) $R_2 \approx \frac{V_2}{I_1} = \frac{6.6V}{10\times10^{-3}} = 660\Omega$ ◁—



$I_{B1} \approx \frac{I_L + I_1 + I_D}{h_{FE1}} = \frac{220}{200} = 1.1\,mA$

$I_3 = I_{B1} + I_{c2} = 1.1 + 10 = 11.1\,mA$

$\therefore R_3 = \frac{V_i - (V_{BE1} + V_o)}{I_3} = \frac{30 - 12.6}{11.1\times10^{-3}}$

$= \frac{17.4}{11.1}K = 1.57\,K$ ◁—

b) From Eq. (20-74), $S_V = \frac{540+660}{660} \cdot \frac{(540 \| 660) + 1K + 201(10\Omega)}{200 \times 1.57K} = 1.82\,\frac{297+1000+2010}{314{,}000}$

or $S_V = \frac{1.82 \times 3307}{314{,}000} = 0.0192$ ◁—

c) R_o is given by Eq. (20-75) where $G_m = \frac{200\times660}{1200} \cdot \frac{1}{297+1000+2010} = 0.0333$

$\therefore R_o = \frac{8 + \frac{2{,}570}{101}}{1 + 0.0333\times(1578)} = \frac{8 + 25.5}{1 + 52.5} = 0.625\,\Omega$ ◁—

(20-33) a) Since $I_{B3} \approx \frac{I_{E1}}{h_{FE1} h_{FE3}}$, then $I_3 = I_{B3} + I_{C2}$ and $R_3 = \frac{V_i - (V_{BE1} + V_{BE3} + V_o)}{I_3}$

Thus, with the Darlington pair it is possible to use a larger value for R_3 which leads to a reduction in S_V (see Eq. 20-74)

b) From Eq. (20-75), $R_o \approx \frac{r_o + \frac{R_3 + h_{ie}'}{1 + h_{fe}'}}{1 + G_m(R_3 + r_o)}$. With the Darlington pair,

$1 + h_{fe}' \approx h_{fe1} h_{fe3}$ and $h_{ie}' = h_{ie1}(1 + h_{fe3}) \approx h_{ie1} h_{fe3}$

Hence, $R_o \approx \frac{r_o + \frac{R_3 + h_{fe3} h_{ie1}}{h_{fe1} h_{fe3}}}{1 + G_m(R_3 + r_o)}$ QED

(20-34) a) From Prob. 20-32, $R_1 = 540\Omega$, $R_2 = 660\Omega$, $I_{B1} = 1.1\,mA$. Then $I_{B3} \approx \frac{1.1}{200}\,mA$ or $I_{B3} \approx 5.5\,\mu A$; $I_3 \approx I_{c2} = 10\,mA$ and $R_3 = \frac{17.4V}{10mA} = 1.74K$

b) $S_V = \frac{1.82 \times 3307}{200 \times 1740} = 0.0173$

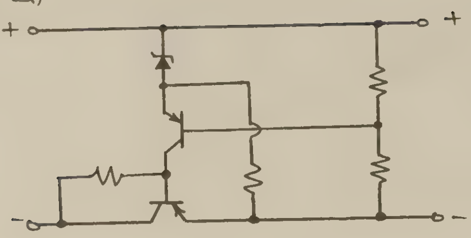
c) $R_o \approx \frac{8 + \frac{1740 + 200{,}000}{100 \times 200}}{1 + 0.0333(1748)} = \frac{18.0}{1 + 58.2} \approx 0.304\,\Omega$

(20-35) a) Instead of connecting R_3 to the collector of Q1, where the voltage varies with load, R_3 is connected to a voltage $(V_o + V_{Z2})$ which is essentially constant. R_4 serves to bias Z_2 appropriately. Thus, Q1 has a nearly constant bias supply and a minimum value for R_3 may not have to be calculated.

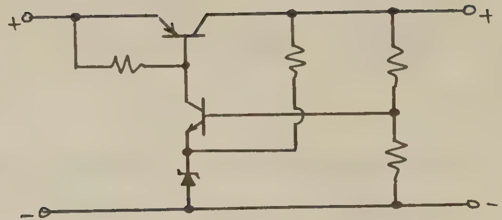
b) If $\Delta V_i >> \Delta V_o$, then the input variation is multiplied by $\frac{R_{Z2}}{R_{Z2} + R_4}$ before appearing at the input side of R_3.

Hence, $S_V = \frac{R_{Z2}}{R_{Z2} + R_4} \cdot \frac{R_1 + R_2}{R_2} \, \frac{(R_1 \| R_2) + h_{ie2} + (1 + h_{fe2}) R_{Z1}}{h_{fe2} R_3}$

(20-36) a)



b)

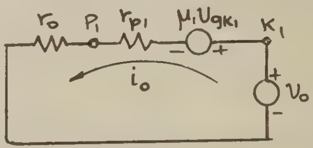


(20-37) a) One possibility is to flip the circuit of Prob. 20-36 a over, so that the + terminals become the ground.

b) If in the circuit of Prob. 20-36 a, we substitute the p-n-p transistors with n-p-n, reverse the Zener diode and the polarity of the applied voltage, we obtain a regulated power supply which uses n-p-n transistors and provides negative voltage.

c) Flip the circuit of Prob. 20-36 b over.

(20-38) From the equivalent circuit, we have:

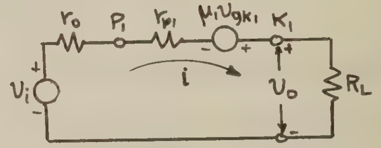


$i_o = \dfrac{v_o - \mu_1 v_{gk1}}{r_{p1} + r_o}$ where $v_{gk1} = -A_2 b v_o$

or $i_o = \dfrac{v_o(1+\mu_1 A_2 b)}{r_{p1}+r_o} \approx \dfrac{v_o\,\mu_1 A_2 b}{r_{p1}+r_o}$, if $\mu_1 A_2 b \gg 1$.

Using Eq. (20-78), we have: $R_o = \dfrac{v_o}{i_o} \approx \dfrac{r_{p1}+r_o}{\mu_1 A_2 b} = S_v(r_{p1}+r_o) \quad \ldots (20\text{-}79)$

(20-39) From Fig. 20-26, $v_{gk1} = -A_2 b v_o - v_o = -(1+A_2 b) v_o$ where A_2 is the magnitude of the gain of V2.



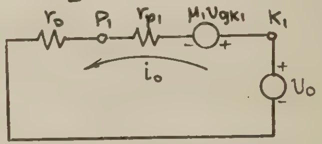
From the equivalent circuit shown, we have:

$i = \dfrac{v_i + \mu_1 v_{gk1} - v_o}{r_{p1}+r_o} = \dfrac{v_i - \mu_1 v_o(1+A_2 b) - v_o}{r_{p1}+r_o}$

where $v_o = i R_L = \dfrac{R_L}{r_{p1}+r_o}\left[v_i - v_o(1+\mu_1+\mu_1 A_2 b)\right]$

or $v_o\left[1 + \dfrac{R_L}{r_{p1}+r_o}(1+\mu_1+\mu_1 A_2 b)\right] = \dfrac{R_L}{r_{p1}+r_o} v_i \qquad \therefore S_v = \dfrac{v_o}{v_i} = 1\Big/\left(\dfrac{r_o+r_{p1}}{R_L}+1+\mu_1+\mu_1 A_2 b\right)$

(20-40) From the equivalent circuit shown, we have:



$v_o = \mu_1 v_{gk1} + i_o(r_{p1}+r_o)$. If R_3 is connected to the plate of V1 instead of the cathode, then when v_o is applied to the output, G_1 drops by $A_2 b v_o$. Because of i_o, the plate of V1 (or G_1) rises by $i_o r_o \dfrac{r_{p2}}{r_{p2}+R_3} = i_o r_o \gamma$ where $\gamma \equiv \dfrac{r_{p2}}{r_{p2}+R_3}$.

Thus, $v_{gk1} = i_o r_o \gamma - A_2 b v_o$

and $v_o = \mu_1(i_o r_o \gamma - A_2 b v_o) + i_o(r_{p1}+r_o)$ or $v_o(1+\mu_1 A_2 b) = [r_{p1} + r_o(1+\mu_1\gamma)]\, i_o$

Hence, $R_o = \dfrac{v_o}{i_o} = \dfrac{r_{p1}+r_o(1+\mu_1\gamma)}{1+\mu_1 A_2 b} \approx \dfrac{r_{p1}+r_o(1+\mu_1\gamma)}{\mu_1 A_2 b} = \dfrac{r_{p1}}{\mu_1 A_2 b} + r_o\dfrac{(1+\mu_1\gamma)}{(\mu_1 A_2 b)}$

Let $S_v \equiv \dfrac{1+\mu_1\gamma}{\mu_1 A_2 b}$. Since $\mu_1\gamma$ is usually small and $\mu_1 A_2 b$ is very large then, $S_v \approx 1/\mu_1 A_2 b$ and $R_o \approx (r_{p1}+r_o) S_v$. QED

— THE END —

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